The Optimal Schedule for the Opening of Buildings in an Office Complex

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The Problem

This model was formulated to analyze a problem presented by a land development firm planning to construct a group of office buildings. The developer has acquired approximately fifty two acres of land in a prime north Dallas location and is planning to construct seven office buildings over the next several years. The goal of this model is to determine the schedule of building openings which will maximize the firm's profit.

The Model

The model deals with two distinct types of office buildings, high rise and garden. The two types each have their own net incomes, demands for space, and construction costs. The model can, in fact, be considered as two independent models. This fact is very helpful and makes finding the optimal solution much easier, due to the size of the final formulated model.

In the formulation of the model, variables are separated by building number, with buildings one, two, and three being high rise, and four, five, six, and seven being garden type. Buildings one and three are essentialy the same,each having 350k sq. ft. of available space. Building two is slightly larger with 450k sq. ft. Buildings four and five are likewise essentially the same each having 60k sq. ft. Buildings six and seven are also

Definition of the Model

This model as formulated is a mixed integer linear programming model with 105 variables and 85 constraints. The objective function finds.the optimal solution using present value of the rent plus the present value of the sale value minus the present value of the construction cost for each of the seven buildings. The model is constrained by the demand for each type of office space, by the maximum capacity of each building, and by the obvious requirement that a building must be opened before it is rented out.

Definition of Variables

There are three sets of variables in this model, Z_{iy}, B_{iy}, and SALEi.

Z_{iy} integer variable either 0 or 1. A.1 indicates that building i is to be opened in year y. A 0 indicates no change in building's status.

B_{iy} continuous variable indicating the number of square feet to be rented in building i in year y

SALEi continuous variable indicating the sale value of building i in.year 7. It is found by multiplying the total number of sq. ft. rented out by the NOIi divided by the capitalization rate of 10%.

Other constraining variables:

CC_{iy} - construction cost for building i in year y.

$D1y$ - demand for high rise space in year y.

$D2y$ - demand for garden space in year y.

MCi - Maximum capacity of building i.

essentially the same with 75k sq. ft. each.

The demand for each type of office space is based on projections for Dallas as a whole, and this location in particular. The actual figures were provided by the firm involved.

It should be noted the firm involved is primarily involved in land development and is, therefore, not interested in owning the buildings for an extended period of time. They have specified that the buildings be sold by the model at the end of the time period, in this case seven years.

Assumptions

There are several basic assumptions incorporated into the formulation of this model.

The first assumption is that once a square foot of space is rented, it will remain rented for the remainder of the seven year time span. This is used so that income can be considered as a cash stream for n years, thus eliminating the need to keep track of rent for each year. This way it can be considered as a lump sum.

The second assumption was one stipulated by the firm involved; that is, there is to be no capital restriction placed on the opening of the buildings. They have enough capital to cover the construction of any or all of the buildings at any time.

Definition of Coefficients

The coefficient on the Zi_y variable is the preserit value of the construction cost for building i in year y, adjusted of inflation of 8% per year.

The coefficient on the Biy variable is the present value of the cash stream of rent for building i in year y, adjusted for inflation of 8% per year.

The coefficient on the SALEi variable is the present value of the sale value of building i in year seven.

The present value discount rate is 20% in all three cases.

The Formulated Model

The formulated model can be mathematically represented in the following way:

Maximize Profit=

$$\sum_{r1}^{7} \sum_{i1}^{7} (PVy*Bi_y* OIiy+PV7*SALEi-PV_y*Zi_y*CCi_y)$$

Subject To:

1) $Z1\ Bi_y \leq D1_y$ (High rise space rented less than or equal to demand)

$Bi_y \leq D2_y$ (Garden space rented less than or equal to demand)

For y from 1 to 7

2) $\Sigma\ (Zi_y*Di_y) \leq Bi_y$

For i from 1 to 7

$\sum^{7} (Ziy*Diy) \geq Bly$ (The building must be opened before space is rented out)

For i from 1 to 7

3) $\sum_{y=1}^{7} Bi_y=MC\ i$ (Total space leased equal to building capacity)

Vor i from 1 to 7

4) $\sum_{y=1}^{7} Ziy=1$ (The building is to be opened once and only once)

?nr i from 1 +n 7

The Optimal Solution

The formulated model was solved using the LINDO software package. LINDO was used because of its ability to optimize the model using 0,1 variables.

Due to the model's size and the increased complexity of solving an integer model, it was necessary to optimize the model in two parts. The was acomplished by setting specific Zi_y values for i=1,2,3 and allowing the software to optimize for buildings four, five, six, and seven. The optimal values for Ziy $i=4,5,6,7$ were then set, and the software was allowed to optimize for buildings one, two, and three. The optimal value yields a present value net profit of $37,118,515.

The optimal opening schedule is as follows:

	Building	Year Opened
High rise	1	5
	2	3
	3	1
	4	2
Garden	5	2
	6	3
	7	1

It can be seen that the buildings should be opened as soon as demand constraints allow. Further, with the given demand there is room for additional high rise space, and especially for garden space.

It should be noted that the model is solving for a schedule of building openings. Actual construction takes from twelve to eighteen months and must be started accordingly. The lag time between start and opening will have no effect on the outcome of the optimal solution due to the fact that all present values are related to the same year 0 starting point and the lag time "slide" will have the same effect on all variables.

This slide, while not effecting the actual optimal solution, may have some effect on the net profit, however, the effect should be relatively small.

In the course of finding an optimal value, it was discovered that the initial revenues collected from the garden offices were too low, making the garden buildings unprofitable. When the firm was notified, they instructed the rent to be raised slightly and the resulting optimal value incresed profit by over 20%.

Conclusion

The usefulness and importance of linear programming can be clearly seen in situations such as this. It can give a firm a much clearer picture of the interaction of all factors involved, and which factors are the critical ones. In this case, the rent on the garden offices proved too low, a fact which had not been seen before this model was formulated.

Further, a model of this nature provides a great deal of flexibility in exploring various changes to situation. Such factors as rent, demand, and changes in construction can be analyzed with relative ease. To make such changes and evaluate by hand would be extemely tedious, and often virtually impossible.

TABLE I

The optimal solution

Total net profit: $37,118,515

Space Rented in Year
(000 sq.ft.)

		1	2	3	4	5	6
High rise Building						293	57
				242	208		
	3	200	150				
	4		60				
Garden Buildings	5		50	10			
	6			75			
		75					

TABLE I I

Net profit per building

High rise buildings:

(000)	Building 1	2	3
Rent	$ 7,754.26	$17,550.16	$22,211.50
PV. sale	24,657.68	31,702.73	24,657.68
Con. cost	(23,870.00)	(37,884.00)	(36,400.00)
Net profit	8,541.85	11,368.89	10,469.18

Garden buildings:

(000)	Building 1	2	3	4
Rent	$2340.00	$2256.80	$2301.00	$3618.75
PV. sale	2950.68	2950.68	*3688.35*	*3688.35*
Con. cost	(3778.00)	(3778.00)	(4250.00)	(5250.00)
Net profit	1512.00	1429.48	*1739.35*	2057.10

Total net profit:

High rise :	$30,379.92
Garden :	6,738.61
Total :	37,116.53

Note: This figure is off in the last digit due to roundoff error.

TABLF III

Demand for Office Space

	Square . eet in Year (000)						
	1	2	3	4	5	6	7
High rise	200	220	242	266	293	322	354
Garden	100	110	121	133	146	161	177

Demand for each type is expected to increase at a rate of 10% per year for the seven years.

TABTIF IV

Sale Value of the Buildings

Present value factor: 0.335

Building		Sale Value(000)	PV Sale Value(000)
High rise	1	$73,605.00	$24,657.68
	2	94,635.00	31,702.73
	3	73,605.00	24,657.68
Garden	4	8,808.00	2,950.68
	5	8,808.00	2,950.68
	6	11,010.00	3,688.35
	7	11,010.00	3,688.35

TABLE V

Net Operating Income

High rise office space:

Inflation:	8% per year	Gross rent:	$18.00/sq. ft.
Present Value:	20% per year	Expenses:	4.75
		NOI:	13.25

Year	Infl. Factor	Adj. NOI	PV Factor	PV Adj. NOI	PV Cash Stream
1	1.000	$13.25	1.000	$13.25	$69.14
2	1.080	14.31	0.833	11.93	55.89
3	1.166	15.45	0.694	10.73	43.96
4	1.260	16.69	0.579	9.66	33.23
5	1.360	18.03	0.482	8.69	23.57
6	1.469	19.47	0.402	7.83	14.88
7	1.587	21.03	0.335	7.05	7.05

Garden office space:

Inflation :	8% per year	Gross rent:	$13.00/sq. ft.
Present Value:	20% per year	Expenses:	3.75
		NOI:	V.25

Year	Infl. Factor	Adj. NOI	PV Factor	PV Adj. NOI	PV Cash Stream
1	1.000	$ 9.25	1.000	$9.25	$48.25
2	1.080	9.99	0.833	8.32	39.00
3	1.166	10.79	0.694	7.49	30.68
4	1.260	11.65	0.579	6.75	23.19
5	1.360	12.58	0.482,	6.o6	16.44
6	1.469	13.59	0.402	5.46	10.38
7	1.587	14.68	0.335	4.92	4.92

Note: The inflation factor listed is accurate only to 3 decimal places. The actual adj. NOI is calculated with the non-rounded inflation and PV factor

TABLE VI

<u>Construction Costs</u>

High rise office buildings:

Inflation: 8% per year
Present Value: 20% per year

Cost: $104.00/sq. ft.

Building	sq. ft. (000)	cost(000)
1,3	350	$36,400
2	450	$46,800

Year	Infl. Factor	Adj. Cost (000) Building 1,3	Adj. Cost (000) Building 2	PV Factor	Adj. Cost (000) Building: 1,3	Adj. Cost (000) Building: 2
1	1.000	$36,400	$46,800	1.000	36,400	$46,800
2	1.080	39,312	50,544	0.833	32,747	42,103
3	1.166	42,457	54,588	0.694	29,465	37,884
4	1.260	45,854	58,955	0.579	26,549	34,135
5	1.360	49,522	63,671	0.482	23,870	30,689
6	1.469	53,484	68,765	0.402	21,501	27,644
7	1.587	57,762	74,266	0.335	19,350	24,879

Garden office buildings:

Inflation: 8% per year
Present Value: 20% per year

Cost: $70.00/sq. ft.

Building	sq. ft. (000)	Cost(000)
4,5	60	$4200
6,7	75	$5250

Year	Infl. Factor	Adj. Cost(000) Building: 4,5	Adj. Cost(000) Building: 6,7	PV Factor	Adj. Cost(000) Building: 4,5	Adj. Cost(000) Building: 6,7
1	1.000	$ 4200	$ 5250	1.000	4200	$ 5250
2	1.080	4536	5670	0.833	3778	4723
3	1.166	4899	6124	0.694	3400	4250
4	1.260	5291	6613	0.579	3o63	3829
5	1.360	5714	7143	0.482	2754	3443
6	1.469	6171	7714	0.402	2481	3101
7	1.587	6665	8331	0.335	2233	2791

Note: The inflation factor listed is accurate only to 3 decimal places. The actual adj. cost is calculated with the non-rounded inflation and PV factor.