Optimizing vs. Heuristic Methods for Vehicle Routing and Scheduling Problems

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OPTIMIZATION VS. HEURISTIC

METHODS FOR VEHICLE

ROUTING AND SCHEDULING PROBLEMS

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Distribution costs add about $400 billion each year to the cost of purchased goods in the U.S. alone. The costs associated with operating vehicles and crews for delivery purposes form an important component of total distribution costs. Therefore, even small percentage savings in these expenses could result in substantial total savings over a number of years. And in an increasingly technological society which must take advantage of economies of scale, the relative importance of transportation will continue to grow, and thus routing and scheduling applications will increase in importance.

"Routing and Scheduling" problems refer to the effective management of a fleet of vehicles and associated crews. Private firms that undertake the distribution of their goods to customer locations, and public transportation authorities responsible for providing transportation services to users both rely on routing and scheduling. "Routing problems" give solutions concerning the spatial configuration of vehicle movements, and these problems usually specify a sequence of locations that a vehicle must visit. "Scheduling problems" explicitly consider the times at which various locations are visited. However, in many instances the spatial and temporal characteristics interact and result in "combined routing and scheduling" problems.

An extensive list of applications of routing and scheduling models include mass transit scheduling of vehicles and crews, pick-up and delivery distribution systems, design of dial-a-ride systems, and school-bus routing and scheduling. The area of routing and scheduling has recently been the focus of intensive research activity. Major advances in this area have been made along both theoretical and applied dimensions.

CLASSIFICATION OF ROUTING AND SCHEDULING PROBLEMS

Basically, the output of all routing and scheduling problems is the same. For each vehicle, a route and a schedule is produced. In general, the route specifies the order that each location will be visited and the schedule designates the times at which the activities at these locations are to be performed.

However, routing and scheduling problems may be classified into one of three major groups: routing, scheduling, or combined
routing and scheduling. Combined routing and scheduling problems may be further subdivided according to a more detailed classification system. Each particular problem is distinguished by characteristics and assumptions that pertain to the given situation. The characteristics, restrictions, and assumptions associated with each problem result in different categories of problems requiring different modeling assumptions. For example, figure 1 (1, p. 73) reveals various combinations of options for eleven characteristics of routing and scheduling problems. Each combination of these various options would result in a problem with unique characteristics.

COMPLEXITY OF ROUTING AND SCHEDULING PROBLEMS

In the formulation and solution of Routing and Scheduling problems, it is important to consider the computational burden associated with various solution techniques. The growth in computation time clearly increases as a function of problem size. In a realistic environment where typical routing and scheduling problems are extremely large, the computation time is prohibitive. Therefore, the applicability of some solution techniques is extremely limited in real-world application.

Since the computational burden in solving these problems grows almost exponentially with problem size, methods of approximation are often resorted to as an alternative to pursuing an optimal solution. The use of heuristics allows for a near-optimal solution. "A heuristic algorithm is a procedure that uses the problem structure in a mathematical (and usually intuitive) way to provide feasible or near-optimal solutions. A heuristic is considered effective if the solutions it provides are consistently close to the optimal solution." (1, p. 76) A powerful heuristic method is defined on the basis of the statistical distribution of answers produced over a range of problems.

EXAMPLES OF HEURISTICS METHODS

A variety of heuristic approaches have found wide use. Here we describe two routing problems and heuristic procedures that have been developed to aid in obtaining near optimal solutions for each type.
TRAVELING SALESMAN PROBLEM

The traveling salesman problem determines the minimal cost cycle that passes through each node in a network exactly once.

1. select any node as the central depot (denoted as 1).
2. compute savings $S_{ij} = C_{i1} + C_{j1} - C_{ij}$ for $i, j = 2, 3, \ldots, n$.
3. order the savings from largest to smallest.
4. begin at the top of the savings list and move downwards, forming larger subtours by linking appropriate nodes i and j. Repeat until a tour is formed.

VEHICLE ROUTING PROBLEM

The vehicle routing problem requires a set of delivery routes from a central depot to various demand centers, each having service requirements, in order to minimize the total distance covered by the entire fleet. The vehicles have specified capacities, and each starts and terminates at the central depot. Two heuristic solution strategies for vehicle routing problems are the "cluster first-route second" approach, and the "route first-cluster second" approach.

"Cluster First-Route Second" Approach:
groups demand nodes and arcs first and then designs economical routes over each cluster as a second step.

"Route First-Cluster Second" Approach:
works in reverse. A large, and usually infeasible route is constructed which includes all nodes and arcs. Then the large route is partitioned into a number of smaller, feasible routes.

OTHER HEURISTICS (1, p. 99)

MATHEMATICAL PROGRAMMING APPROACHES are based on mathematical programming formulations for the problem.

INTERACTIVE OPTIMIZATION is a general-purpose approach in which a high degree of human interaction is
incorporated into the problem solving process. The idea is that experienced decision-makers have the capability of setting the revising parameters and injecting subjective assessments based on knowledge and intuition into the optimization model.

THE AVAILABLE TECHNOLOGY TO SOLVE VEHICLE ROUTING AND SCHEDULING PROBLEMS IN COMMERCIAL APPLICATIONS

Until recently, only the larger companies have been able to utilize computer systems in the distribution area. These systems were very effective, but they were expensive to design and maintain. Most of these systems were individually designed and implemented for particular companies, and demanded expensive, ongoing, in-house support.

However, with advancing computer technology, smaller, more affordable computers have been designed to meet the needs of smaller companies. Off-the-shelf computer programs currently exist and are less expensive, simpler, and do not demand professional in-house support. These developments have made automated distribution systems feasible for companies of all sizes.

Various software packages designed for the PC help companies solve distribution problems. For example, ROUTEMASTER, offered by Applied Operations Research, Inc., lists for approximately $200. It optimizes the sequence of stops in a single truck route, minimizing time or mileage, and cost. However, ROUTEMASTER is not as intelligent as some other packages, and it ignores several important variables. A more complex package is TRUCKSTOPS offered by MicroAnalytics for a little over $900. Although TRUCKSTOPS is more expensive, it is also much more intelligent. This package designs a route for each truck and sequences the stops on that route to minimize the total route cost in terms of mileage, time, and overtime.

TRUCKS, offered by STSC, is perhaps one of the most complex and intelligent of the software packages available for solving routing and scheduling problems. This remarkable package is used by companies such as Frito-Lay, Safeway, and Martin-Brower, which services McDonald's. TRUCKS sells for approximately $175,000. The package is heuristics-based, but it takes into consideration almost any constraining condition imaginable. The system parameters that have a significant influence on the routing
operation of the package are listed and described below. (2, pp. 631-641) These parameters are divided into three groups according to the level of importance in the routing operation. Type A constraints are the most fundamental to model, Type B constraints are somewhat difficult to model and moderate in importance, and Type C constraints are the most luxurious and the most difficult to model. And some constraints range from Type A to Type C depending on the complexities involved.

**TYPE A CONSTRAINTS**

**MAXIMUM ROUTE ON-DUTY-TIME**

This constraint limits the number of hours a driver may be on duty. On-duty time is the sum of all load and unload times, (pre-trip and post-trip), all driver wait times, and total route drive time. In our model we limit each driving team's on-duty time to a 48-hour maximum.

**MAXIMUM ROUTE DISTANCE**

This constraint limits the total distance of any single route. In our model we put a different limit on each route. We limit route A to 500 miles, route B to 600 miles, and route C to 450 miles.

**SERVICE TIME AT EACH LOCATION**

This constraint limits the service time at each stop along a route, and is based on some base unload rate multiplied by the quantity unloaded. Our model assumes the base unload rate is six minutes/pallet, which converts to 0.10 hour/pallet. Multiply the unload rate by the quantity unloaded to obtain the service time at each location.

**PRE-TRIP TIME**

This time is actually considered part of a route. It is the time at the beginning of a route that accounts for administrative overhead, equipment inspections, etc. Our model assumes pre-trip time to be 30 minutes.

**POST-TRIP TIME**

This time is also considered part of a route. It is the time at the end of a route allocated for a driver to end a route. Our model assumes post-trip time is 15 minutes.

**LENGTH OF ROUTING CYCLE**

This constraint limits the length of time within which all vehicles used in the routing cycle must be dispatched and returned. Our model assumes the maximum length of the routing cycle to be five days.
MAXIMUM STOPS PER ROUTE
This constraint sets a maximum number of stops allowed for each route. The constraint is used to control the maximum size of any route generated. The domicile stop at each end of the route is included in the number of stops in a route. Our model sets a maximum of three stops per route.

MAXIMUM ORDERS PER ROUTE
This constraint sets a maximum number of orders allowed for a single route. Our model assumes that an order equals a stop. Therefore, there is a maximum of three stops per route also.

LOCATION WINDOWS
These constraints limit the times that a particular location is open to receive an order. This window applies to all orders picked up or delivered at a particular location. For example, location X is open to receive orders only from 6 a.m. to 8 a.m. every day.

ASSIGNED DOMICILE
A specific domicile (or central depot) can be assigned to certain distribution areas. This forces these distribution areas to be serviced by vehicles originating only from the designated domicile. Our model assumes that one domicile (Dallas) services six customers.

TYPE B CONSTRAINTS

CAPACITY VS. TRAILER TYPE
Our model assumes one unit of measure for the loads (pallets) as opposed to multiple units of measure (weight, volume, pallets). Our model assumes that we have multiple trailer types each with a different capacity. The capacity of vehicle A is 30 pallets, vehicle B is 40 pallets, and vehicle C is 25 pallets.

MAXIMUM CAPACITY PERCENTAGE or MAXIMUM TRAILER LOAD
This constraint limits the capacity on each vehicle during a route. The capacity of the vehicle type is considered when determining the total load on a trailer as stops are added to the route. We assume that all three vehicles are only loaded to 90 percent capacity. Therefore the maximum load on vehicle A is 27 pallets, on vehicle B it is 36 pallets, and on vehicle C it is 22 pallets.

MAXIMUM SHIFT TIME
This constraint limits the maximum number of on-duty hours
(including pre-trip time, post-trip time, load and unload times, and driver wait times) a driver may have before a lay-over is required. This constraint applies only to domicile locations and restricts shift times for routes run from that domicile.

MAXIMUM DRIVE TIME
This constraint limits the maximum number of driving hours only before a lay-over is required. This constraint also applies only to domicile locations and restricts drive times for routes run from that domicile.

MAXIMUM WAIT TIME AT A STOP
This constraint limits the amount of time a vehicle may wait at a location before the time windows are open. Wait time provides some flexibility, especially when time windows tend to be very tight. Our model assumes that maximum wait time at a stop must not exceed four hours.

MAXIMUM TOTAL WAIT TIME IN A ROUTE
This constraint limits the total sum of all wait times for all stops in a route.

MAXIMUM ROUTES AVAILABLE
This constraint limits the maximum number of routes that may be dispatched in a given time period.

ORDER WINDOWS
This constraint specifies the earliest and latest pick-up and delivery dates and times allowed for each order. Order windows are used to further restrict the handling of an order because they restrict the time a particular order is handled at a location.

TYPE C CONSTRAINTS

ORDER WANDER LIMIT
The order wander measures for each step the ratio between a particular route distance and a direct line distance from the route origin. A low ratio acts as a limit on the amount of "wandering" the Router can do to fit new routes into the schedule.

ROUTE WANDER LIMIT
The route wander measures for each stop the ratio between the total route distance developed so far and the minimum route distance to service the orders. The ratio helps keep a route from crossing over itself and increasing mileage.
MINIMUM DISTANCE FOR SMOTR/T unlawful ROUTES

This constraint limits the total route distance before a route will be considered no longer a "local" route, but a single-man-over-the-road (SMOTR) route or a team route.

SINGLE-TO-TEAM CUTOVER TIME

The cutover time constrains the maximum allowable length of a route run by a single driver, ignoring the length of a shift. Once a route is no longer considered "local" and has been assigned as a SMOTR, if the single-to-team cutover time limit is exceeded, the route is considered for assignment to a team.

SINGLE-TO-TEAM CUTOVER DISTANCE

If either the single-to-team cutover time limit or the single-to-team cutover distance limit is exceeded, the SMOTR will become a team.

ADDITIONAL CAPACITY PERCENTAGE

This parameter measures how much more capacity a trailer can have during a route if it is not already empty. It permits control over the amount of cargo shifting necessary during a route with mixed pick-up and deliveries.

MINIMUM REDISPATCH TIME

This constraint sets the time to be allowed unscheduled between successive dispatches of the same vehicle and driver. This constraint does not include the time at the domicile.

BASE COST MARGIN

The base cost for an order is calculated for each available domicile based on that domicile's stated equipment cost per mile and the distance necessary to service the order. Any domicile whose base cost falls outside this limit will not be considered as a domicile for this order. This margin allows the router to choose a suitable domicile to service a set of orders in a multiple domicile situation. This constraint would not apply to our model since we assume a single assigned domicile.

ROUTING CYCLE OVERLAP
Routing cycle overlap allows a route to be dispatched in one cycle and return in the next cycle as long as the total route length plus any redispatch time requirement does not exceed the length of one full cycle.

MINIMUM LAYOVER

This constraint sets a minimum on the amount of lay-over time of routes originating at the corresponding domicile location.
MAXIMUM LAYOVER
This constraint sets a maximum on the above.

TIME ZONES
This parameter takes into account the fact that routes may cross into different time zones. This constraint would not apply to our model since we are routing only through the Texas, Louisiana, Arkansas, Oklahoma areas.

PRODUCT CLASS VS. TRAILER TYPE
The product class of an order must be compatible with the classes of other orders on the same trailer. This constraint would not apply to our model since we assume only a single product class, OR1.

ORDER FREQUENCY AND MINIMUM SPACING
Order frequency is the number of times the order is to be serviced in a single routing cycle, and the spacing is the minimum time to be allotted between deliveries. This creates multiple duplicate orders of the same kind.

RANGES OF CONSTRAINT TYPES

Some of the Type A constraints may range to Type B and Type C constraints with added complexities.

LOCATION WINDOWS
A Type A constraint on location windows assumes that there is one window open to receive an order at the same time each day. However, a Type B constraint would occur with multiple windows open any day, and a Type C constraint would occur with multiple windows open only on specific days.

CAPACITY VS. TRAILER TYPE
Our constraint is a Type B. We assume one unit of measure (pallets, and multiple trailer types. A simpler Type A constraint would assume one unit of measure and one trailer type, while a more complex Type C constraint would assume multiple units of measure and multiple trailer types.

DOMICILE
Our model assumes the Type A constraint that we have a single domicile. However, a more complex Type B or C constraint could assume multiple domicile locations.

SERVICE TIME AT EACH LOCATION
Our model's Type B constraint assumes the service time is
determined by the set unload rate multiplied by the quantity unloaded. A simpler Type A constraint would assume a base unload time not related to the quantity unloaded. A more complex Type C constraint would assume various unload rates for various product classes at various delivery locations.

OBJECTIVE FUNCTION

Our model assumes a Type A objective function that we minimize cost (with cost in terms of miles). More complex objective functions would result if we minimized more than one variable type. For example, we could minimize cost or miles, hours, and number of vehicles required, all in the same objective function.

OUR MIXED INTEGER PROGRAMMING MODEL

Our simplified model of a routing and scheduling problem is based on a distribution network with one central depot which distributes goods to a large region. The input data is a set of orders from six customers during one business week. (See figure 2) The distribution center has three trucks available to make deliveries, each with a different capacity and route-mileage limit.

In the model, we formulated first what we believed to be the most fundamental constraints, such as the maximum distance each truck can travel, and "subtouring" constraints to insure that the trucks return home to the central depot. With that accomplished, we sought to add constraints which would be valuable in making our model simulate a real-world situation. Our choice was limited by the fact that we were programming in LINDO, which processes a maximum of 400 constraints. The most difficult but necessary constraint of those we chose was the arrival-time constraint, which forces delivery to occur within a time interval specified by the customer. This feature alone represents over 200 constraints in the mixed integer program; over half of the total number of constraints.

An example of a feature we did not include because of the complexity it would add to the model is a maximum-number-orders for trucks' routes. We assume in our model that one stop is equivalent to one order, but in a real-world situation, one stop may have multiple orders. Another example is having both location windows and delivery windows; our model is limited to one set of windows.
The variables, constants, and assumptions of the model are as follows:

**VARIABLES**

- $X_{ij}^v$: 0-1 integer variable representing an arc from node (customer) $i$ to node $j$ for truck $v$. For seven nodes (one depot and six customers) and three trucks, there are 126 $X_{ij}^v$'s.
- $A_j$: Arrival time at node $j$; $A_i$ represents arrival time at the previous node $i$.
- $W_j^v$: Wait time at node $j$ for truck $v$.
- $U_i^v, U_j^v$: Variables associated with nodes $i$ and $j$, employed in the subtouring-elimination constraints.

**CONSTRAINT PARAMETERS**

- $n = 7$: Number of nodes.
- $V = 3$: Number of vehicles (trucks) available.
- $C_v$: Capacity of truck $V$ in pallets.
  - $C_1 = 30$, $C_2 = 40$, $C_3 = 25$
- $P = 90\%$: Percent loading capacity of each truck, effectively making $C_1 = 27$, $C_2 = 36$, $C_3 = 22$.
- $q_i$: Demand (quantity ordered) at node $i$ in pallets.
  - $q_1 = 0$
  - $q_2 = 12$
  - $q_3 = 18$
  - $q_4 = 18$
- $d_{ij}$: Distance from node $i$ to node $j$ in miles.
- $D_v$: Maximum route distance for truck $v$.
  - $D_1 = 500$, $D_2 = 600$, $D_3 = 450$
- $T = 48$: Maximum route-time allowed in hours.
- $t_{pre} = 0.5$ (30 minutes): pre-trip time for each route.
\( t_{pos} = 0.25 \) (15 minutes): post-trip time for each route.

\( A_{j}^e = \) Earliest delivery date/hour for each node \( j \).

\( A_{j}^l = \) Latest delivery time for node \( j \).

**ASSUMPTIONS**

1) The product being delivered is homogeneous; it is the same for all customers.

2) The product is unloaded at a constant rate. We assume 10 pallets per hour.

3) The trucks' travelling speeds are the same, and assumed to be 50 mph.

4) All deliveries take place within a time-frame of five consecutive days (one business week).

5) Team drivers are used, so that many single-driver work-rules (i.e. 8-hour shift limit) can be disregarded.

**MATHEMATICAL FORMULATION**

The mathematical formulation of the mixed integer program involves ten basic constraints, which expand to 388 constraints when coded.

**OBJECTIVE FUNCTION:**

\[
\text{MINIMIZE} \quad \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{v=1}^{V} d_{ij} x_{ij}^v \\
\text{(minimize total distance travelled to make all deliveries)}
\]

Such That:

1) \( \sum_{i=1}^{n} \sum_{v=1}^{V} x_{ij}^v = 1 \) \( \quad (j = 2, \ldots, n) \)

\( \sum_{i=2}^{n} \sum_{v=1}^{V} x_{ij}^v = 1 \) \( \quad (i = 2, \ldots, n) \)

(each customer must be serviced by one truck)
(route continuity must be preserved; i.e. if a truck visits customer i, it must leave customer i)

3) \[ \sum_{i=1}^{n} q_i \left( \sum_{j=1}^{n} x_{ij}^v \right) \leq C_v \quad (v = 1, \ldots, V) \]

(the trucks' capacities cannot be exceeded for deliveries on an assigned route)

4) \[ \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij}^v \leq D_v \quad (v = 1, \ldots, V) \]

(maximum route distance for each truck cannot be exceeded)

5) \[
\sum_{i=1}^{n} 0.10 q_i \left( \sum_{j=1}^{n} x_{ij}^v \right) + \sum_{i=1}^{n} \sum_{j=1}^{n} 0.02 d_{ij} x_{ij}^v + \sum_{j=1}^{n} \omega_j^v \leq T - t_{PREF} - t_{POST} \quad (v = 1, \ldots, V)
\]

Assuming unload rate = 6 min/pallet, driving speed = 50 mph.
(total route-time must not exceed maximum T)

6) \[ A_j^e \leq A_j \leq A_j^g \quad (j = 1, \ldots, n) \]

(arrival time must fall within window specified by customer)

7) \[ \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij}^v - 1 \leq 3 \quad (v = 1, \ldots, V) \]

(maximum number of stops per route is 3)

8) \[ A_j \geq (A_i + 0.10 q_i + 0.02 d_{ij} + \omega_j^v) - (1 - x_{ij}^v) T \]
\[ A_j \leq (A_i + 0.10 q_i + 0.02 d_{ij} + \omega_j^v) + (1 - x_{ij}^v) T \]

for all i, j, v
(arrival time formulation)
\[ \omega^v_j \leq 4 \quad (j = 2, \ldots, n; \quad v = 1, \ldots, V) \]

(maximum wait-time at a customer stop is 4 hours)

\[ u_i - u_j + n \chi^v_{ij} \leq n - 1 \]

(i = 2, \ldots, n; \quad j = 2, \ldots, n; \quad v = 1, \ldots, V)

(subtours are not allowed; truck must return to central depot for route completion)

PROBLEM SOLUTION---MIP

When we attempted to run our program on LINDO, we found that although we had stayed within the limits for the total number of constraints and variables, it could not solve the problem. The obstacle was our 126 integer variables. Although LINDO has a capacity of 599 variables, it cannot handle nearly as many integer variables.

Our alternative was to solve the problem on the MPSX (Mathematical Programming System) package on the IBM computer. LINDO has a feature to convert a mixed integer problem to the MPS input format required. After converting our problem, MPS was able to solve it successfully.

RESULT SUMMARY

The MIP delivered all six orders with three truck dispatches. The total distance travelled for all three routes was 1376 miles, the optimal integer solution. The first LP solved gave an objective function value of 942 miles.

The total run time for 5706 iterations was 1.37 minutes. (For examples of iterations performed by MPSX and a summary of program results, see figures 3, 4, and 5.)
PROBLEM SITUATION—"TRUCKS"

We simulated our problem on the TRUCKS software package as closely as possible. We included the same constraints in its system parameters, and "turned off" all other parameters built into the TRUCKS database that would have given our MIP an unfair advantage.

We were not, however, able to simulate our model as accurately as we wanted to. TRUCKS' heuristics have assumed two objectives that we did not want: to place a priority on using the largest-capacity trucks, and to minimize the number of different trucks used.

RESULTS SUMMARY

TRUCKS delivered all six orders with five truck dispatches. The total distance travelled for all five routes was 1476 miles, 100 miles more than the MIP optimum. (See figures 6 and 7 for examples of TRUCKS screens detailing individual route information; see figure 8 for route results summary.)

DETERMINISTIC VS. HEURISTICS

A deterministic solution to a routing and scheduling problem using mixed integer programming gives an optimal solution, clearly defines the constraints, and leaves no room for approximation. However, real-world problems quickly become computationally burdensome. A realistic problem would model a network of hundreds of distribution locations, and would certainly necessitate the inclusion of more complex parameters and constraints than our model did. And the problem matrix increases exponentially with each added variable and constraint. As a result, computer time becomes infeasibly expensive. It is also difficult to simulate real-world situations using linear programming because the real world is not always linear. It becomes impractical to use such MIP solutions in large commercial applications because either a large computer system or expensive rented computer time would be required.

A heuristics-based solution, on the other hand, is much
better able to approximate real-world situations. A heuristics-based package such as TRUCKS can quickly solve extremely large problems. And more complex constraints that cannot be modeled in a linear program can be approximated using heuristic methods. However, a heuristics-based solution is not optimal, and it becomes difficult to assess how close to optimal the solution actually is. Since LP models for solving routing and scheduling problems are so limited in size, (the largest problem solved to optimality has considered 318 cities), heuristics-based results of extremely large problems cannot be compared to an optimal result. This presents problems in the marketability of software packages such as TRUCKS because prospective clients are usually interested in bottom-line dollar figures. Naturally, a client would want to know how statistically confident the heuristics-based results are before making such a large investment. Further limitations of heuristics are the assumptions that a heuristic model must make. Intuition is applied in heuristics to shape the mathematical model, and there is assuredly some built-in bias towards certain objectives. These assumptions and biases make the model less flexible for the user unless these assumptions are standard to the industry.

Weighing the pros and cons of both heuristic models and deterministic models, heuristic methods appear to be the only feasible line of attack. By sacrificing some effectiveness, running times for larger problems can be decreased. Frequently all that matters is that good answers are obtained in feasible running times.
Table I. Characteristics of routing and scheduling problems.

<table>
<thead>
<tr>
<th>CHARACTERISTICS</th>
<th>POSSIBLE OPTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Size of Available Fleet</td>
<td>one vehicle</td>
</tr>
<tr>
<td></td>
<td>multiple vehicles</td>
</tr>
<tr>
<td>2. Type of Available Fleet</td>
<td>homogeneous (only one vehicle type)</td>
</tr>
<tr>
<td></td>
<td>heterogeneous (multiple vehicle types)</td>
</tr>
<tr>
<td></td>
<td>special vehicle types (commandmented, etc.)</td>
</tr>
<tr>
<td>3. Housing of Vehicles</td>
<td>single depot (domicile)</td>
</tr>
<tr>
<td></td>
<td>multiple depots</td>
</tr>
<tr>
<td>4. Nature of Demands</td>
<td>deterministic (known) demands</td>
</tr>
<tr>
<td></td>
<td>stochastic demand requirements</td>
</tr>
<tr>
<td></td>
<td>partial satisfaction of demand allowed</td>
</tr>
<tr>
<td>5. Location of Demands</td>
<td>at nodes (not necessarily all)</td>
</tr>
<tr>
<td></td>
<td>on area (&quot;&quot;&quot;&quot;&quot;&quot;)</td>
</tr>
<tr>
<td></td>
<td>mixed</td>
</tr>
<tr>
<td>6. Underlying Network</td>
<td>undirected</td>
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<tr>
<td></td>
<td>directed</td>
</tr>
<tr>
<td></td>
<td>mixed</td>
</tr>
<tr>
<td></td>
<td>euclidean</td>
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<tr>
<td>7. Vehicle Capacity Restrictions</td>
<td>imposed (all the same)</td>
</tr>
<tr>
<td></td>
<td>imposed (different vehicle capacities)</td>
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<tr>
<td></td>
<td>not imposed (unlimited capacity)</td>
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<tr>
<td>8. Maximum Route Times</td>
<td>imposed (same for all routes)</td>
</tr>
<tr>
<td></td>
<td>imposed (different for different routes).</td>
</tr>
<tr>
<td></td>
<td>not imposed</td>
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<tr>
<td>9. Operations</td>
<td>pickups only</td>
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<tr>
<td></td>
<td>drop-offs (deliveries) only</td>
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<tr>
<td></td>
<td>mixed (pick ups and deliveries)</td>
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<tr>
<td></td>
<td>split deliveries (allowed or disallowed)</td>
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<tr>
<td>10. Costs</td>
<td>variable or routing costs</td>
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<tr>
<td></td>
<td>fixed operating or vehicle acquisition costs</td>
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<tr>
<td></td>
<td>common carrier costs (for unserviced demands)</td>
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<tr>
<td>11. Objectives</td>
<td>minimize total routing costs</td>
</tr>
<tr>
<td></td>
<td>minimize sum of fixed and variable costs</td>
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<tr>
<td></td>
<td>minimize number of vehicles required</td>
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<tr>
<td></td>
<td>maximize utility function based on service or convenience.</td>
</tr>
<tr>
<td></td>
<td>maximize utility function based on customer priorities</td>
</tr>
</tbody>
</table>
FIGURE 3
# MPSXI370 R1.6 MPSCL EXECUTION

## INTEGER NODES

<table>
<thead>
<tr>
<th>NODE</th>
<th>57</th>
<th>59</th>
<th>321</th>
<th>79</th>
</tr>
</thead>
<tbody>
<tr>
<td>FUNCTIONAL</td>
<td>1367.0000</td>
<td>1367.0000</td>
<td>1367.0000</td>
<td>1367.0000</td>
</tr>
<tr>
<td>ESTIMATION</td>
<td>INTEGER</td>
<td>INTEGER</td>
<td>INTEGER</td>
<td>INTEGER</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Node</th>
<th>INTEGER</th>
<th>INTEGER</th>
<th>INTEGER</th>
<th>INTEGER</th>
</tr>
</thead>
<tbody>
<tr>
<td>383 = X12A</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>385 = X12C</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>390 = X14A</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>392 = X15A</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>394 = X15C</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>395 = X16A</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>397 = X16C</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>415 = X27A</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>418 = X27C</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>420 = X31B</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>444 = X43B</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>455 = X51A</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>457 = X51C</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>467 = X55A</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>469 = X56C</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>475 = X61A</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>475 = X61C</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>491 = X71A</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>493 = X71C</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

## FIGURE 4
**PROBLEM STATISTICS**

<table>
<thead>
<tr>
<th>ROWS</th>
<th>382</th>
</tr>
</thead>
<tbody>
<tr>
<td>COLUMNS</td>
<td>162</td>
</tr>
<tr>
<td>VARIABLES</td>
<td>544</td>
</tr>
<tr>
<td>INTEGER VARIABLES</td>
<td>126</td>
</tr>
<tr>
<td>ELEMENTS</td>
<td>2506</td>
</tr>
<tr>
<td>DENSITY</td>
<td>1.20</td>
</tr>
</tbody>
</table>

**FUNCTIONAL ELEMENTS**

<table>
<thead>
<tr>
<th>FUNCTIONAL (MIN)</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**TIME**

<table>
<thead>
<tr>
<th>TIME SINCE MIXSTART</th>
<th>ITERATION NO. SINCE SETUP</th>
<th>NODE NO.</th>
<th>FUNCTIONAL VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONTINUOUS OPTIMUM</td>
<td>0</td>
<td>1</td>
<td>920.0000</td>
</tr>
<tr>
<td>FIRST INTEGER SOLUTION</td>
<td>0.17</td>
<td>710</td>
<td>1367.0000</td>
</tr>
<tr>
<td>OPTIMAL INTEGER SOLUTION</td>
<td>0.25</td>
<td>1084</td>
<td>1367.0000</td>
</tr>
<tr>
<td>OPTIMALITY PROVED</td>
<td>1.34</td>
<td>5706</td>
<td>378</td>
</tr>
<tr>
<td>TIME OF SEARCH</td>
<td>1.34</td>
<td>5706</td>
<td>378</td>
</tr>
</tbody>
</table>

NUMBER OF INTEGER VARIABLES NOT INTEGER AT CONTINUOUS OPTIMUM = 14

NUMBER OF INTEGER SOLUTIONS FOUND = 4

BRANCHES ABANDONED WHILE COMPUTING = 371

**FIGURE 5**
### Figure 6

<table>
<thead>
<tr>
<th>Route ID...</th>
<th>SAH-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type...</td>
<td>T (TEAM)</td>
</tr>
<tr>
<td>Total Dist.</td>
<td>416</td>
</tr>
<tr>
<td>Total Time.</td>
<td>9:41</td>
</tr>
<tr>
<td>Arrival Time</td>
<td>8:20</td>
</tr>
<tr>
<td>Stay Time.</td>
<td>1:21</td>
</tr>
<tr>
<td>Load/Unload</td>
<td></td>
</tr>
<tr>
<td>Driver 1...</td>
<td></td>
</tr>
<tr>
<td>Driver 2...</td>
<td></td>
</tr>
<tr>
<td>Driver 3...</td>
<td></td>
</tr>
<tr>
<td>Actor ID...</td>
<td><em>001</em></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TRAILER</th>
<th>BEG-OD-</th>
<th>END-OD-</th>
<th>AUX EQPT</th>
<th>BEG-</th>
<th>END-</th>
</tr>
</thead>
</table>

### Route 2.2

<table>
<thead>
<tr>
<th>Route ID...</th>
<th>SAH-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type...</td>
<td>T (TEAM)</td>
</tr>
<tr>
<td>Total Dist.</td>
<td>352</td>
</tr>
<tr>
<td>Total Time.</td>
<td>9:23</td>
</tr>
<tr>
<td>Arrival Time</td>
<td>7:02</td>
</tr>
<tr>
<td>Stay Time.</td>
<td>2:21</td>
</tr>
<tr>
<td>Load/Unload</td>
<td></td>
</tr>
<tr>
<td>Driver 1...</td>
<td></td>
</tr>
<tr>
<td>Driver 2...</td>
<td></td>
</tr>
<tr>
<td>Driver 3...</td>
<td></td>
</tr>
<tr>
<td>Actor ID...</td>
<td><em>001</em></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TRAILER</th>
<th>BEG-OD-</th>
<th>END-OD-</th>
<th>AUX EQPT</th>
<th>BEG-</th>
<th>END-</th>
</tr>
</thead>
</table>

F1=HELP 2=TOP 3=EXIT 6=PRINT 9=DELETE 10=BACKWARD 11=FORWARD 12=HOME

Row 23 Col 6 PROCEED HALF Duplex
<table>
<thead>
<tr>
<th>ACT</th>
<th>LOCATION</th>
<th>CT</th>
<th>ARRIVAL</th>
<th>DEPARTURE</th>
<th>DRIVE</th>
<th>LAYOV</th>
<th>WAIT</th>
<th>LD/UN</th>
<th>DIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>300-TX</td>
<td>01</td>
<td>05/09/85</td>
<td>09:59</td>
<td>05/09/85</td>
<td>10:29</td>
<td>3:31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>02</td>
<td>328-AR</td>
<td>02</td>
<td>05/09/85</td>
<td>14:00</td>
<td>05/09/85</td>
<td>15:36</td>
<td>3:31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>03</td>
<td>300-TX</td>
<td>03</td>
<td>05/09/85</td>
<td>19:07</td>
<td>05/09/85</td>
<td>19:22</td>
<td>3:31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**FIGURE 7**
## FIGURE 8

<table>
<thead>
<tr>
<th>Domicile</th>
<th>String</th>
<th>Route</th>
<th>Type</th>
<th>Dispatch</th>
<th>Return</th>
<th>Driv</th>
</tr>
</thead>
<tbody>
<tr>
<td>300-TX</td>
<td>001</td>
<td>SAH-4</td>
<td>T</td>
<td>05/06/85 13:32</td>
<td>05/06/85 17:25</td>
<td>1:5</td>
</tr>
<tr>
<td>300-TX</td>
<td>001</td>
<td>SAH-1</td>
<td>T</td>
<td>05/06/85 21:20</td>
<td>05/07/85 07:01</td>
<td>8:2</td>
</tr>
<tr>
<td>300-TX</td>
<td>001</td>
<td>SAH-3</td>
<td>T</td>
<td>05/08/85 07:54</td>
<td>05/08/85 19:08</td>
<td>7:3</td>
</tr>
<tr>
<td>300-TX</td>
<td>001</td>
<td>SAH-2</td>
<td>T</td>
<td>05/09/85 10:59</td>
<td>05/09/85 20:22</td>
<td>7:0</td>
</tr>
<tr>
<td>300-TX</td>
<td>002</td>
<td>SAH-5</td>
<td>T</td>
<td>05/08/85 06:09</td>
<td>05/08/85 13:24</td>
<td>4:4</td>
</tr>
</tbody>
</table>

REPORT NO: TK-501

DATE/TIME: 05/08/85 19:59

ROUTER LOG FOR ROUTE GROUP: SA

TEST COMPANY, INC.

TRACTOR USAGE SUMMARY

<table>
<thead>
<tr>
<th>Non-Sleeper Pool Used</th>
<th>Sleeper Pool Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domicile</td>
<td></td>
</tr>
<tr>
<td>pool</td>
<td>used</td>
</tr>
<tr>
<td>1A</td>
<td></td>
</tr>
<tr>
<td>row 2 col 7</td>
<td>proceed half duplex</td>
</tr>
</tbody>
</table>

ROUTER LOG A1 F 131

3 BLKS 85/05/08 LINE 60 OF 82

ORDERS INPUT: 6
ORDER FREQUENCIES INPUT: 6
ORDERS THAT FAILED TO BE ROUTED: 0
ORDER FREQUENCIES THAT FAILED: 0
LOCATION RECORDS IN THE DATABASE: 90
DOMICILE LOCATIONS: 1
LOCAL ROUTES GENERATED: 0
SMOTR ROUTES GENERATED: 0
TEAM ROUTES GENERATED: 5
TOTAL ROUTES GENERATED: 5

TOTAL DRIVE TIME: 29:34
TOTAL LAYOVER TIME: 
TOTAL WAIT TIME: :31
TOTAL LOAD/UNLOAD TIME: 11:21
TOTAL ROUTE TIME: 41:26
TOTAL ROUTE DISTANCE: 1476
BIBLIOGRAPHY


2. STSC user's guide for TRUCKS.


physical distr. costs account for about 16% of the sales value of an item

costs associated with operating vehicles and crews for delivery purposes are a major component of total distr. costs.

cost minimization is the primary objective of most E+R problems, but other objectives may be important, such as safety, service convenience, satisfaction of labor union agreements.

When such service criterion are met satisfactorily, the routes & schedules
Inputs into VRP problem:

- Locations of domiciles
- Delivery destinations (customers/nodes)
- Truck Fleet (Truck type/Capacity)
- Driving crew availability
- Customer demands (Orders)
- Driver work rules: Time/Mileage limits

Routing and Scheduling

Scheduling

Routing

T.S.P.: Vehicle Routing Problem

T.S.P.: single domicile, single vehicle w/ unlimited capacity

Reliance on heuristics; lack of exact algorithm

\[ p = 74 \]
OPTIMIZATION VS. HEURISTIC
METHODS FOR VEHICLE
ROUTING AND SCHEDULING PROBLEMS

Christi Hale
Sharon Hatcher

for
OREM 4390, SENIOR DESIGN
May, 1985
ACKNOWLEDGEMENTS

We wish to thank STSC, Inc., of Rockville, Maryland, for their generous support of this project. In particular, we thank Mr. Nanlal Singh for initiating this idea and his encouragement throughout.
DETERMINISTIC VS. HEURISTIC METHODS FOR SOLVING VEHICLE ROUTING AND SCHEDULING PROBLEMS

Distribution costs add about $400 billion each year to the cost of purchased goods in the U.S. alone. The costs associated with operating vehicles and crews for delivery purposes form an important component of total distribution costs. Therefore, even small percentage savings in these expenses could result in substantial total savings over a number of years. And in an increasingly technological society which must take advantage of economies of scale, the relative importance of transportation will continue to grow, and thus routing and scheduling applications will increase in importance.

"Routing and Scheduling" problems refer to the effective management of a fleet of vehicles and associated crews. Private firms that undertake the distribution of their goods to customer locations, and public transportation authorities responsible for providing transportation services to users both rely on routing and scheduling. "Routing problems" give solutions concerning the spatial configuration of vehicle movements, and these problems usually specify a sequence of locations that a vehicle must visit. "Scheduling problems" explicitly consider the times at which various locations are visited. However, in many instances the spatial and temporal characteristics interact and result in "combined routing and scheduling" problems.

An extensive list of applications of routing and scheduling models include mass transit scheduling of vehicles and crews, pick-up and delivery distribution systems, design of dial-a-ride systems, and school-bus routing and scheduling. The area of routing and scheduling has recently been the focus of intensive research activity. Major advances in this area have been made along both theoretical and applied dimensions.

CLASSIFICATION OF ROUTING AND SCHEDULING PROBLEMS

Basically, the output of all routing and scheduling problems is the same. For each vehicle, a route and a schedule is produced. In general, the route specifies the order that each location will be visited and the schedule designates the times at which the activities at these locations are to be performed.

However, routing and scheduling problems may be classified into one of three major groups: routing, scheduling, or combined
Combined routing and scheduling problems may be further subdivided according to a more detailed classification system. Each particular problem is distinguished by characteristics and assumptions that pertain to the given situation. The characteristics, restrictions, and assumptions associated with each problem result in different categories of problems requiring different modeling assumptions. For example, figure 1 (1, p. 73) reveals various combinations of options for eleven characteristics of routing and scheduling problems. Each combination of these various options would result in a problem with unique characteristics.

**COMPLEXITY OF ROUTING AND SCHEDULING PROBLEMS**

In the formulation and solution of Routing and Scheduling problems, it is important to consider the computational burden associated with various solution techniques. The growth in computation time clearly increases as a function of problem size. In a realistic environment where typical routing and scheduling problems are extremely large, the computation time is prohibitive. Therefore, the applicability of some solution techniques is extremely limited in real-world application.

Since the computational burden in solving these problems grows almost exponentially with problem size, methods of approximation are often resorted to as an alternative to pursuing an optimal solution. The use of heuristics allows for a near-optimal solution. "A heuristic algorithm is a procedure that uses the problem structure in a mathematical (and usually intuitive) way to provide feasible or near-optimal solutions. A heuristic is considered effective if the solutions it provides are consistently close to the optimal solution." (1, p. 76) A powerful heuristic method is defined on the basis of the statistical distribution of answers produced over a range of problems.

**EXAMPLES OF HEURISTICS METHODS**

A variety of heuristic approaches have found wide use. Here we describe two routing problems and heuristic procedures that have been developed to aid in obtaining near optimal solutions for each type.
TRAVELING SALESMAN PROBLEM

The traveling salesman problem determines the minimal cost cycle that passes through each node in a network exactly once.

(1) select any node as the central depot (denoted as 1).
(2) compute savings \( S_{ij} = C_i + C_j - C_k \) for \( i, j = 2, 3, \ldots, n \).
(3) order the savings from largest to smallest.
(4) begin at the top of the savings list and move downwards, forming larger subtours by linking appropriate nodes \( i \) and \( j \). Repeat until a tour is formed.

VEHICLE ROUTING PROBLEM

The vehicle routing problem requires a set of delivery routes from a central depot to various demand centers, each having service requirements, in order to minimize the total distance covered by the entire fleet. The vehicles have specified capacities, and each starts and terminates at the central depot. Two heuristic solution strategies for vehicle routing problems are the "cluster first-route second" approach, and the "route first-cluster second" approach.

"Cluster First-Route Second" Approach:
groups demand nodes and arcs first and then designs economical routes over each cluster as a second step.

"Route First-Cluster Second" Approach:
works in reverse. A large, and usually infeasible route is constructed which includes all nodes and arcs. Then the large route is partitioned into a number of smaller, feasible routes.

OTHER HEURISTICS (1, p. 99)

MATHEMATICAL PROGRAMMING APPROACHES are based on mathematical programming formulations for the problem.

INTERACTIVE OPTIMIZATION is a general-purpose approach in which a high degree of human interaction is
incorporated into the problem solving process. The idea is that experienced decision-makers have the capability of setting the revising parameters and injecting subjective assessments based on knowledge and intuition into the optimization model.

THE AVAILABLE TECHNOLOGY TO SOLVE VEHICLE ROUTING AND SCHEDULING PROBLEMS IN COMMERCIAL APPLICATIONS

Until recently, only the larger companies have been able to utilize computer systems in the distribution area. These systems were very effective, but they were expensive to design and maintain. Most of these systems were individually designed and implemented for particular companies, and demanded expensive, ongoing, in-house support.

However, with advancing computer technology, smaller, more affordable computers have been designed to meet the needs of smaller companies. Off-the-shelf computer programs currently exist and are less expensive, simpler, and do not demand professional in-house support. These developments have made automated distribution systems feasible for companies of all sizes.

Various software packages designed for the PC help companies solve distribution problems. For example, ROUTEMASTER, offered by Applied Operations Research, Inc., lists for approximately $200. It optimizes the sequence of stops in a single truck route, minimizing time or mileage, and cost. However, ROUTEMASTER is not as intelligent as some other packages, and it ignores several important variables. A more complex package is TRUCKSTOPS offered by MicroAnalytics for a little over $900. Although TRUCKSTOPS is more expensive, it is also much more intelligent. This package designs a route for each truck and sequences the stops on that route to minimize the total route cost in terms of mileage, time, and overtime.

TRUCKS, offered by STSC, is perhaps one of the most complex and intelligent of the software packages available for solving routing and scheduling problems. This remarkable package is used by companies such as Frito-Lay, Safeway, and Martin-Brower, which services McDonald's. TRUCKS sells for approximately $175,000. The package is heuristics-based, but it takes into consideration almost any constraining condition imaginable. The system parameters that have a significant influence on the routing
operation of the package are listed and described below. (2, pp. 631-641) These parameters are divided into three groups according to the level of importance in the routing operation. Type A constraints are the most fundamental to model, Type B constraints are somewhat difficult to model and moderate in importance, and Type C constraints are the most luxurious and the most difficult to model. And some constraints range from Type A to Type C depending on the complexities involved.

**TYPE A CONSTRAINTS**

**MAXIMUM ROUTE ON-DUTY-TIME**

This constraint limits the number of hours a driver may be on duty. On-duty time is the sum of all load and unload times, (pre-trip and post-trip), all driver wait times, and total route drive time. In our model we limit each driving team's on-duty time to a 48-hour maximum.

**MAXIMUM ROUTE DISTANCE**

This constraint limits the total distance of any single route. In our model we put a different limit on each route. We limit route A to 500 miles, route B to 600 miles, and route C to 450 miles.

**SERVICE TIME AT EACH LOCATION**

This constraint limits the service time at each stop along a route, and is based on some base unload rate multiplied by the quantity unloaded. Our model assumes the base unload rate is six minutes/pallet, which converts to 0.10 hour/pallet. Multiply the unload rate by the quantity unloaded to obtain the service time at each location.

**PRE-TRIP TIME**

This time is actually considered part of a route. It is the time at the beginning of a route that accounts for administrative overhead, equipment inspections, etc. Our model assumes pre-trip time to be 30 minutes.

**POST-TRIP TIME**

This time is also considered part of a route. It is the time at the end of a route allocated for a driver to end a route. Our model assumes post-trip time is 15 minutes.

**LENGTH OF ROUTING CYCLE**

This constraint limits the length of time within which all vehicles used in the routing cycle must be dispatched and returned. Our model assumes the maximum length of the routing cycle to be five days.
MAXIMUM STOPS PER ROUTE
This constraint sets a maximum number of stops allowed for each route. The constraint is used to control the maximum size of any route generated. The domicile stop at each end of the route is included in the number of stops in a route. Our model sets a maximum of three stops per route.

MAXIMUM ORDERS PER ROUTE
This constraint sets a maximum number of orders allowed for a single route. Our model assumes that an order equals a stop. Therefore, there is a maximum of three stops per route also.

LOCATION WINDOWS
These constraints limit the times that a particular location is open to receive an order. This window applies to all orders picked up or delivered at a particular location. For example, location X is open to receive orders only from 6 a.m. to 8 a.m. every day.

ASSIGNED DOMICILE
A specific domicile (or central depot) can be assigned to certain distribution areas. This forces these distribution areas to be serviced by vehicles originating only from the designated domicile. Our model assumes that one domicile (Dallas) services six customers.

TYPE B CONSTRAINTS

CAPACITY VS. TRAILER TYPE
Our model assumes one unit of measure for the loads (pallets) as opposed to multiple units of measure (weight, volume, pallets). Our model assumes that we have multiple trailer types each with a different capacity. The capacity of vehicle A is 30 pallets, vehicle B is 40 pallets, and vehicle C is 25 pallets.

MAXIMUM CAPACITY PERCENTAGE or MAXIMUM TRAILER LOAD
This constraint limits the capacity on each vehicle during a route. The capacity of the vehicle type is considered when determining the total load on a trailer as stops are added to the route. We assume that all three vehicles are only loaded to 90 percent capacity. Therefore the maximum load on vehicle A is 27 pallets, on vehicle B it is 36 pallets, and on vehicle C it is 22 pallets.

MAXIMUM SHIFT TIME
This constraint limits the maximum number of on-duty hours
(including pre-trip time, post-trip time, load and unload times, and driver wait times) a driver may have before a lay-over is required. This constraint applies only to domicile locations and restricts shift times for routes run from that domicile.

MAXIMUM DRIVE TIME
This constraint limits the maximum number of driving hours only before a lay-over is required. This constraint also applies only to domicile locations and restricts drive times for routes run from that domicile.

MAXIMUM WAIT TIME AT A STOP
This constraint limits the amount of time a vehicle may wait at a location before the time windows are open. Wait time provides some flexibility, especially when time windows tend to be very tight. Our model assumes that maximum wait time at a stop must not exceed four hours.

MAXIMUM TOTAL WAIT TIME IN A ROUTE
This constraint limits the total sum of all wait times for all stops in a route.

MAXIMUM ROUTES AVAILABLE
This constraint limits the maximum number of routes that may be dispatched in a given time period.

ORDER WINDOWS
This constraint specifies the earliest and latest pick-up and delivery dates and times allowed for each order. Order windows are used to further restrict the handling of an order because they restrict the time a particular order is handled at a location.

TYPE C CONSTRAINTS

ORDER WANDER LIMIT
The order wander measures for each step the ratio between a particular route distance and a direct line distance from the route origin. A low ratio acts as a limit on the amount of "wandering" the Router can do to fit new routes into the schedule.

ROUTE WANDER LIMIT
The route wander measures for each stop the ratio between the total route distance developed so far and the minimum route distance to service the orders. The ratio helps keep a route from crossing over itself and increasing mileage.
MINIMUM DISTANCE FOR SMOTR/TEAM ROUTES
This constraint limits the total route distance before a route will be considered no longer a "local" route, but a single-man-over-the-road (SMOTR) route or a team route.

SINGLE-TO-TEAM CUTOVER TIME
The cutover time constrains the maximum allowable length of a route run by a single driver, ignoring the length of a shift. Once a route is no longer considered "local" and has been assigned as a SMOTR, if the single-to-team cutover time limit is exceeded, the route is considered for assignment to a team.

SINGLE-TO-TEAM CUTOVER DISTANCE
If either the single-to-team cutover time limit or the single-to-team cutover distance limit is exceeded, the SMOTR will become a team.

ADDITIONAL CAPACITY PERCENTAGE
This parameter measures how much more capacity a trailer can have during a route if it is not already empty. It permits control over the amount of cargo shifting necessary during a route with mixed pick-up and deliveries.

MINIMUM REDISPATCH TIME
This constraint sets the time to be allowed unscheduled between successive dispatches of the same vehicle and driver. This constraint does not include the time at the domicile.

BASE COST MARGIN
The base cost for an order is calculated for each available domicile based on that domicile's stated equipment cost per mile and the distance necessary to service the order. Any domicile whose base cost falls outside this limit will not be considered as a domicile for this order. This margin allows the router to choose a suitable domicile to service a set of orders in a multiple domicile situation. This constraint would not apply to our model since we assume a single assigned domicile.

ROUTING CYCLE OVERLAP
Routing cycle overlap allows a route to be dispatched in one cycle and return in the next cycle as long as the total route length plus any redispatch time requirement does not exceed the length of one full cycle.

MINIMUM LAYOVER
This constraint sets a minimum on the amount of lay-over time of routes originating at the corresponding domicile location.
MAXIMUM LAYOVER
This constraint sets a maximum on the above.

TIME ZONES
This parameter takes into account the fact that routes may cross into different time zones. This constraint would not apply to our model since we are routing only through the Texas, Louisiana, Arkansas, Oklahoma areas.

PRODUCT CLASS VS. TRAILER TYPE
The product class of an order must be compatible with the classes of other orders on the same trailer. This constraint would not apply to our model since we assume only a single product class, OR1.

ORDER FREQUENCY AND MINIMUM SPACING
Order frequency is the number of times the order is to be serviced in a single routing cycle, and the spacing is the minimum time to be allotted between deliveries. This creates multiple duplicate orders of the same kind.

RANGES OF CONSTRAINT TYPES

Some of the Type A constraints may range to Type B and Type C constraints with added complexities.

LOCATION WINDOWS
A Type A constraint on location windows assumes that there is one window open to receive an order at the same time each day. However, a Type B constraint would occur with multiple windows open any day, and a Type C constraint would occur with multiple windows open only on specific days.

CAPACITY VS. TRAILER TYPE
Our constraint is a Type B. We assume one unit of measure (pallets, and multiple trailer types. A simpler Type A constraint would assume one unit of measure and one trailer type, while a more complex Type C constraint would assume multiple units of measure and multiple trailer types.

DOMICILE
Our model assumes the Type A constraint that we have a single domicile. However, a more complex Type B or C constraint could assume multiple domicile locations.

SERVICE TIME AT EACH LOCATION
Our model's Type B constraint assumes the service time is
determined by the set unload rate multiplied by the quantity unloaded. A simpler Type A constraint would assume a base unload time not related to the quantity unloaded. A more complex Type C constraint would assume various unload rates for various product classes at various delivery locations.

**OBJECTIVE FUNCTION**

Our model assumes a Type A objective function that we minimize cost (with cost in terms of miles). More complex objective functions would result if we minimized more than one variable type. For example, we could minimize cost or miles, hours, and number of vehicles required, all in the same objective function.

**OUR MIXED INTEGER PROGRAMMING MODEL**

Our simplified model of a routing and scheduling problem is based on a distribution network with one central depot which distributes goods to a large region. The input data is a set of orders from six customers during one business week. (See figure 2) The distribution center has three trucks available to make deliveries, each with a different capacity and route-mileage limit.

In the model, we formulated first what we believed to be the most fundamental constraints, such as the maximum distance each truck can travel, and "subtouring" constraints to insure that the trucks return home to the central depot. With that accomplished, we sought to add constraints which would be valuable in making our model simulate a real-world situation. Our choice was limited by the fact that we were programming in LINDO, which processes a maximum of 400 constraints. The most difficult but necessary constraint of those we chose was the arrival-time constraint, which forces delivery to occur within a time interval specified by the customer. This feature alone represents over 200 constraints in the mixed integer program; over half of the total number of constraints.

An example of a feature we did not include because of the complexity it would add to the model is a maximum-number-orders for trucks' routes. We assume in our model that one stop is equivalent to one order, but in a real-world situation, one stop may have multiple orders. Another example is having both location windows and delivery windows; our model is limited to one set of windows.
The variables, constants, and assumptions of the model are as follows:

**VARIABLES**

\( X_{ij}^v \) 0-1 integer variable representing an arc from node (customer) \( i \) to node \( j \) for truck \( v \). For seven nodes (one depot and six customers) and three trucks, there are 126 \( X_{ij}^v \)'s.

\( A_j \) Arrival time at node \( j \); \( A_i \) represents arrival time at the previous node \( i \).

\( W_j^v \) Wait time at node \( j \) for truck \( v \).

\( U_i, U_j \) Variables associated with nodes \( i \) and \( j \), employed in the subtour-elimination constraints.

**CONSTRAINT PARAMETERS**

\( n = 7 \) : Number of nodes.

\( V = 3 \) : Number of vehicles (trucks) available.

\( C_v = \) Capacity of truck \( V \) in pallets. \( C_1 = 30, \ C_2 = 40, \ C_3 = 25 \)

\( P = 90\% \) : Percent loading capacity of each truck, effectively making \( C_1 = 27, \ C_2 = 36, \ C_3 = 22 \).

\( q_i = \) Demand (quantity ordered) at node \( i \) in pallets. \( q_1 = 0 \), \( q_2 = 12 \), \( q_3 = 18 \), \( q_4 = 18 \), \( q_5 = 6 \), \( q_6 = 16 \), \( q_7 = 6 \)

\( d_{ij} \) Distance from node \( i \) to node \( j \) in miles.

\( D_v = \) Maximum route distance for truck \( v \). \( D_1 = 500, \ D_2 = 600, \ D_3 = 450 \)

\( T = 48 \) : Maximum route-time allowed in hours.

\( t_{pre} = 0.5 \) (30 minutes): pre-trip time for each route.
\[ t_{\text{post}} = 0.25 \text{ (15 minutes): post-trip time for each route.} \]
\[ A_j^e = \text{Earliest delivery date/hour for each node } j. \]
\[ A_j^l = \text{Latest delivery time for node } j. \]

**ASSUMPTIONS**

1) The product being delivered is homogeneous; it is the same for all customers.

2) The product is unloaded at a constant rate. We assume 10 pallets per hour.

3) The trucks' travelling speeds are the same, and assumed to be 50 mph.

4) All deliveries take place within a time-frame of five consecutive days (one business week).

5) Team drivers are used, so that many single-driver work-rules (i.e. 8-hour shift limit) can be disregarded.

**MATHEMATICAL FORMULATION**

The mathematical formulation of the mixed integer program involves ten basic constraints, which expand to 388 constraints when coded.

**OBJECTIVE FUNCTION:**

\[
\text{MINIMIZE} \quad \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{v=1}^{V} d_{ij} x_{ij}^v
\]

(minimize total distance travelled to make all deliveries)

Such That:

1) \[ \sum_{j=1}^{n} \sum_{i=1}^{n} x_{ij}^v = 1 \quad (j = 2, \ldots, n) \]
2) \[ \sum_{j=1}^{n} \sum_{i=1}^{n} x_{ij}^v = 1 \quad (i = 2, \ldots, n) \]

(each customer must be serviced by one truck)
2) \[ \sum_{i=1}^{n} x_{i\rho}^v - \sum_{j=1}^{n} x_{\rho j}^v = 0 \] 
\[ (v = 1, \ldots, \nu; \] 
\[ \rho = 1, \ldots, n) \]

(route continuity must be preserved; i.e. if a truck visits customer i, it must leave customer i)

3) \[ \sum_{i=1}^{n} q_i \left( \sum_{j=1}^{n} x_{ij}^v \right) \leq C_v \] 
\[ (v = 1, \ldots, \nu) \]

(the trucks' capacities cannot be exceeded for deliveries on an assigned route)

4) \[ \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij}^v \leq D_v \] 
\[ (v = 1, \ldots, \nu) \]

(maximum route distance for each truck cannot be exceeded)

5) \[ \sum_{i=1}^{n} 0.10 q_i \left( \sum_{j=1}^{n} x_{ij}^v \right) + \sum_{i=1}^{n} \sum_{j=1}^{n} 0.02 d_{ij} x_{ij}^v \]
\[ + \sum_{j=1}^{n} W_j^v \leq T - t_{PRE} - t_{POST} \] 
\[ (v = 1, \ldots, \nu) \]

Assuming unload rate = 6 min/pallet, driving speed = 50 mph.
(total route-time must not exceed maximum T)

6) \[ A_j^e \leq A_j \leq A_j^g \] 
\[ (j = 1, \ldots, n) \]

(arrival time must fall within window specified by customer)

7) \[ \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij}^v - 1 \leq 3 \] 
\[ (v = 1, \ldots, \nu) \]

(maximum number of stops per route is 3)

8) \[ A_j \geq \left( A_i + 0.10 q_i + 0.02 d_{ij} + W_j^v \right) - (1 - x_{ij}^v) T \]
\[ A_j \leq \left( A_i + 0.10 q_i + 0.02 d_{ij} + W_j^v \right) + (1 - x_{ij}^v) T \]

for all i, j, v
(arrival time formulation)
9) \[ \omega_j^v \leq 4 \quad (j = 2, \ldots, n \quad v = 1, \ldots, V) \]
(maximum wait-time at a customer stop is 4 hours)

10) \[ u_i - u_j + n x_{ij}^v \leq n-1 \]
\( (i = 2, \ldots, n; \quad j = 2, \ldots, n; \quad v = 1, \ldots, V) \)
(subtours are not allowed; truck must
truck must return to central depot for
route completion)

PROBLEM SOLUTION---MIP

When we attempted to run our program on LINDO, we found that
although we had stayed within the limits for the total number of
constraints and variables, it could not solve the problem. The
obstacle was our 126 integer variables. Although LINDO has a
capacity of 599 variables, it cannot handle nearly as many
integer variables.

Our alternative was to solve the problem on the MPSX
(Mathematical Programming System) package on the IBM computer.
LINDO has a feature to convert a mixed integer problem to the MPS
input format required. After converting our problem, MPS was
able to solve it successfully.

RESULT SUMMARY

The MIP delivered all six orders with three truck
dispatches. The total distance travelled for all three routes
was 1376 miles, the optimal integer solution. The first LP
solved gave an objective function value of 942 miles.
The total run time for 5706 iterations was 1.37 minutes.
(For examples of iterations performed by MPSX and a summary of
program results, see figures 3, 4, and 5.)
PROBLEM SITUATION—"TRUCKS"

We simulated our problem on the TRUCKS software package as closely as possible. We included the same constraints in its system parameters, and "turned off" all other parameters built into the TRUCKS database that would have given our MIP an unfair advantage.

We were not, however, able to simulate our model as accurately as we wanted to. TRUCKS' heuristics have assumed two objectives that we did not want: to place a priority on using the largest-capacity trucks, and to minimize the number of different trucks used.

RESULTS SUMMARY

TRUCKS delivered all six orders with five truck dispatches. The total distance travelled for all five routes was 1476 miles, 100 miles more than the MIP optimum. (See figures 6 and 7 for examples of TRUCKS screens detailing individual route information; see figure 8 for route results summary.)

DETERMINISTIC VS. HEURISTICS

A deterministic solution to a routing and scheduling problem using mixed integer programming gives an optimal solution, clearly defines the constraints, and leaves no room for approximation. However, real-world problems quickly become computationally burdensome. A realistic problem would model a network of hundreds of distribution locations, and would certainly necessitate the inclusion of more complex parameters and constraints than our model did. And the problem matrix increases exponentially with each added variable and constraint. As a result, computer time becomes infeasibly expensive. It is also difficult to simulate real-world situations using linear programming because the real world is not always linear. It becomes impractical to use such MIP solutions in large commercial applications because either a large computer system or expensive rented computer time would be required.

A heuristics-based solution, on the other hand, is much
better able to approximate real-world situations. A heuristics-based package such as TRUCKS can quickly solve extremely large problems. And more complex constraints that cannot be modeled in a linear program can be approximated using heuristic methods. However, a heuristics-based solution is not optimal, and it becomes difficult to assess how close to optimal the solution actually is. Since LP models for solving routing and scheduling problems are so limited in size, (the largest problem solved to optimality has considered 318 cities), heuristics-based results of extremely large problems cannot be compared to an optimal result. This presents problems in the marketability of software packages such as TRUCKS because prospective clients are usually interested in bottom-line dollar figures. Naturally, a client would want to know how statistically confident the heuristics-based results are before making such a large investment. Further limitations of heuristics are the assumptions that a heuristic model must make. Intuition is applied in heuristics to shape the mathematical model, and there is assuredly some built-in bias towards certain objectives. These assumptions and biases make the model less flexible for the user unless these assumptions are standard to the industry.

Weighing the pros and cons of both heuristic models and deterministic models, heuristic methods appear to be the only feasible line of attack. By sacrificing some effectiveness, running times for larger problems can be decreased. Frequently all that matters is that good answers are obtained in feasible running times.
Routing and scheduling of vehicles and crews

Table 1.1. Characteristics of routing and scheduling problems.

<table>
<thead>
<tr>
<th>CHARACTERISTICS</th>
<th>POSSIBLE OPTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Size of Available Fleet</td>
<td>one vehicle</td>
</tr>
<tr>
<td></td>
<td>multiple vehicles</td>
</tr>
<tr>
<td>2. Type of Available Fleet</td>
<td>homogeneous (only one vehicle type)</td>
</tr>
<tr>
<td></td>
<td>heterogeneous (multiple vehicle types)</td>
</tr>
<tr>
<td></td>
<td>special vehicle types (compartmentalized, etc.)</td>
</tr>
<tr>
<td>3. Housing of Vehicles</td>
<td>single depot (domicile)</td>
</tr>
<tr>
<td></td>
<td>multiple depots</td>
</tr>
<tr>
<td>4. Nature of Demands</td>
<td>deterministic (known) demands</td>
</tr>
<tr>
<td></td>
<td>stochastic demand requirements</td>
</tr>
<tr>
<td></td>
<td>partial satisfaction of demand allowed</td>
</tr>
<tr>
<td>5. Location of Demands</td>
<td>at nodes (not necessarily all)</td>
</tr>
<tr>
<td></td>
<td>on arcs (&quot;&quot;&quot;&quot;)</td>
</tr>
<tr>
<td></td>
<td>mixed</td>
</tr>
<tr>
<td>6. Underlying Network</td>
<td>undirected</td>
</tr>
<tr>
<td></td>
<td>directed</td>
</tr>
<tr>
<td></td>
<td>mixed</td>
</tr>
<tr>
<td></td>
<td>euclidean</td>
</tr>
<tr>
<td>7. Vehicle Capacity Restrictions</td>
<td>imposed (all the same)</td>
</tr>
<tr>
<td></td>
<td>imposed (different vehicle capacities)</td>
</tr>
<tr>
<td></td>
<td>not imposed (unlimited capacity)</td>
</tr>
<tr>
<td>8. Maximum Route Times</td>
<td>imposed (same for all routes)</td>
</tr>
<tr>
<td></td>
<td>imposed (different for different routes)</td>
</tr>
<tr>
<td></td>
<td>not imposed</td>
</tr>
<tr>
<td>9. Operations</td>
<td>pickups only</td>
</tr>
<tr>
<td></td>
<td>drop-offs (deliveries) only</td>
</tr>
<tr>
<td></td>
<td>mixed (pick ups and deliveries)</td>
</tr>
<tr>
<td></td>
<td>split deliveries (allowed or disallowed)</td>
</tr>
<tr>
<td>10. Costs</td>
<td>variable or routing costs</td>
</tr>
<tr>
<td></td>
<td>fixed operating or vehicle acquisition costs</td>
</tr>
<tr>
<td></td>
<td>common carrier costs (for unserviced demands)</td>
</tr>
<tr>
<td>11. Objectives</td>
<td>minimize total routing costs</td>
</tr>
<tr>
<td></td>
<td>minimize sum of fixed and variable costs</td>
</tr>
<tr>
<td></td>
<td>minimize number of vehicles required</td>
</tr>
<tr>
<td></td>
<td>maximize utility function based on service or convenience,</td>
</tr>
<tr>
<td></td>
<td>maximize utility function based on customer priorities</td>
</tr>
</tbody>
</table>

**FIGURE 1**
**FIGURE 3**

MPSX/370 R1.6  MPSCL EXECUTION

INVERT - TIME = 1.07 - ITERATION..4424

MIXFLOW - TIME = 1.07

*** ANY FURTHER SOLUTION CANNOT BE BETTER THAN 1130.00 - ESTIMATION OF THE BEST ONE IS 1364.74

<table>
<thead>
<tr>
<th>ITP</th>
<th>VECTOR</th>
<th>VECTOR REDUCED</th>
<th>NUMBER</th>
<th>FUNCTION</th>
<th>NUMBER</th>
<th>SUM</th>
<th>NONOPT</th>
<th>VALUE</th>
<th>INFEAS</th>
<th>INFEAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>4500</td>
<td>507</td>
<td>446</td>
<td>0</td>
<td>1260.7370</td>
<td>30</td>
<td>132.0832</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

INVERT - TIME = 1.09 - ITERATION..4510

MIXFLOW - TIME = 1.09

*** ANY FURTHER SOLUTION CANNOT BE BETTER THAN 1130.00 - ESTIMATION OF THE BEST ONE IS 1371.33

INVERT - TIME = 1.10 - ITERATION..4578

MIXFLOW - TIME = 1.11

INVERT - TIME = 1.11 - ITERATION..4606

MIXFLOW - TIME = 1.11

*** ANY FURTHER SOLUTION CANNOT BE BETTER THAN 1130.00 - ESTIMATION OF THE BEST ONE IS 1378.10

INVERT - TIME = 1.12 - ITERATION..4634

MIXFLOW - TIME = 1.12

*** ANY FURTHER SOLUTION CANNOT BE BETTER THAN 1130.00 - ESTIMATION OF THE BEST ONE IS 1389.61

INVERT - TIME = 1.12 - ITERATION..4652

MIXFLOW - TIME = 1.12

*** ANY FURTHER SOLUTION CANNOT BE BETTER THAN 1130.00 - ESTIMATION OF THE BEST ONE IS 1403.19

INVERT - TIME = 1.13 - ITERATION..4686

MIXFLOW - TIME = 1.13

- - - - - - INTEGER SOLUTION OBTAINED AT NODE 321 AND ITER 4703 - ITS FUNCTIONAL VALUE IS 1367.0000

XOOPRINT DEMAND SET
### MPSX/370 R1.6  
**MPSCL EXECUTION**

#### INTEGER NODES

<table>
<thead>
<tr>
<th>NODE</th>
<th>57</th>
<th>59</th>
<th>321</th>
<th>79</th>
</tr>
</thead>
<tbody>
<tr>
<td>FUNCTIONAL</td>
<td>1367.0000</td>
<td>1367.0000</td>
<td>1367.0000</td>
<td>1367.0000</td>
</tr>
<tr>
<td>ESTIMATION</td>
<td>INTEGER</td>
<td>INTEGER</td>
<td>INTEGER</td>
<td>INTEGER</td>
</tr>
</tbody>
</table>

| 383=X12A | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 385=X12C | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 390=X14B | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 392=X15A | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 394=X15C | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 397=X16A | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 397=X16C | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 415=X27A | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 418=X27C | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 420=X31B | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 444=X43B | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 455=X51A | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 457=X51C | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 467=X56A | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 469=X56C | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 473=X61A | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 475=X61C | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 485=X65A | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 487=X65C | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 491=X71A | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 493=X71C | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

---

**FIGURE 4**
### Problem Statistics

<table>
<thead>
<tr>
<th>Rows</th>
<th>Columns</th>
<th>Integer Variables</th>
<th>Elements</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>382</td>
<td>162</td>
<td>544</td>
<td>1,26</td>
<td>2506</td>
</tr>
</tbody>
</table>

### Computational Elements

<table>
<thead>
<tr>
<th>Functional (Min)</th>
<th>Restraints</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time Since Mixstart</th>
<th>Iteration No. Since Setup</th>
<th>Node No.</th>
<th>Functional Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous Optimum</td>
<td>0</td>
<td>1</td>
<td>920.0000</td>
</tr>
<tr>
<td>First Integer Solution</td>
<td>0.17</td>
<td>710</td>
<td>1367.0000</td>
</tr>
<tr>
<td>Optimal Integer Solution</td>
<td>0.25</td>
<td>1084</td>
<td>1367.0000</td>
</tr>
<tr>
<td>Optimality Proved</td>
<td>1.34</td>
<td>5706</td>
<td>378</td>
</tr>
<tr>
<td>Time of Search</td>
<td>1.34</td>
<td>5706</td>
<td>378</td>
</tr>
</tbody>
</table>

Number of Integer Variables Not Integer at Continuous Optimum: 14

Number of Integer Solutions Found: 4

Branches Abandoned While Computing: 371

**Figure 5**
FIGURE 6
<table>
<thead>
<tr>
<th>ACT</th>
<th>LOCATION</th>
<th>DATE</th>
<th>TIME</th>
<th>DATE</th>
<th>TIME</th>
<th>DRIVE</th>
<th>LAYOV</th>
<th>WAIT</th>
<th>LD/UN</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>300-TX</td>
<td>05/09/85</td>
<td>09:59</td>
<td>05/09/85</td>
<td>10:29</td>
<td>3:30</td>
<td>1:36</td>
<td>176</td>
<td></td>
</tr>
<tr>
<td>02</td>
<td>328-AR</td>
<td>05/09/85</td>
<td>14:00</td>
<td>05/09/85</td>
<td>15:36</td>
<td>3:31</td>
<td>1:36</td>
<td>176</td>
<td></td>
</tr>
<tr>
<td>03</td>
<td>300-TX</td>
<td>05/09/85</td>
<td>19:07</td>
<td>05/09/85</td>
<td>19:22</td>
<td>3:31</td>
<td>1:15</td>
<td>176</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ACT</th>
<th>LOCATION</th>
<th>DATE</th>
<th>TIME</th>
<th>DATE</th>
<th>TIME</th>
<th>DRIVE</th>
<th>LAYOV</th>
<th>WAIT</th>
<th>LD/UN</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>300-TX</td>
<td>05/08/85</td>
<td>06:54</td>
<td>05/08/85</td>
<td>07:24</td>
<td>1:48</td>
<td>1:48</td>
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<td>02</td>
<td>316-TX</td>
<td>05/08/85</td>
<td>09:12</td>
<td>05/08/85</td>
<td>11:00</td>
<td>1:59</td>
<td>0:31</td>
<td>99</td>
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<tr>
<td>03</td>
<td>324-LA</td>
<td>05/08/85</td>
<td>13:30</td>
<td>05/08/85</td>
<td>14:06</td>
<td>1:59</td>
<td>0:36</td>
<td>99</td>
<td></td>
</tr>
<tr>
<td>04</td>
<td>300-TX</td>
<td>05/08/85</td>
<td>17:53</td>
<td>05/08/85</td>
<td>18:08</td>
<td>3:47</td>
<td>0:15</td>
<td>189</td>
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</table>

**FIGURE 7**
**FIGURE 8**

<table>
<thead>
<tr>
<th>DOMICILE</th>
<th>STRING</th>
<th>ROUTE</th>
<th>TYPE</th>
<th>DISPATCH</th>
<th>RETURN</th>
<th>DRIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>300-TX</td>
<td>001</td>
<td>SAH-4</td>
<td>T</td>
<td>05/06/85 13:32</td>
<td>05/06/85 17:25</td>
<td>1:5</td>
</tr>
<tr>
<td>300-TX</td>
<td>001</td>
<td>SAH-1</td>
<td>T</td>
<td>05/06/85 21:20</td>
<td>05/07/85 07:01</td>
<td>8:2</td>
</tr>
<tr>
<td>300-TX</td>
<td>001</td>
<td>SAH-3</td>
<td>T</td>
<td>05/08/85 07:54</td>
<td>05/08/85 19:08</td>
<td>7:3</td>
</tr>
<tr>
<td>300-TX</td>
<td>001</td>
<td>SAH-2</td>
<td>T</td>
<td>05/09/85 10:59</td>
<td>05/09/85 20:22</td>
<td>7:0</td>
</tr>
<tr>
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<td>002</td>
<td>SAH-5</td>
<td>T</td>
<td>05/08/85 06:09</td>
<td>05/08/85 13:24</td>
<td>4:4</td>
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</table>

**ROUTE GROUP:** SA

**ORDER NO:** TK-501

**DATE/TIME:** 05/08/85 19:59

**ROUTER LOG FOR ROUTE GROUP: SA**

**TRACTOR USAGE SUMMARY**

<table>
<thead>
<tr>
<th>Non-Sleeper</th>
<th>Sleeper</th>
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</thead>
<tbody>
<tr>
<td>Domicile</td>
<td>Pool</td>
</tr>
<tr>
<td>Pool</td>
<td>Pool</td>
</tr>
</tbody>
</table>

**ROUTER LOG**

A1 F131 3 BLKS 85/05/08

LINES 60 OF 82

**BROWSE**

- Orders Input: 6
- Order Frequencies Input: 6
- Orders That Failed To Be Routed: 0
- Order Frequencies That Failed: 0
- Location Records in the Database: 90
- Domicile Locations: 1

- Local Routes Generated: 0
- Smotr Routes Generated: 0
- Team Routes Generated: 5
- Total Routes Generated: 5

- Total Drive Time: 29:34
- Total Layover Time: 
- Total Wait Time: 31
- Total Load/Unload Time: 11:21
- Total Route Time: 41:26
- Total Route Distance: 1476
BIBLIOGRAPHY


2. STSC user's guide for TRUCKS.
