Phasing of Income-Producing Real Estate

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Integer programming techniques were used to determine the order in which to build office buildings and when to put the space on the market for a seven-building, 90-acre, mixed-use real estate project in Texas. The output of the optimization provided development managers with the schedule for opening each building, the amount of space to be leased each year in each building, and the annual cash flows to the owner.

In large, multi-phase real estate developments, a major planning problem is deciding on the order in which to build buildings and when to put the space on the market. The problem is a common one in urban development where either a single developer or a planning authority, as in England, controls development for a number of buildings. Although integer programming applications to real estate are few in number [see Orne, Rao and Wallace 1975; Patterson and Huber 1974], a number of recent linear programming papers have addressed the problem of land allocation — how to subdivide a given parcel of land so as to maximize profits [Boaden 1977; Gau and Kohlhepp 1980; and Peiser 1982]. A key advantage of integer programming over linear programming for real estate applications is that decision units (for example, land and buildings) are not homogeneous and divisible. Also, large fixed costs are often associated with opening new subdivisions and buildings.

While the integer programming application described here is to office development, analogous applications can be drawn to other types of urban development such as residential subdivisions or

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Background

The property is a 90-acre mixed-use development in northwest Dallas metropolitan area called Texas Plaza. It is a joint venture of the Murchison family interests and MEPC American Properties, Incorporated, a subsidiary of the second largest property company in England.

The property is adjacent to Texas Stadium in Irving, Texas, which is home of the Dallas Cowboys, another company owned by the Murchison family. This site, which fronts on two freeways, was purchased over a 15-year period beginning at the time the stadium site was obtained. The project will ultimately include 4.5 million square feet of office space, a 400-room hotel, and 40,000 square feet of retail space.

The primary issue was phasing the construction of the office space which was to be divided among seven buildings, three high-rise and four low-rise buildings. The key decision to be made was the year in which each building should be opened for occupancy, and which market (luxury-office or garden-office) it should serve. If a building were opened prematurely or if too large a building were built, profits would suffer from inadequate demand to fill the building. If a building were opened too late, profits would be lost to other competing developments (unfilled demand in one period can not be accumulated into the next). We sought an optimal phasing sequence and schedule to maximize the present value of profits.

The Problem

Texas Plaza was in the early planning stages of development when the scheduling problem was first addressed. A land-use master plan had been agreed upon and initial costs had been projected based on engineering studies. We became involved with Texas Plaza when construction needed to be scheduled in order to project future financing requirements.

As with any large development, planning for Texas Plaza was an evolutionary process. Schedules were revised as master plans were changed and as market absorption rates and rental estimates were improved. At any given time, an optimal schedule would exist for which development profits were maximized based on the set of available information.

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Texas Plaza's particular problem was determining the time when construction should be *started* on each building rather than when each building should be opened (that is, construction completed). However, because construction duration is known, if one can find the opening-date (certificate of occupancy), the construction start-date can be found simply by subtracting the construction time from the opening date. Building data including square footages, rent, and operating expenses for the first year are shown in Table 1. Construction costs were estimated to be $104 per square foot for the high-rise buildings and $70 per square foot for the low-rise buildings based on current bids for comparable buildings in

Building Number	Building Type	Square Footage	Rent	($ per square foot) Operating Expense	Net Operating Income
1	High Rise	350,000	$18.00	$4.75	$13.25
2	High Rise	450,000	18.00	4.75	13.25
3	High Rise	350,000	18.00	4.75	13.25
4	Garden	60,000	13.00	3.75	9.25
5	Garden	60,000	13.00	3.75	9.25
6	Garden	75,000	13.00	3.75	9.25
7	Garden	75,000	13.00	3.75	9.25

Table 1. Summary of data on the seven office buildings planned by Texas Plaza.

Dallas.

Demand for high-rise luxury office space was considered to be independent of demand for low-rise garden office space. Furthermore, demand was assumed exogenous. This latter assumption may be somewhat of a simplification in that it ignores the possibility that "successful projects beget success." In other words, successful projects may be able to induce higher demand than would be forecast by market demand studies for the area.

Demand estimates were derived from market studies performed for the Texas Plaza area by a national market research firm (Table 2). Any demand not met in a given year was presumed lost to competing projects; it was not accumulated into succeeding years.

Year	Demand for High Rise Space	Demand for Garden Space
1	200,000	100,000
2	220,000	110,000
3	242,000	121,000
4	266,000	133,000
5	293,000	146,000
6	322,000	161,000
7	354,000	177,000

Table 2. Estimated demand for office space in square feet per year.

Other key assumptions included the following: Rents as well as construction costs were expected to rise at the rate of 8% per year. A seven-year horizon was assumed — long enough for the space in all seven buildings to be absorbed by the market. All buildings were assumed to be sold at the end of the seven-year period, at a price determined by capitalizing each building's net operating income (NOI) in Year 7 at 10 percent. This procedure for determining sale value is a common one in real estate. Since rents are increasing at a stipulated rate, the capitalized value takes inflation into account. The capitalization rate is determined by the market for similar buildings.

The model has 98 variables with 77 constraints. The objective function maximizes the aggregated net present values for each of the seven buildings over the seven-year holding period. The net present value of profit includes the annual net operating incomes plus sale values less initial construction costs for each building.

The model determines the year in which each building should be opened, the amount of space to be leased in each building each year, and the sale value of each building. The model is constrained by the demand for each type of office space each year, by the maximum capacity of each building, and by the necessary requirement that a building must be opened before it can be rented.

This formulation of the model excludes financial variables which are normally included in a real estate decision model concerning income property. In particular, buildings are usually financed by a combination of cash equity and long-term mortgages. Annual mortgage payments are made out of annual cash flows from operations, that is, net operating income (NOI).

It can be shown that the present model, which excludes mortgage payments, is identical to typical real estate financial models which include mortgage payments for the case where mortgage interest rates equal present value discount rates. Such cases are known as "leverage neutral cases" since economic returns are unaffected by the degree of leverage. While the assumption of leverage-neutrality is restrictive in that it removes the potential leverage benefits from owning real estate, it is necessary to make the problem tractable. With the omission of financial variables, the model represents a form of economic model in that it relies strictly on economic information such as revenues and expenses and disregards specific situations with respect to financing and taxation, which would be found in a financial model.

The Optimal Solution

The mixed integer program was solved using LINDO software [Schrage 1981]. Because of the assumption that demand for high-rise office space is independent of demand for garden space, the model decomposes into two separate problems, thus simplifying the solution. First, values were chosen arbitrarily for the years in which high-rise buildings 1, 2, and 3 were opened, and then optimal opening dates for the low-rise buildings were found. Next, the calculated values for opening the low-rise buildings were specified, and then the optimal opening dates for the high-rise buildings were found. The solution produced a net present value for profit of $37.1 million, excluding land cost which is treated as a "sunk cost" for purposes of the optimization.

Table 3 illustrates the solution for one of the high-rise buildings, Building 3. It is completed at the beginning of Year 5 and space is rented over a two-year period: 293,000 square feet in Year 5 and the remaining 57,000 square feet in Year 6. The initial building cost in current dollars (Year 0) is $36,400,000. Assuming an inflation rate of 8% per year, the building would cost $49,522,000 if completed at the end of Year 4. Similarly, net operating in-

	Present Value at 20%	Year 1	2	3	4	5	6	7
Building Cost	−$23,882				−$49,522			
Net Operating Income	6,458					5,282	6,814	7,359
Sale Value	20,538							73,591
Total	$ 3,114	0	0	0	−49,522	5,282	6,814	80,950

Table 3. The profit calculation for building 3 if built in Year 4 ($000). All figures are "end-of-period" payments. Eight percent annual inflation in building costs and net operating income is assumed.

come of $13.25 per square foot in Year 1 is inflated to $18.03 in Year 5, $19.47 in Year 6, and $21.03 in Year 7. The building's NOI in Year 7 is $7,359,100 which yields a sale value of $73,591,000 when capitalized at 10%. The net present value of Building 3, discounted at 20%, is $3,114,000 which represents its contribution to total profits.

One measure of the benefit provided by the solution is found by comparing it to a tentative scheduling plan that Texas Plaza was considering prior to this study (Table 4). Under that plan (Schedule I), the net present value was only $30.8 million.

		Year Opened	
Building Type	Building	Schedule I	Optimal Solution
High Rise	1	1	1
	2	2	3
	3	4	5
Low Rise	4	1	2
	5	2	2
	6	3	3
	7	4	1
Profit NPV ($ millions)		$30.8	$37.1

Table 4. Comparison of schedule I and optimization solution showing the year in which buildings were opened under each schedule and the profit expected with each schedule.

The solution alters both the order and timing of building development relative to Schedule I. Among the high-rise buildings, a slower construction schedule is recommended with opening dates for Buildings 2 and 3 moved back one year. Among the low-rise buildings, the optimal solution not only indicates that buildings should be opened over a three-year period instead of a four-year period, but also, a 75,000 square foot building (Building 6 or 7) should be built first instead of a 60,000 square foot building (Building 4).

In addition to the order and timing of development, the optimization indicates how much space will be rented in each building each year (Table 5). Demand is the overriding constraint. In most years, the space rented is equal to demand for that type of space in that year. Where space rented is less than total demand, it is the result of the "partial year problem" — a problem discussed below. Based on the given demand, there is room for both additional high-rise and garden office space. Available garden office space is in fact completed and occupied by Year 3.

		Incremental Space Rented in Year (000 sq. ft.)							Total
		1	2	3	4	5	6	7	Space
	1	200	150						350
High Rise	2			242	208				450
Building	3					293	57		350
	4		60						60
Garden	5		50	10					60
Buildings	6			75					75
	7	75							75

Table 5. How much space will be rented each year under the optimal solution.

Caveats

While the optimal scheduling offers vital information for large-scale development projects, several problems should be noted. First, because of the nature of integer programming, there may be surplus demand in a given year which remains unmet. It was noted earlier that demand is the principal constraint. Thus, the second building should be ready for occupancy precisely at the time that the first building is completely occupied so that no demand is lost to other projects. Just as integer programming removes the primary flaw of linear programming — that one cannot build "half a building," it presents a new one — that one cannot open a building for "half a period." A building is either open for the *entire* period or not open at all. For example, Garden Building 5 is opened concurrently with Garden

	Total	Year 1	Year 2	Year 3
Demand for High-Rise (sq. ft.)		200,000	220,000	242,000...
Monthly absorption (sq. ft.)		16,666	18,333	20,166
Supply				
Building 3 (sq. ft.)	350,000	200,000	150,000	
Months of absorption		12	8.18	
Surplus			3.82	
Building 2 (sq. ft.)	450,000		70,000	242,000...

Table 6. The solution to the "partial year problem." The opening for Building 2 should be moved forward to the eighth month of Year 2 from the beginning of Year 3 in order to satisfy the demand remaining in Year 2 after Building 3 is fully occupied.

Building 4, whereas High-Rise Building 2 is not opened until the year after High-Rise Building 3 is fully rented. Clearly, there is some break-even point for building occupancy in a single year that justifies building the next building concurrently. The break-even point is met in Building 5 which is opened in the same year as Building 4, but not in Building 2 which is delayed until Building 1 is fully leased. The problem of opening a building for part of a year may be circumvented by using shorter periods than one year. However, a switch to semi-annual periods, for example, will double the number of variables and constraints.

If demand is assumed to be uniform throughout the year, a simple way to get around the "partial year problem" is to note the percentage of total demand met during any year in which there is excess demand. The next building to be built should then be scheduled for completion at the time when the first building is occupied. This is illustrated for Buildings 3 and 2 in Table 6.

Similarly, when two or more buildings of the same type are scheduled for opening in the same year, any buildings which remain only partially occupied at year-end should be scheduled for opening sometime during the year rather than at the beginning of the year. For example, Building 5 should be opened at the beginning of March in Year 2 (50,000 sq. ft./60,000 sq. ft. × 12 mo. = 10 mo. of absorption) rather than in January. The necessary data to calculate partial-year openings can be obtained directly from the output of the optimization procedure (Table 5).

A second problem concerns the exclusion of land cost and other "sunk costs" from the objective function. Normally, such costs would have no effect on the optimal solution for profit maximization. However, land and other pre-construction expenses involve annual carrying costs which are excluded from the objective function. Since the inclusion of land carrying costs would require the addition of 49 more variables (one per building per year), the problem is simplified considerably by including these costs as part of the building costs. Therefore, land *interest* is implicit in the inflation rate applied to building costs, as is inflation in the building costs themselves. (The 8% inflation rate assumed in the problem may be somewhat understated, if one assumes that land carrying costs are in the range of 16-18% in 1982 and building cost inflation is 5-7%.)

While the problems of partial years and land carrying costs raise important issues, they do not render the results of the optimization any less meaningful. In particular, where buildings of different sizes are programmed to be built, the optimal solution provides critical information on the sequencing of the structures. This sequence will vary depending on demand and on the interaction between building costs, annual rents, inflation, sale values, and the discount rate.

Limitations

The mixed-integer programming used to schedule Texas Plaza can be used whenever several different buildings are planned for a single tract or for different tracts in close proximity to one another.

While the application to real estate development provides superior information on scheduling and profitability, certain limitations should be noted. Special financing arrangements and tenant arrangements frequently override purely economic considerations in determining when a building is started. For example, a termination clause in construction financing or release provisions in a land note may force a developer to begin construction before the optimal solution would indicate. Also, a developer may begin construction early for competitive reasons — to prevent a competitor's building from being built. A developer may prefer to begin one building in a particular location because its success will add value to an adjacent building. Because of these less-easily quantified factors, one must be circumspect in applying mixed-integer programming to real estate development scheduling.

> Any demand not met in a given year was presumed lost to competing projects.

> The model determines the year in which each building should be opened, the amount of space to be leased in each building each year, and the sale value of each building.

Conclusion

The cost of performing the optimization described here is small — approximately $6,000 to $8,000 for the initial set-up and $1,000 to $2,000 for subsequent model runs. Sensitivity tests on key variables such as demand, rents, building costs, and inflation rates can be performed for less than $100 per test.

In the case of Texas Plaza, the optimal solution produced an increase in the net present value of profits compared to the previously proposed schedule of approximately $6.3 million — an increase of 20%. The optimization model has served as one of the inputs to the ongoing planning process for Texas Plaza. Groundwork for the site is underway at this time, with the first building scheduled for completion in 1984. As one result of the study, Texas Plaza noted the magnitude of unsatisfied demand for garden office space after Year 3 and has recently revised the master plan to accommodate an eighth building.

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Robert W. Moss, general manager of Texas Plaza, confirms that: "The approach developed by Professor Peiser played a significant role in the programming and phasing of Texas Plaza; specifically, the study made apparent the need for an additional garden office building, which would have escaped our attention under the typical incremental planning process employed by a developer. Also, Professor Peiser's analytical framework enabled us to optimize scheduling, leasing pace, and annual cash flows in such a manner as to achieve the overall objective of maximizing the net present value of development projects over the development period."

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APPENDIX:

Variables are defined as follows:

Z_{iy} = Integer variable either 0 or 1. A 1 indicates that building i is opened in year y. A 0 indicates no change in the building's status.

B_{iy} = Continuous variable indicating the incremental number of square feet rented in building i in Year y. Once rented, the building, or portion thereof is expected to stay rented.

k = Capitalization rate used for determining property value in the year of sale. Sale Value = $B_{iy} \cdot NOI_7/k$.

NOI_{iy} = Net operating income per square foot in building i in Year y, where NOI = Gross Rent − Expenses. Expenses are treated as being totally variable, linearly related to square feet rented (B_{iy}). NOI is assumed to increase at the rate of inflation, θ: $NOI_{iy} = NOI_{i1} \cdot (1 + \theta)^{i-1}$.

$CONSTR_{iy}$ = Construction cost of building i in Year y. Like NOI_{iy}, construction costs are assumed to increase each year at the rate of inflation, θ: $CONSTR_{iy} = CONSTR_{i1} \cdot (1 + \theta)^{i-1}$.

r = Discount rate.

D_{1y} = Demand for high-rise space in Year y.

D_{2y} = Demand for low-rise space in Year y.

MC_i = Gross potential leasing area of building i.

$$\text{Maximize } \Pi = \sum_{y=1}^{n} \sum_{i=1}^{7} -(1+r)^{-y} Z_{iy} \text{CONST}_{iy} + (1+r)^{-y} B_{iy} \text{NOI}_{iy} + (1+r)^{-n} B_{iy} (\text{NOI}_7/k) \quad (1)$$

subject to 1) $\sum_{i=1}^{3} B_{iy} \leqslant D_{1y}$ (High rise space rented is less than or equal to demand.) (2)

$\sum_{i=4}^{7} B_{iy} \leqslant D_{2y}$ (Garden space rented is less than or equal to demand.) (3)

for $y = 1$ to 7.

2) $B_{iy} \leqslant \sum_{j=1}^{y} D_{1j} \cdot Z_{ij}$ for $i = 1$ to 3 (A building must be opened before space can be rented out.) (4)

$B_{iy} \leqslant \sum_{j=1}^{y} D_{2j} \cdot Z_{ij}$ for $i = 4$ to 7

for $y = 1$ to 7.

3) $\sum_{y=1}^{7} B_{iy} \leqslant MC_i$ (Total space rented in a building does not exceed building capacity.) (5)

for $i = 1$ to 7.

4) $\sum_{y=1}^{7} Z_{iy} = 1$ (A building is opened once and only once.) (6)

for $i = 1$ to 7.