

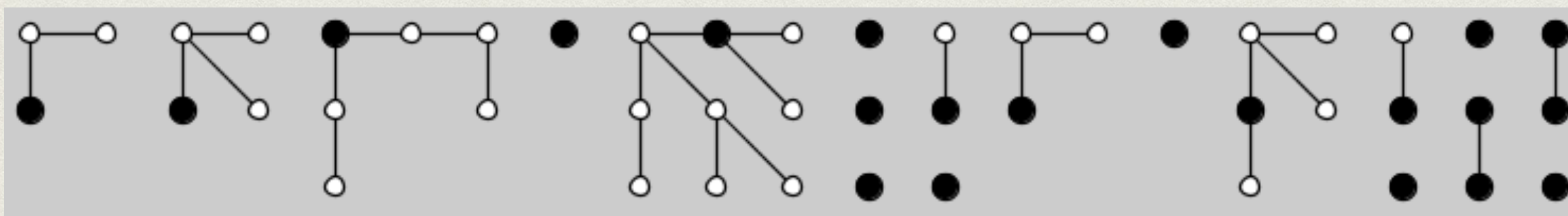
PRECISE AND CONCISE GRAPHICAL REPRESENTATION OF THE NATURAL NUMBERS

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A GRAPHIC IS
WORTH A
THOUSAND
DIGITS



NAMING NUMBERS

Cultural

五十

ごじゅう

오십

पचास

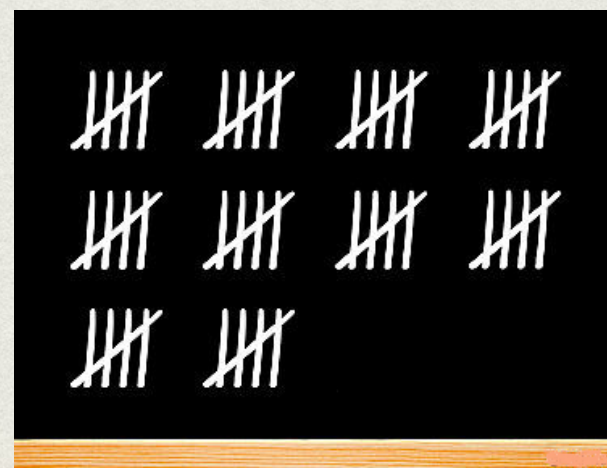
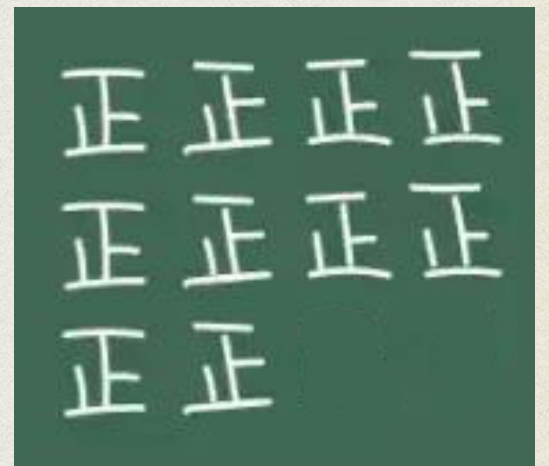
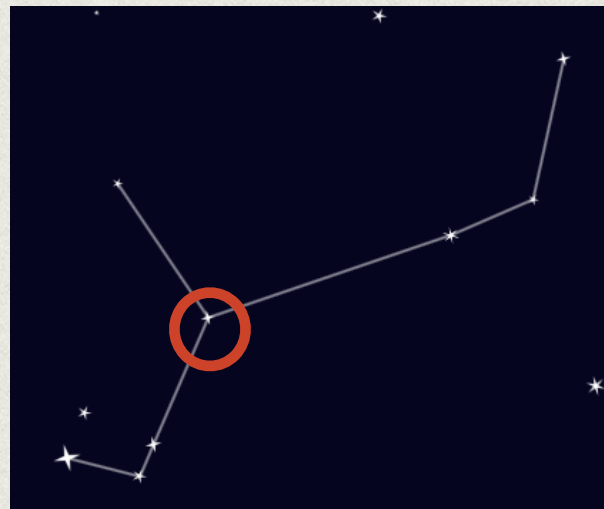
L

50

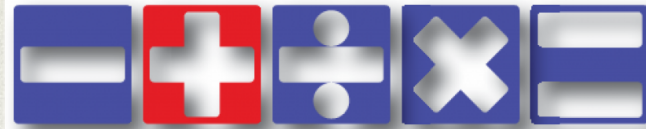
What's so special about "50"?

[It's a round number??]

Natural



五十
ごじゅう
오십
पचास
L



Why is divisible by 10 so special?

ARITH SYMPOSIUM

From 1st to 26th

NAMING NUMBERS

Cultural

四十九

よんじゅう

사십구

उनचास

XLIX

49

五十

ごじゅう

오십

पचास

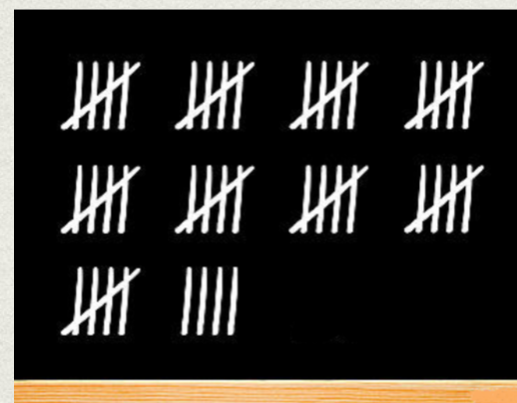
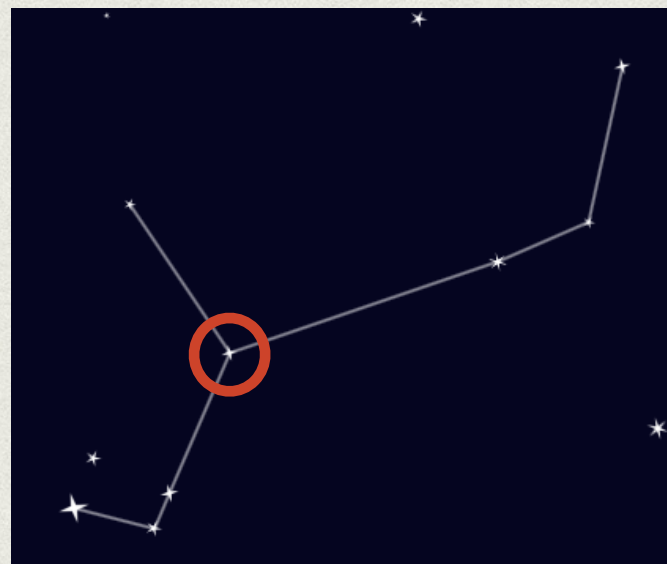
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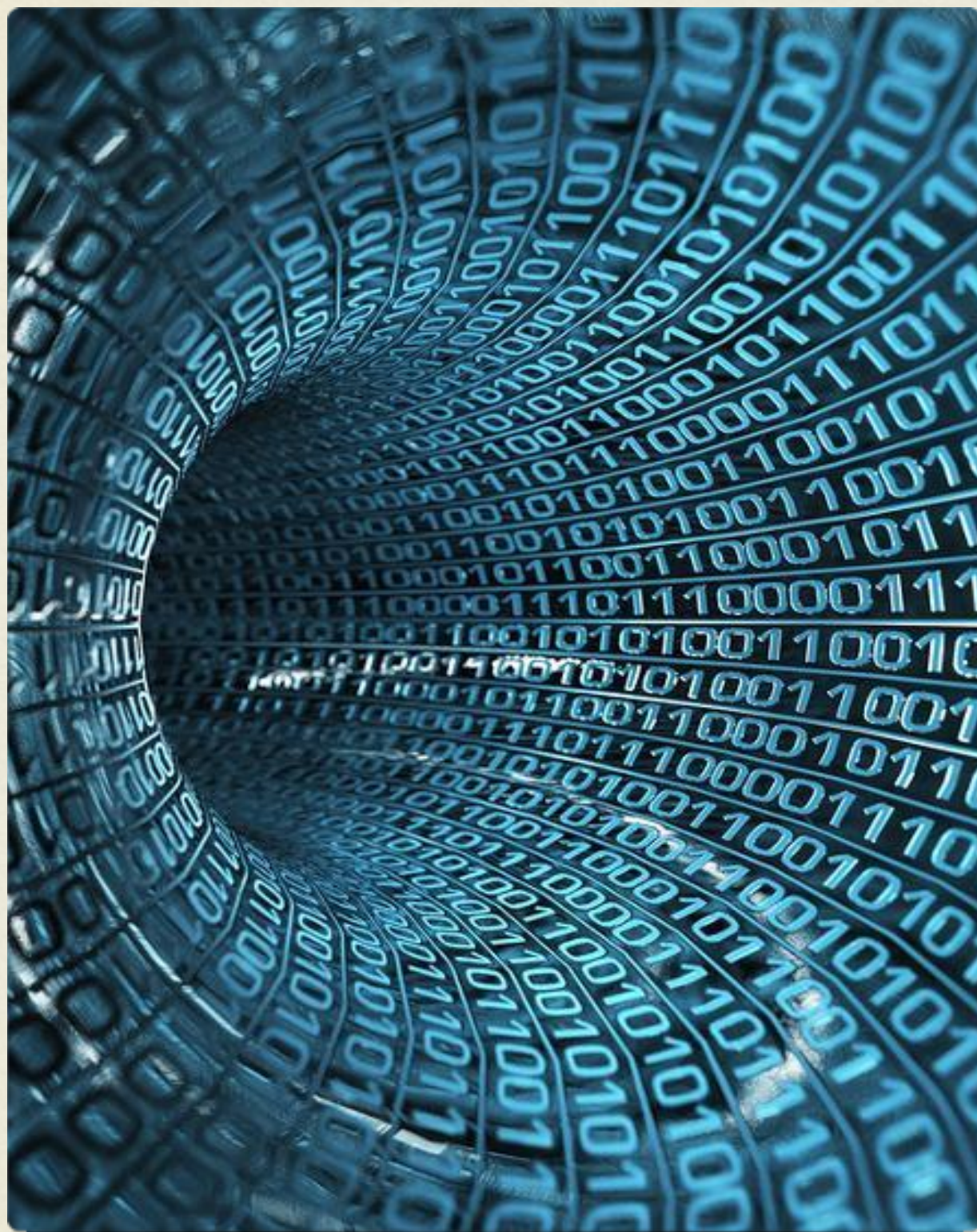
50

Step from 49 to 50

[Protocol or Obvious??]

Natural





Digit (bit) strings suggest??



See the relations?



Feel the music?

ROOTED TREES NATURAL NUMBERS

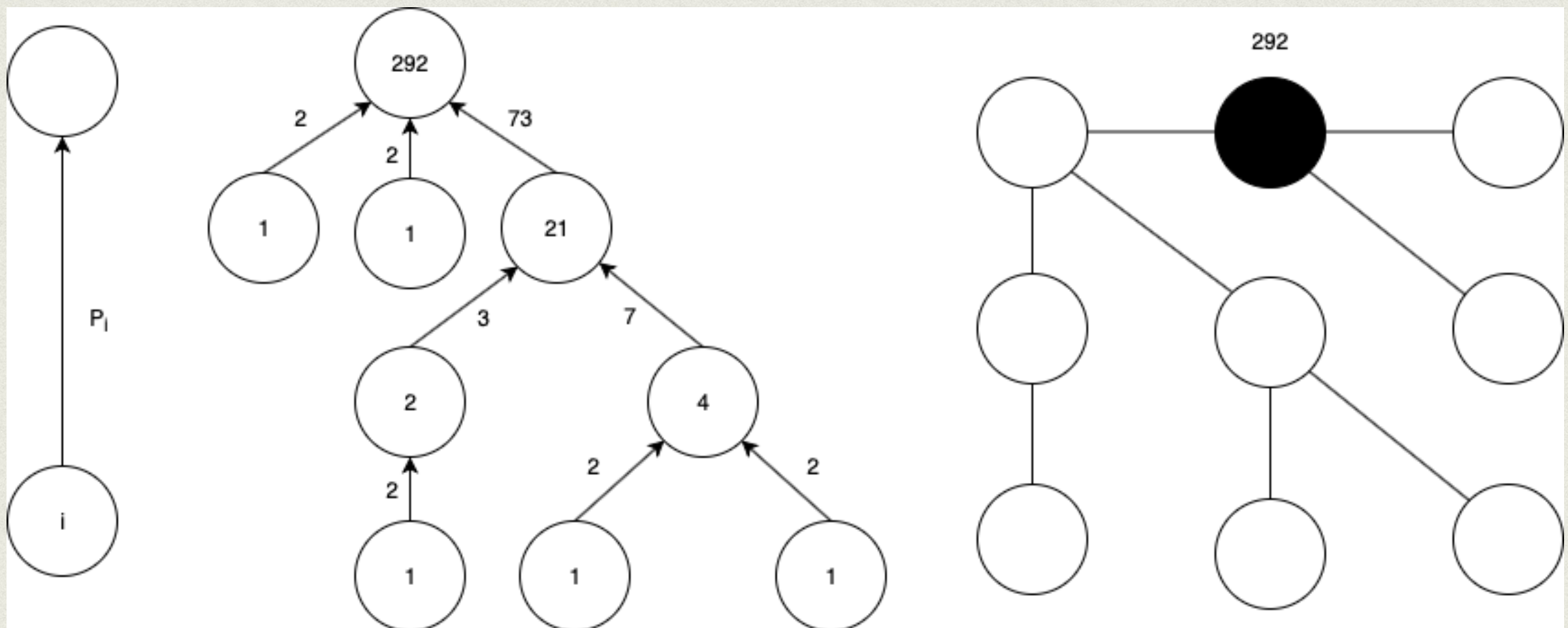
ONE *to* ONE!

Fundamentals of Arithmetic

- Theorem: Unique Prime Factorization
- Operation: Counting (i th prime p_i)
- Procedure: Recursion (finite stopping rule)

ONE-TO-ONE CORRESPONDENCE

A Natural Procedure Over Natural Numbers



ROOTED TREES NATURAL NUMBERS

ONE *to* ONE!

Fundamentals of Arithmetic

- Theorem: Unique Prime Factorization
- Operation: Counting (i th prime p_i)
- Procedure: Recursion (finite stopping rule)

Let's take a look...

C O N C I S E	Structural-e.g. Digital 7 (linear)	Number Fonts
	Artistic-e.g.Chinese, etc. (2D)	
P R E C I S E	Integer $\Leftrightarrow$ One Tree	
	Rational Fraction $\Leftrightarrow$ Two Trees	
	Continued Fraction $\Leftrightarrow$ Sequence of Trees	
	Reals by “Best Rational Approximation”	

STRUCTURAL FONTS

Decimal Digits vs. Rooted Trees



(a) digits in the Digital-7 font



(b) counts 1, 2, 3, ..., 9 in a square grid font



(c) compressed square grid font for selecting counts 9-14 and 17

FIRST 2_1 PARTIAL QUOTIENTS

Everyone looks at

3.7.15.1.292.1.1.2.1.3.1.14.2.1.1.2.2.2.2.1

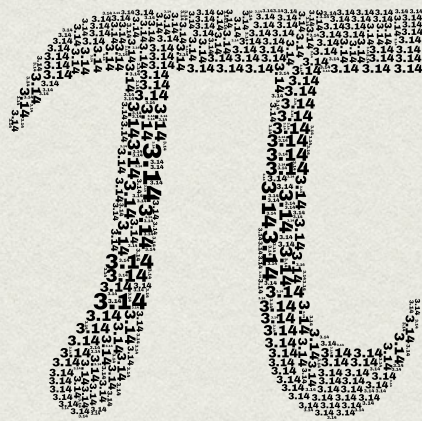
(a) Digital-7 font



(b) square grid font

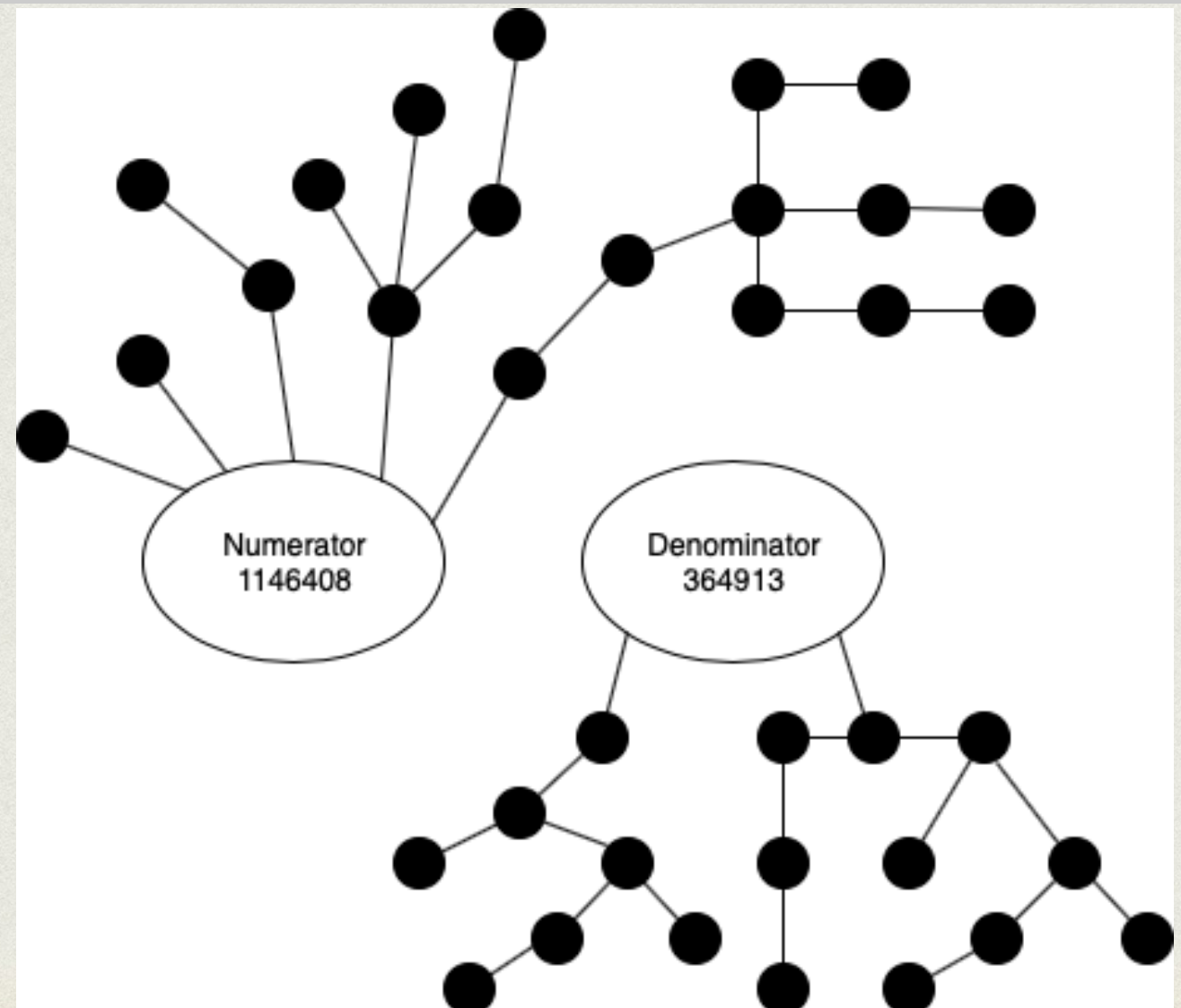
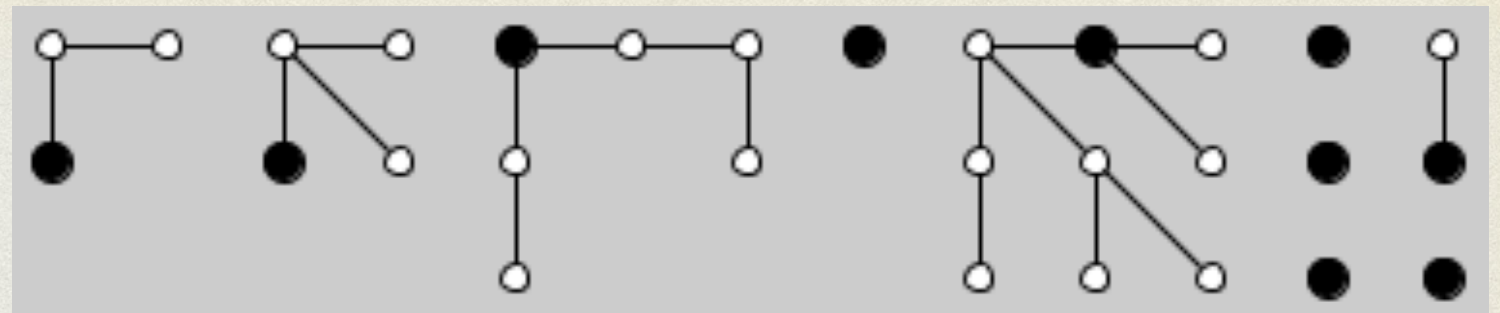
RATIONAL FRACTION FORM

Continued Fraction
(10 partial quotients)

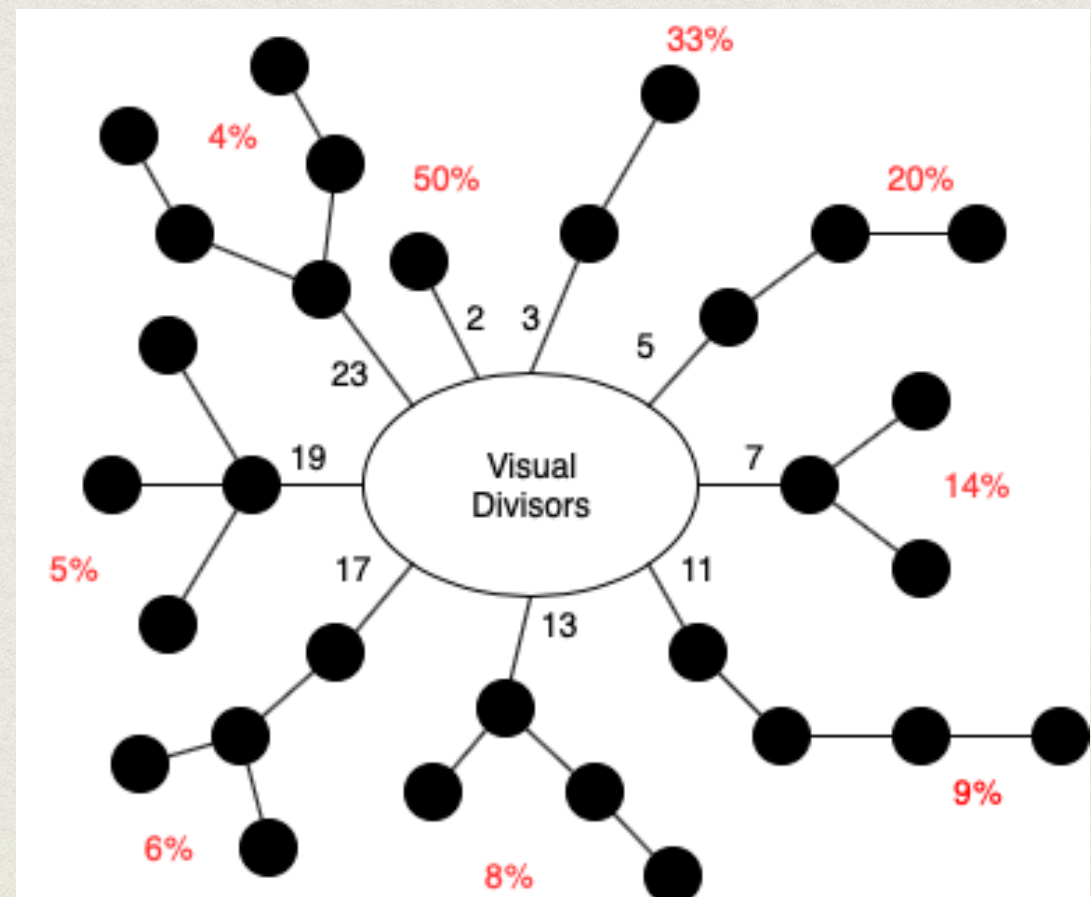
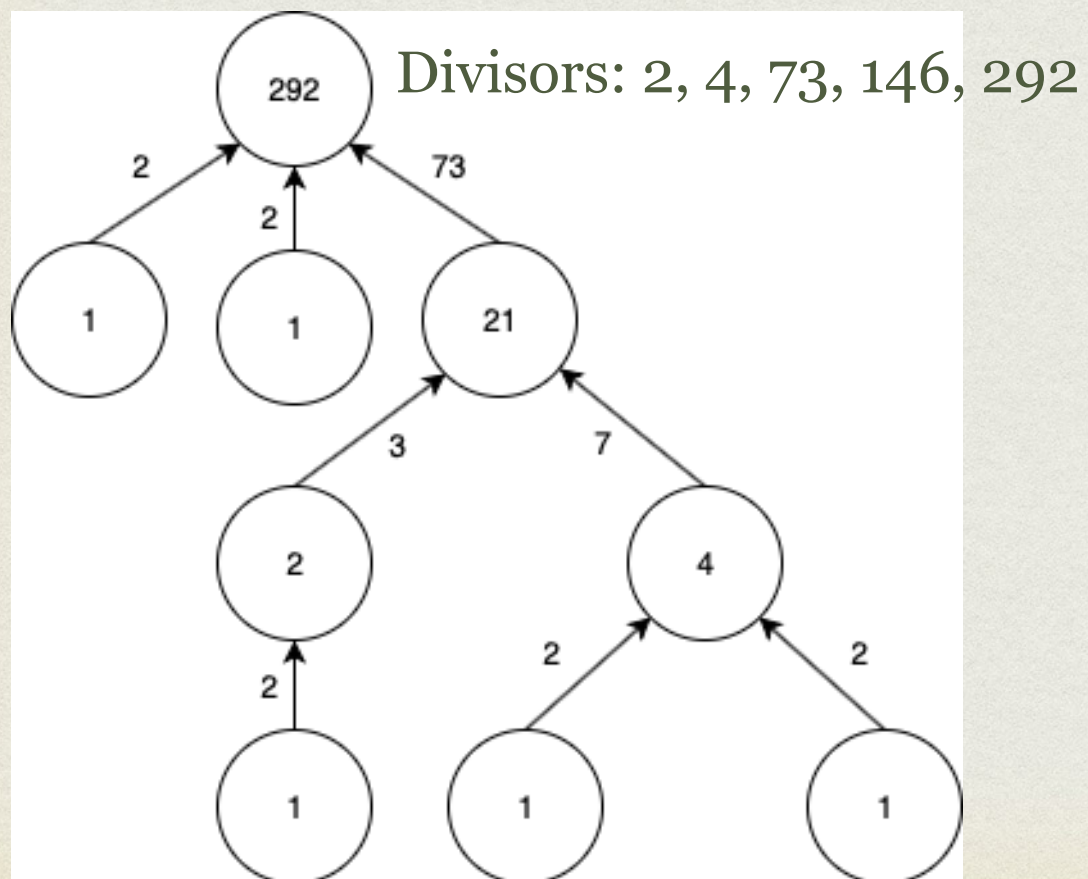
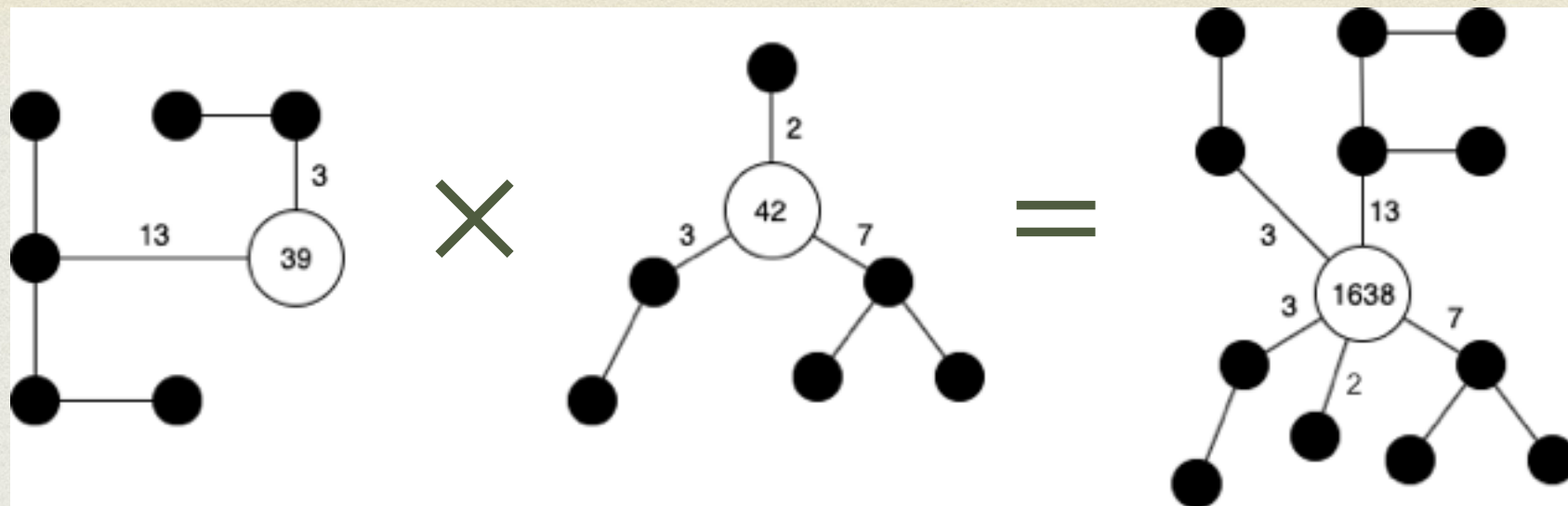


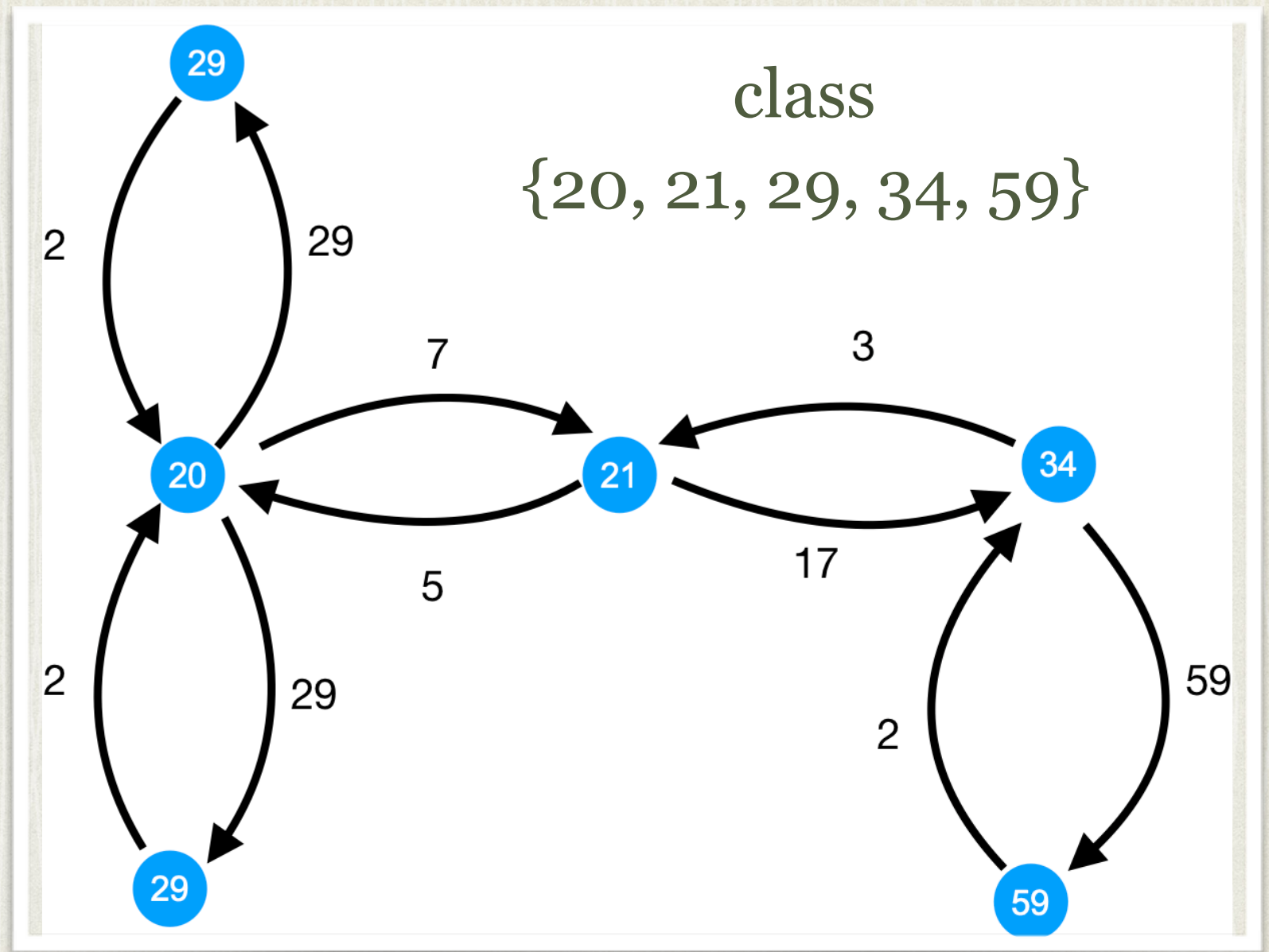
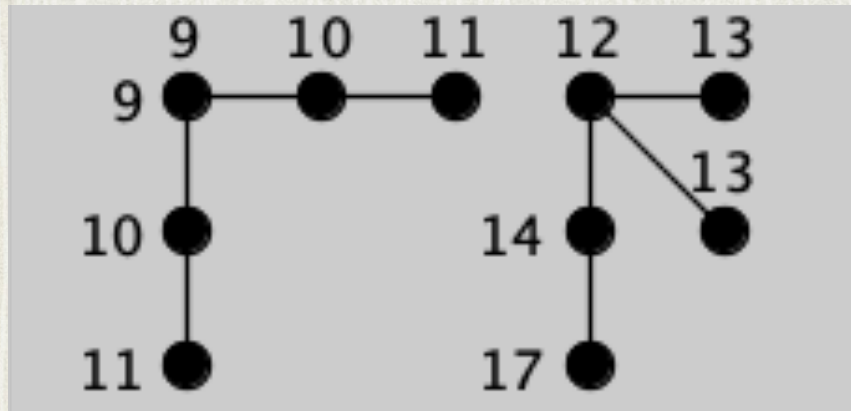
Rational Fraction
(reduced)

1146408/364913
=3.14159265358...
“correct digits”



MULTIPLICATION IS VISUAL





EQUIVALENCE RELATION

$$[j(p_i)] \mathbf{R} [i(p_j)]$$



First 40 classes

$$[j(p_i)] \mathbf{R} [i(p_j)]$$