Wave-driven vortex dynamics in the near-shore region

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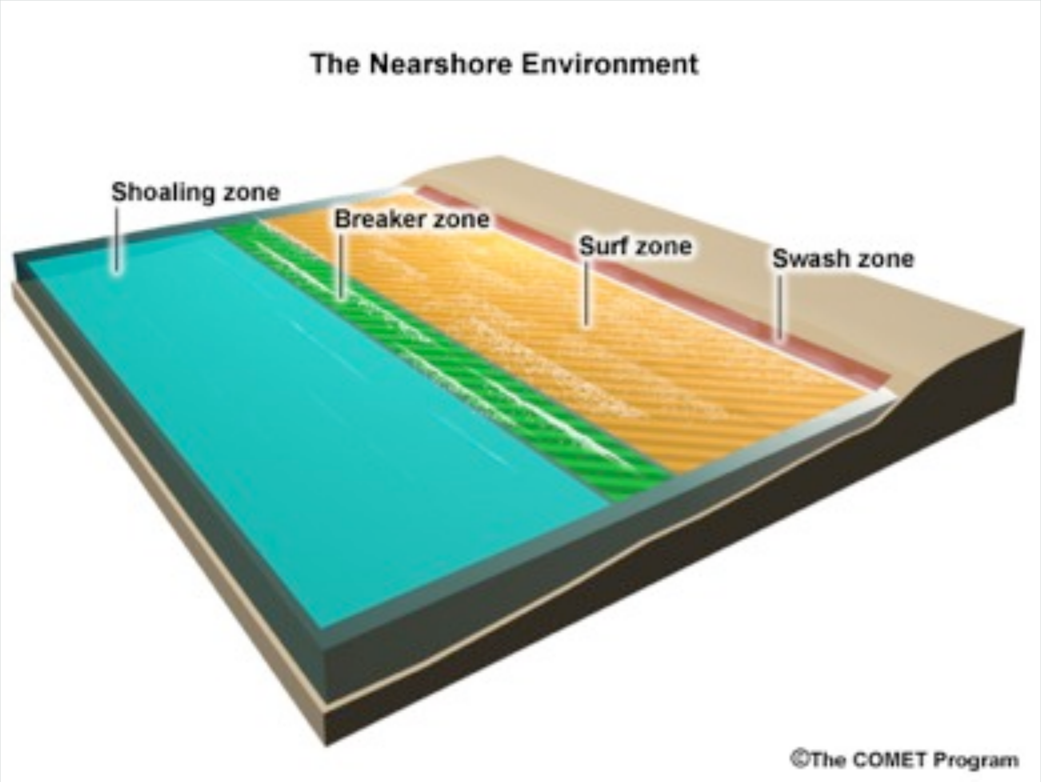
Physical processes in near-shore region

- Within 1 km of coastline
- Typical depth – 1-10m
- Shoaling and breaking of surface waves
- Turbulent (quadratic) drag



The near-shore current system

- Obliquely breaking waves create alongshore current
- Alongshore current transports sediment, can be exploited to prevent erosion
- Displaced/varying alongshore current associated with rip currents

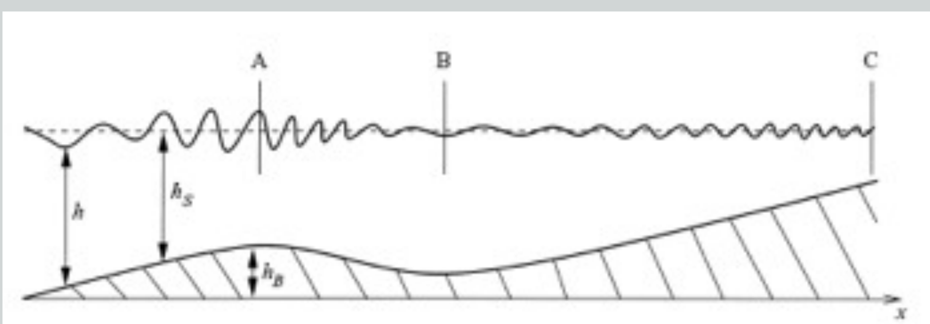
Overview

- Current dislocation on barred beaches is still inadequately explained
- Main idea: Alongshore variation of wave energy on scale of wave groups can produce current dislocation
    - Idealized experiments
    - Near-shore current system is non-turbulent

Alongshore current

- Obliquely breaking waves create alongshore current.
- The momentum transferred shoreward by surface waves is defined as “radiation stress”.
- Convergence of radiation stress transfers momentum to the mean current.
- Longuet-Higgins (1970) used the momentum balance between radiation stress convergence and bottom friction to solve for current.

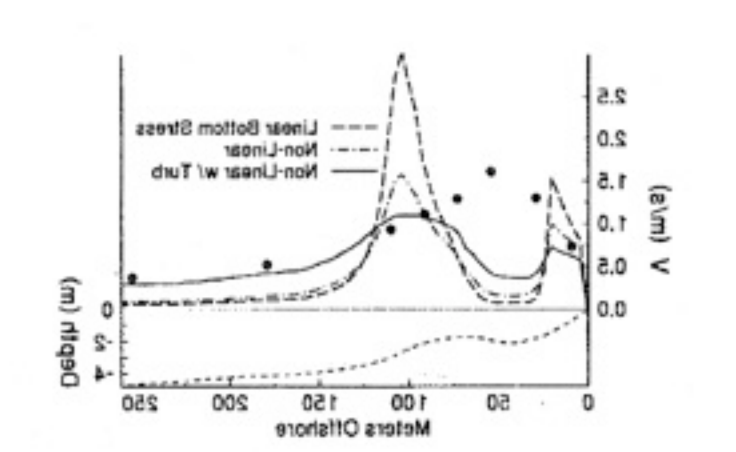
- **Result**: current is strongest at locations of strongest wave breaking

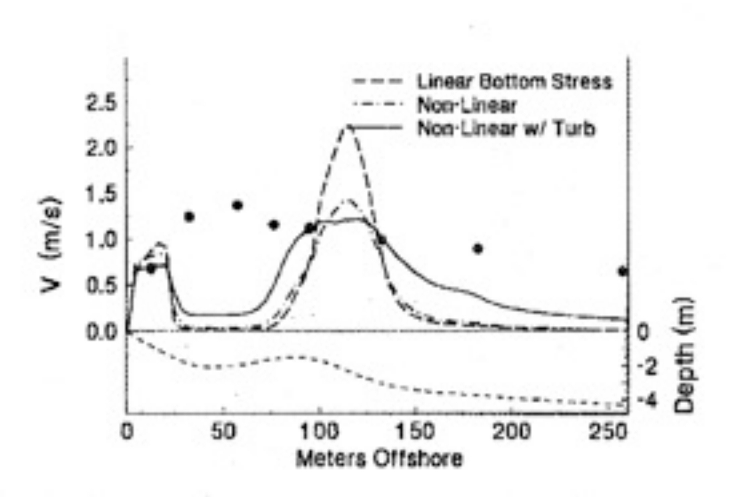


Bühler and Jacobson, 2001

Comparison with experiments (Duck, NC)

- Ok on linear beach (e.g. Santa Barbara)
- Not so good on barred beach…
- Other authors incorporate horizontal mixing (Longuet-Higgins 1970), enhanced friction due to turbulence (Church and Thornton 1993), wave rollers (Ruessink et al 1998, many others)





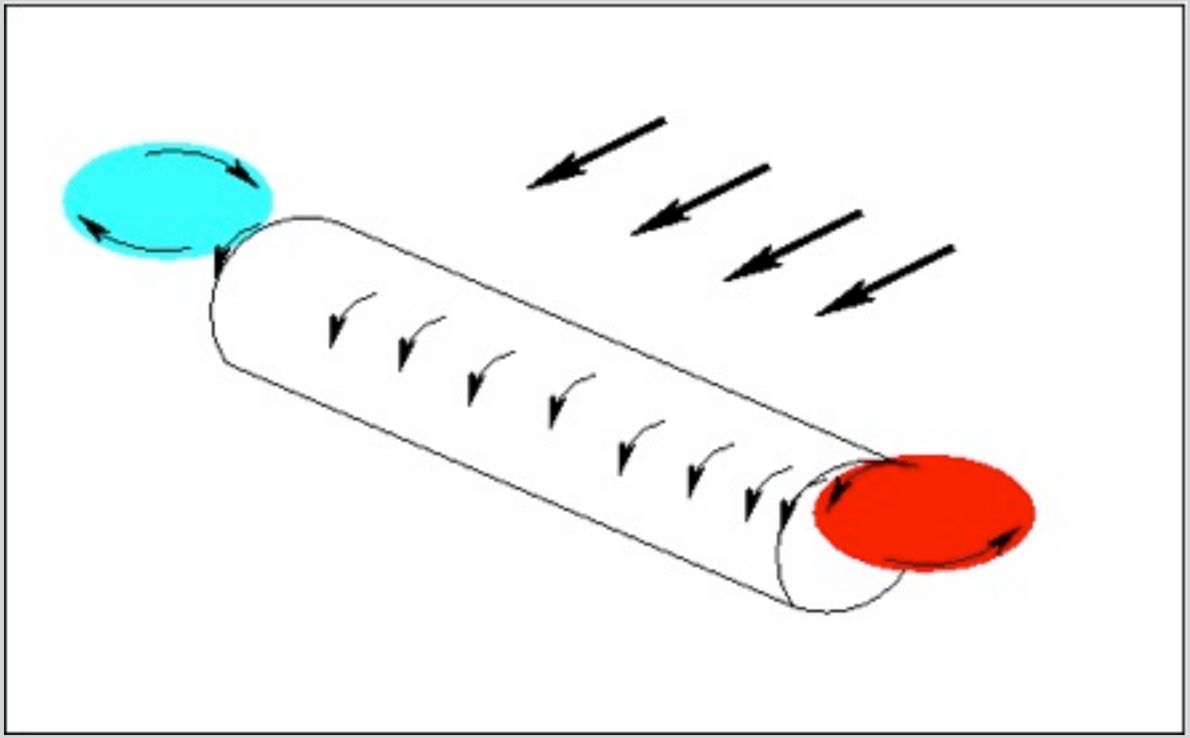
Church and Thornton (1993)

New mechanism for current dislocation

- Directional/frequency spreading can produce alongshore inhomogeneous wave breaking on O (100 m) (Reniers et al. 2002,2004)
- Inhomogeneous wave breaking produces vortex dipoles, which locate current in trough
- Numerical model, idealized studies

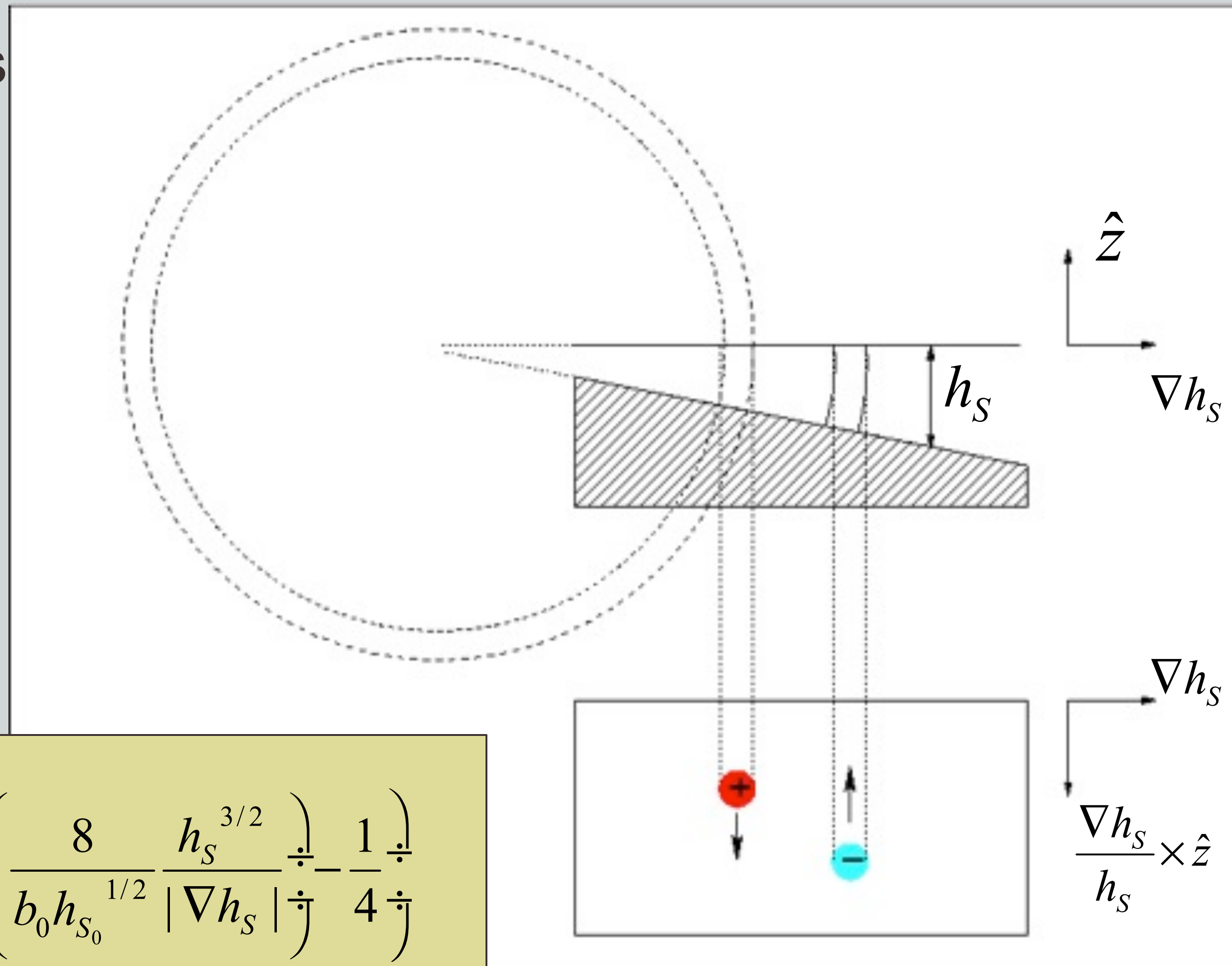
Breaking wave packets produce vortex dipoles

- Peregrine (1998)



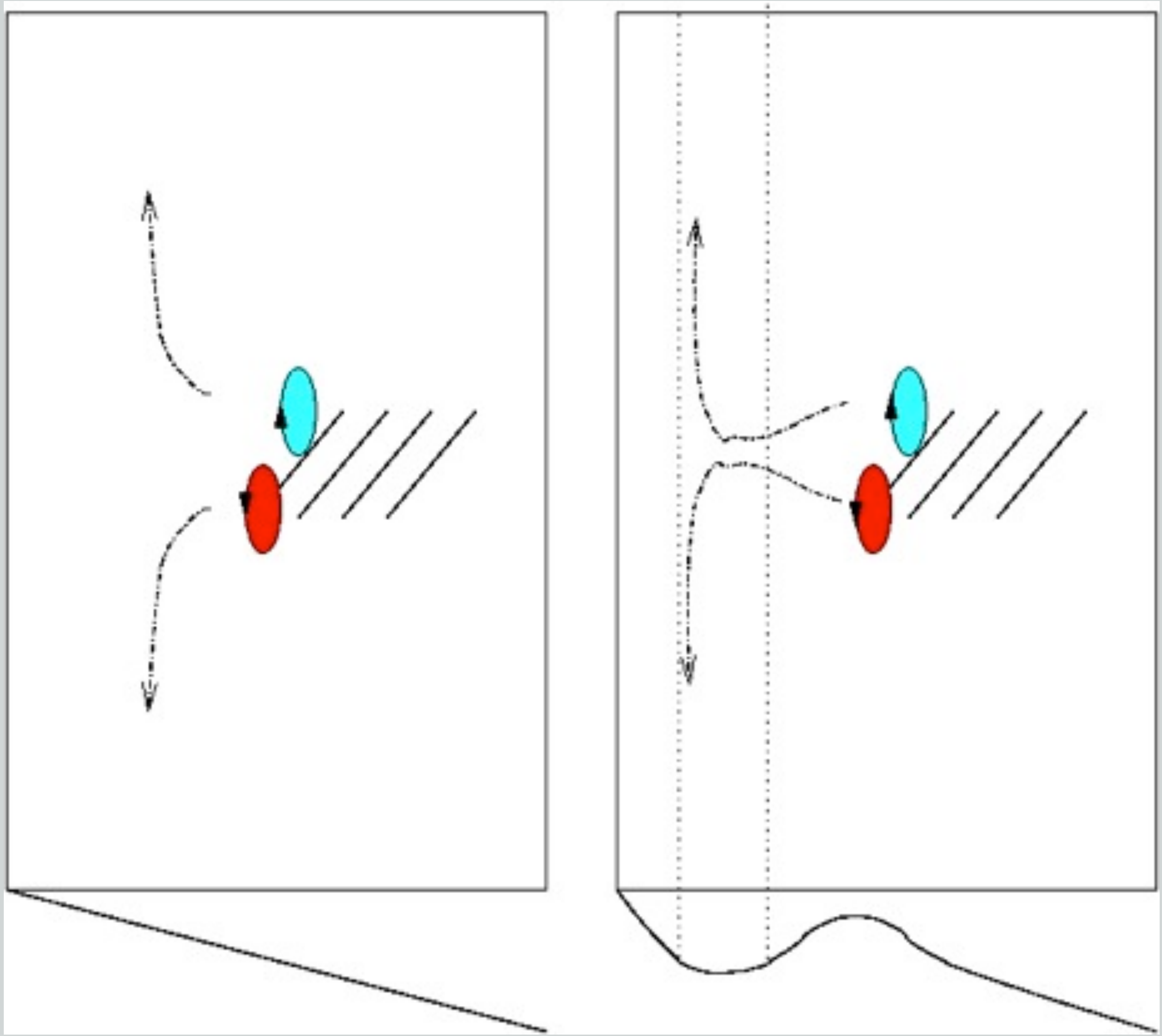
Behavior of vortices on a sloping beach

- Model vortex as axisymmetric vortex ring
- Properties of vortex ring determined by local slope of beach



$$U = \frac{\Gamma}{4\pi}\left(\frac{\nabla h_S}{h_S} \times \hat{z}\right)\left(\ln\left(\frac{8}{b_0 h_{S_0}^{\;1/2}} \frac{h_S^{\;3/2}}{|\nabla h_S|}\right) - \frac{1}{4}\right)$$

Planar vs. barred beach



Rigid-lid approximation

- For low Froude number flow ($U << \sqrt{gh}$) we have

$$\nabla \bullet \left(h_S \mathbf{u}\right) = 0$$

$$\left(\frac{\partial}{\partial t} + \left(\mathbf{u}^{\mathrm{L}} \bullet \nabla\right)\right)\left(\frac{\nabla \times \mathbf{u}}{h_S}\right) = \frac{1}{h_S} \nabla \times \left(\mathbf{B} - \frac{1}{h_S} \nabla \bullet \mathbf{S}\right)$$

- The last term is the “radiation stress” of Longuet-Higgins (defined on next slide)
- We now describe the flow by the single dynamic equation

$$\frac{Dq}{Dt} = -\frac{1}{h} \nabla \times \left(c_f \frac{\mathbf{u}\,|\,\mathbf{u}\,|}{h} + \frac{1}{h} \nabla \bullet \mathbf{S}\right)$$

$$q \equiv \frac{\nabla \times \mathbf{u}}{h}$$

Rotational part of radiation stress (BJ01)

$$\mathbf{S}_{ij} \equiv h_S \overline{u_i' u_j'} + \delta_{ij} \frac{g}{2} \overline{h'^2}$$

$$-\frac{1}{h_S} \nabla \bullet \mathbf{S} = \frac{\partial \mathbf{p}}{\partial t} - \mathbf{F} - \frac{1}{2} \nabla \overline{|u'|^2}$$

$$\mathbf{F} \equiv \frac{\mathbf{k}}{h_S} \nabla \bullet \left(h_S \frac{\mathbf{k}}{\kappa^2} E \right)$$

In the presence of steady waves, only F makes a contribution to the curl of the momentum on the previous slide. F is non-zero in the absence of dissipation.

Wave parameterization

- Geometric ray theory
- Waves “break” when they exceed saturation threshold

$$\frac{d\mathbf{x}}{dt} = \Omega_{\mathbf{k}}$$

$$\frac{d\mathbf{k}}{dt} = -\Omega_{\mathbf{x}}$$

$$\Omega(\mathbf{k},\mathbf{x}) = \sqrt{gh_S}\,\kappa$$

$$\frac{\partial A}{\partial t} + \nabla \bullet \left(c_g A\right) = 0$$

Waves are forced to break when they exceed a saturation threshold: following LH70,

$$A = \min(A^{sat}, A^{unsat})$$

$$A^{sat} = A(\alpha h_S)$$

$$\alpha = 0.41$$

Numerical model: governing equations

$$\frac{Dq}{Dt} = -\frac{1}{h}\nabla\times\mathbf{F} - \frac{c_f}{h}\nabla\times\frac{\mathbf{u}\,|\,\mathbf{u}\,|}{h}$$

$$\nabla\bullet\left(\frac{\nabla\psi}{h}\right) = hq$$

$$\mathbf{u} = \frac{1}{h}\nabla^{\perp}\psi$$

$$0 \le x \le D, 0 \le y \le L$$

$$\psi(x,0) = \psi(x,L)$$

$$\psi(0,y) = 0$$

$$\psi(D,y) = M\psi(D,y)$$

M is the "Dirichlet-to-Neumann" map (DtN) of the operator

$$\nabla\bullet\left(\frac{\nabla\psi}{h}\right)$$

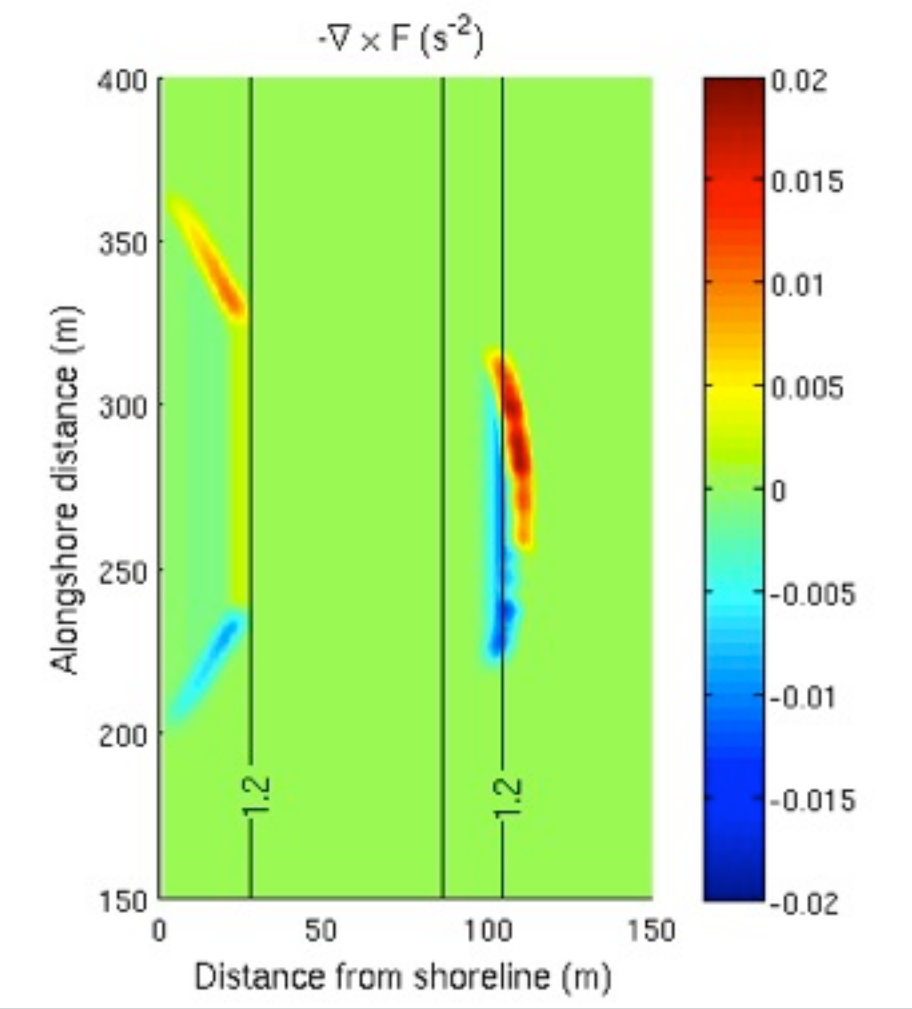
for some specified h(x) and boundary conditions at infinity

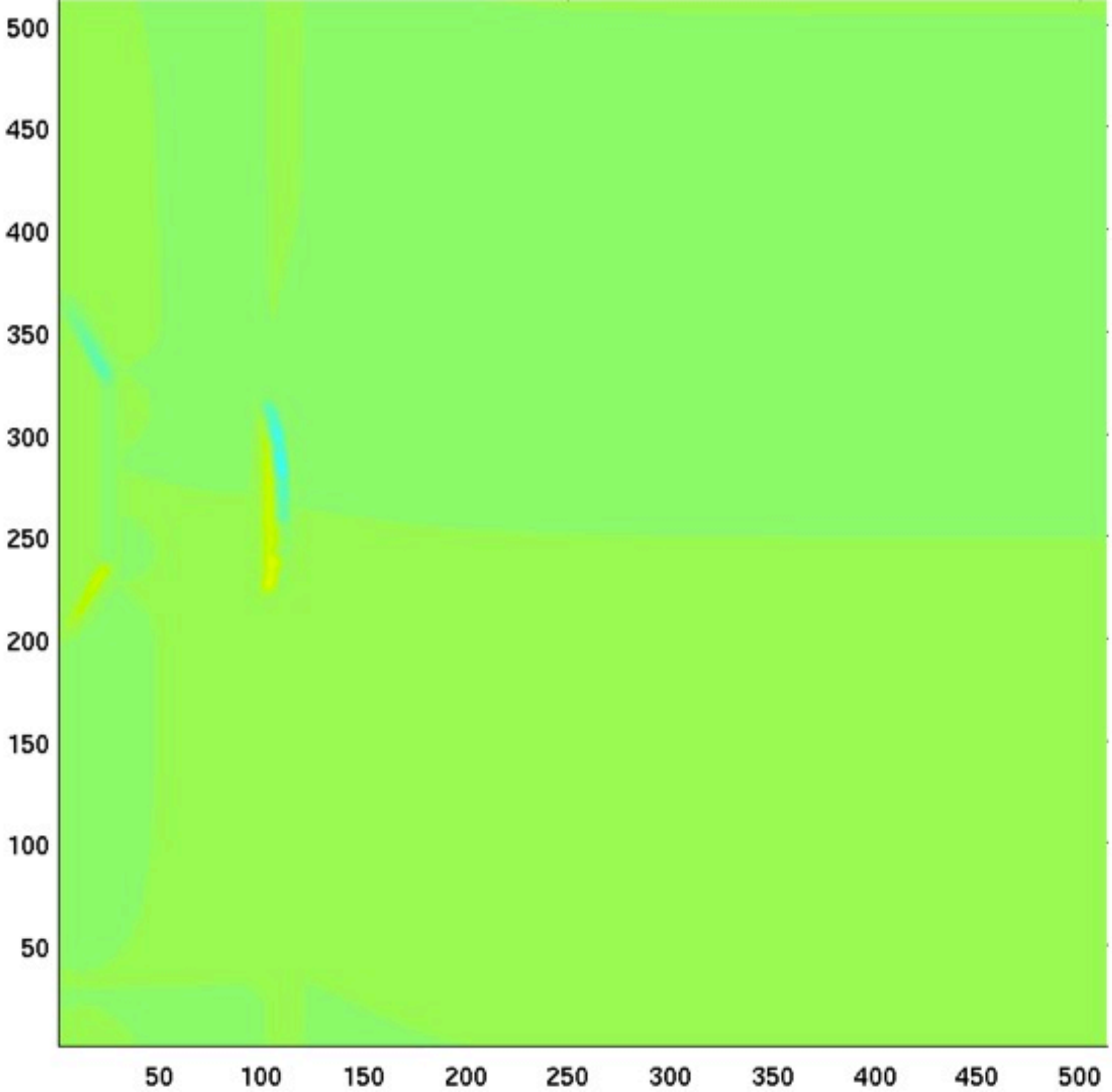
Idealized experiments on current dislocation

- Linear vs. barred topography
- Homogeneous vs. inhomogeneous (packet)

Vortex dipole

- Packet of waves produces vortex dipole





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Idealized study: barred beach, homogeneous waves

- No current dislocation

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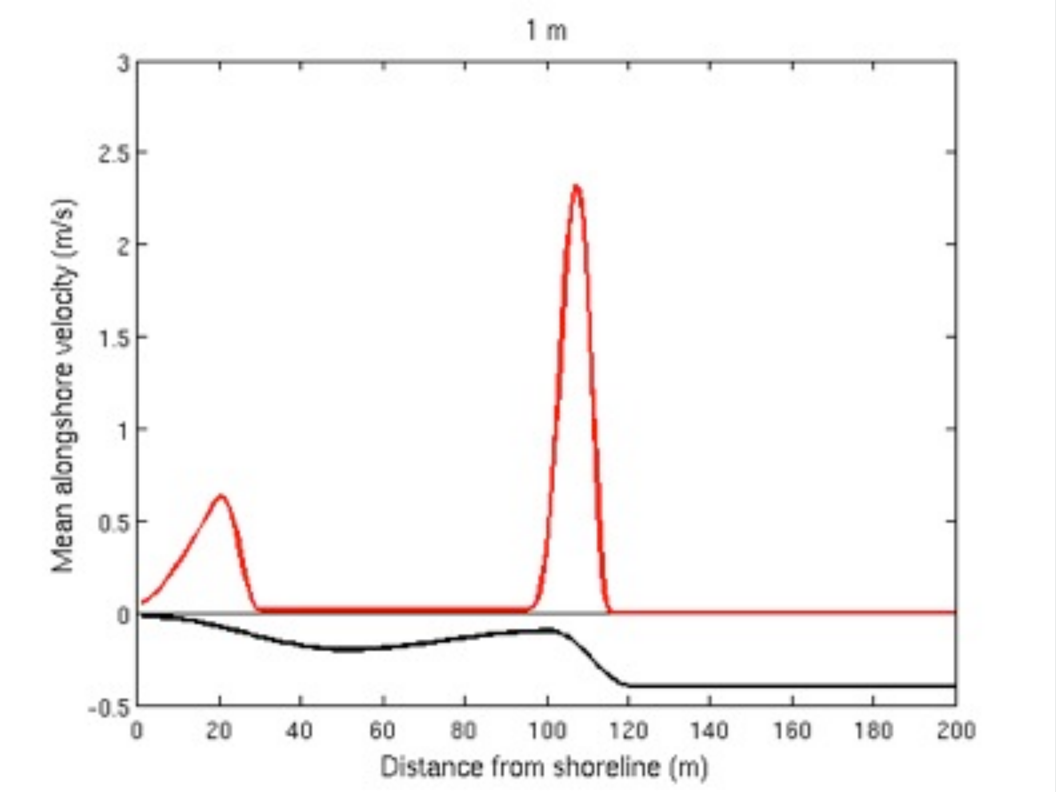

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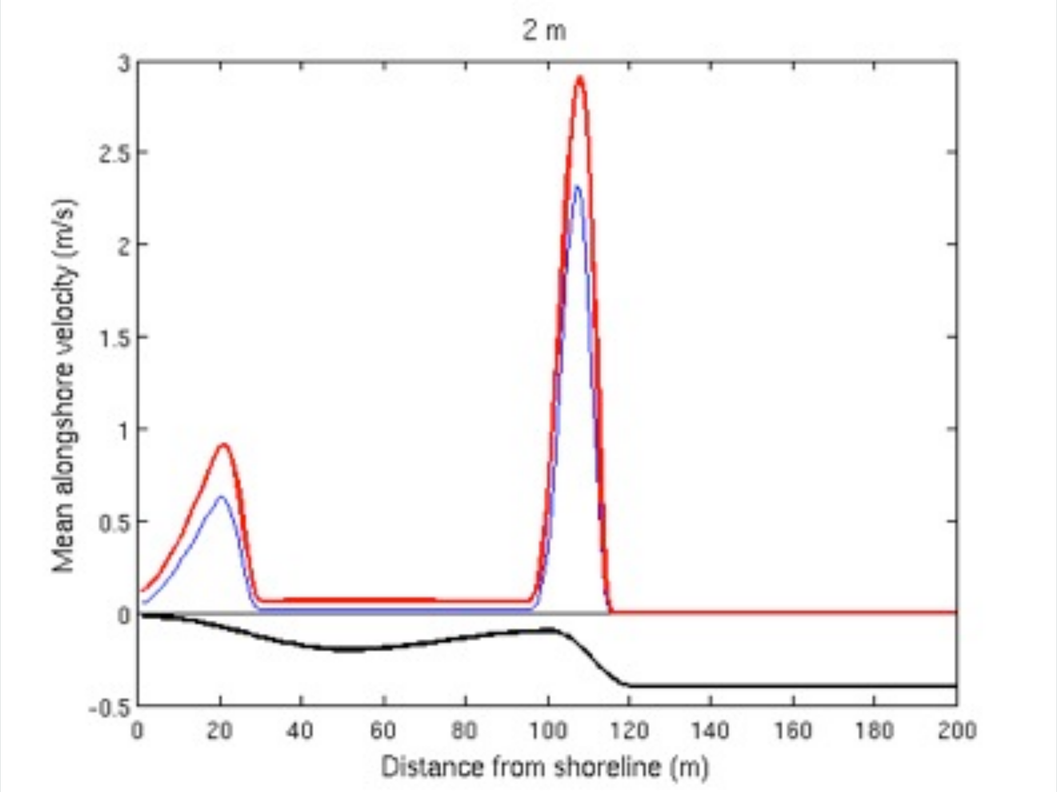

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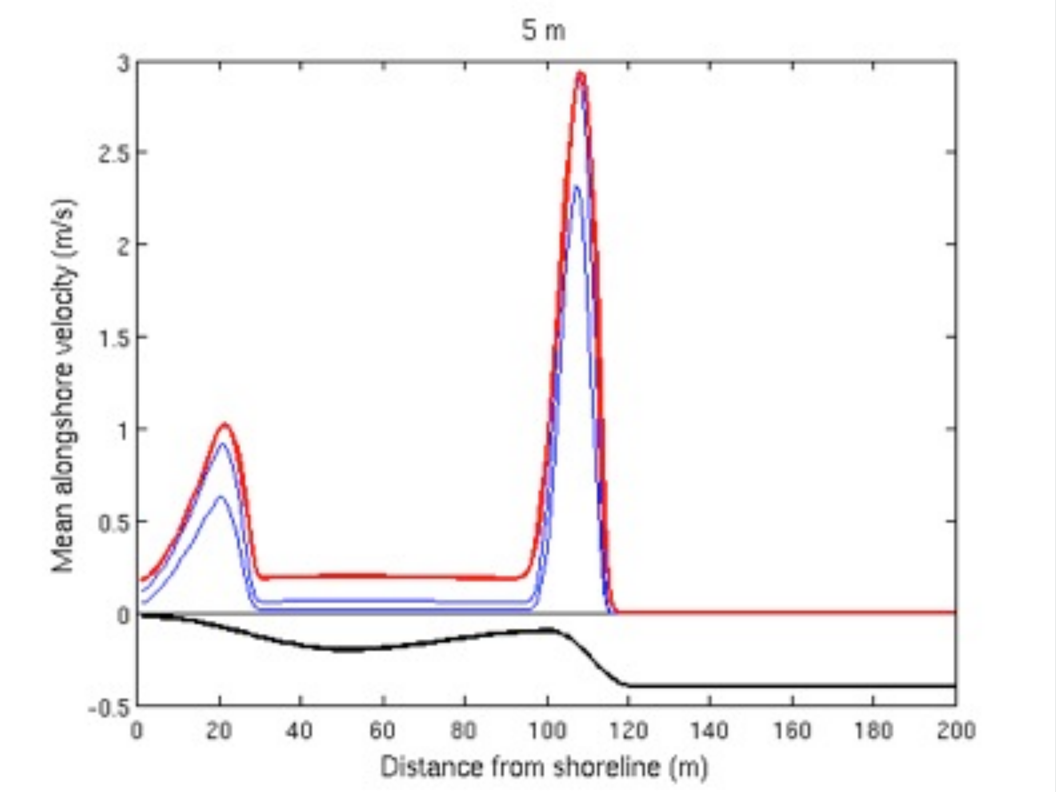

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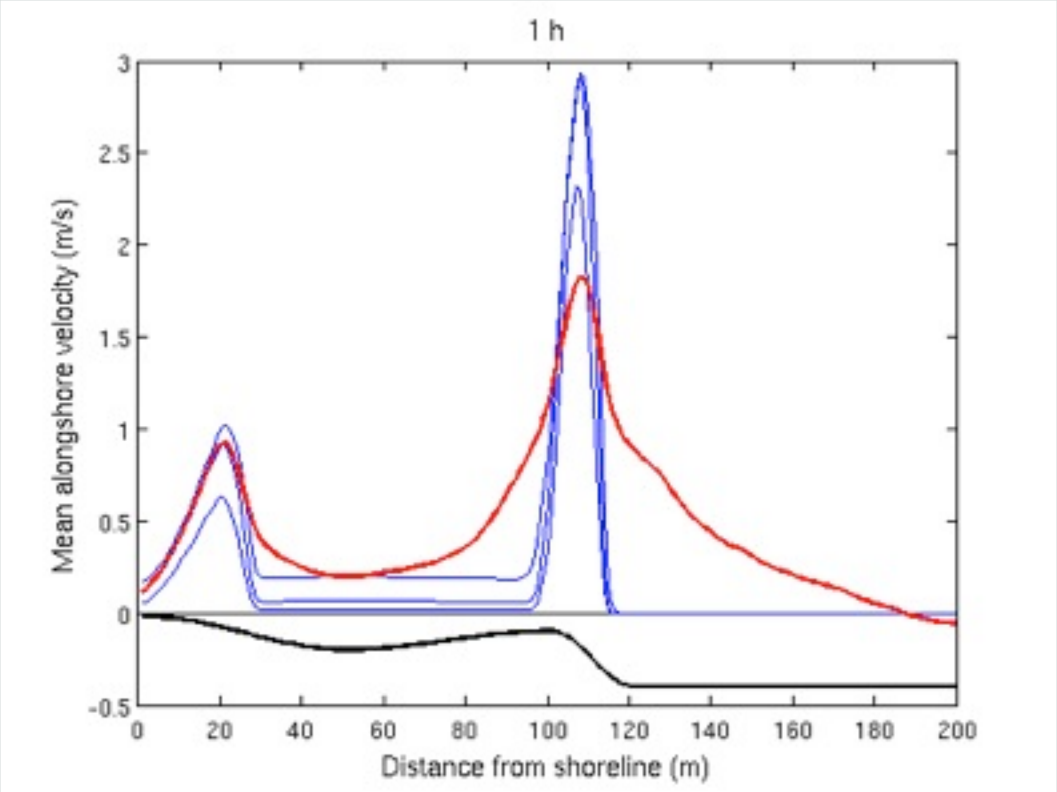

Idealized study: linear beach, packet of waves

- Modest current dislocation

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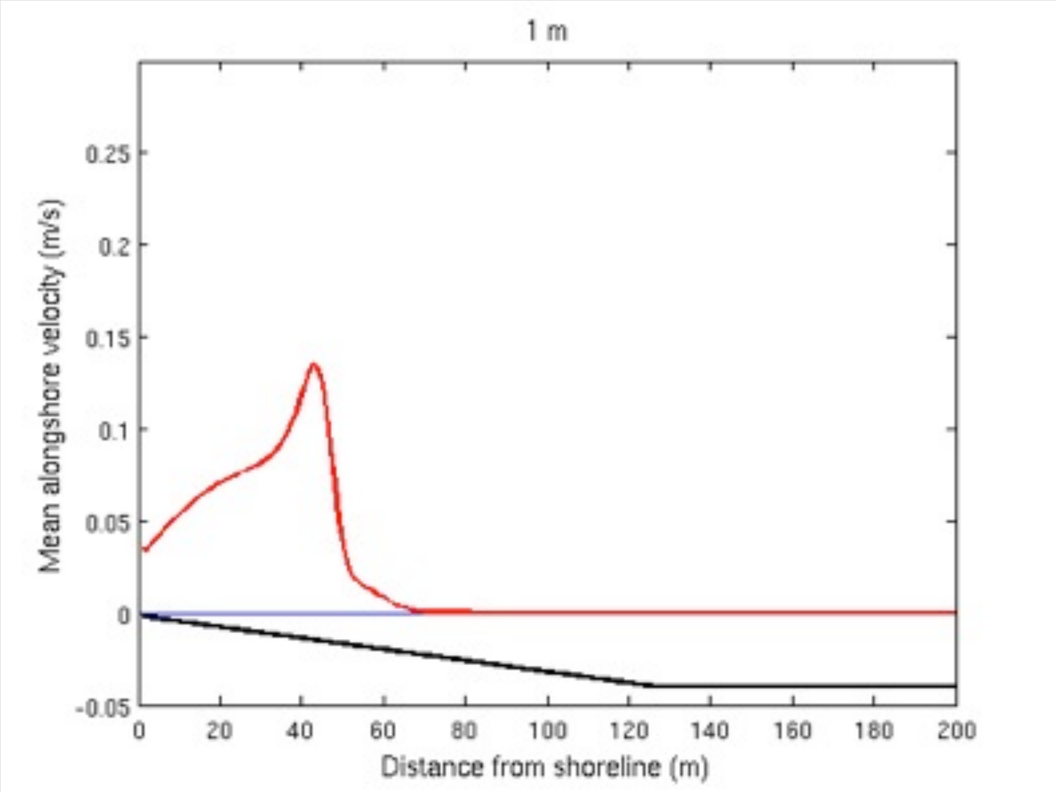

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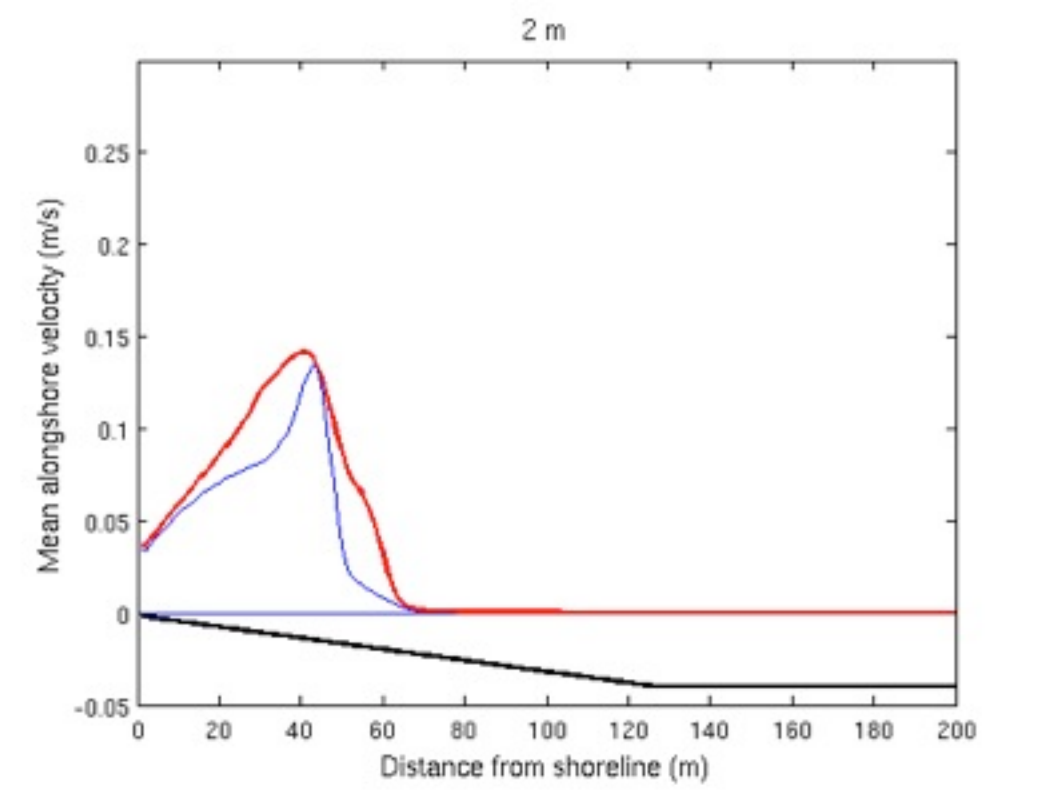

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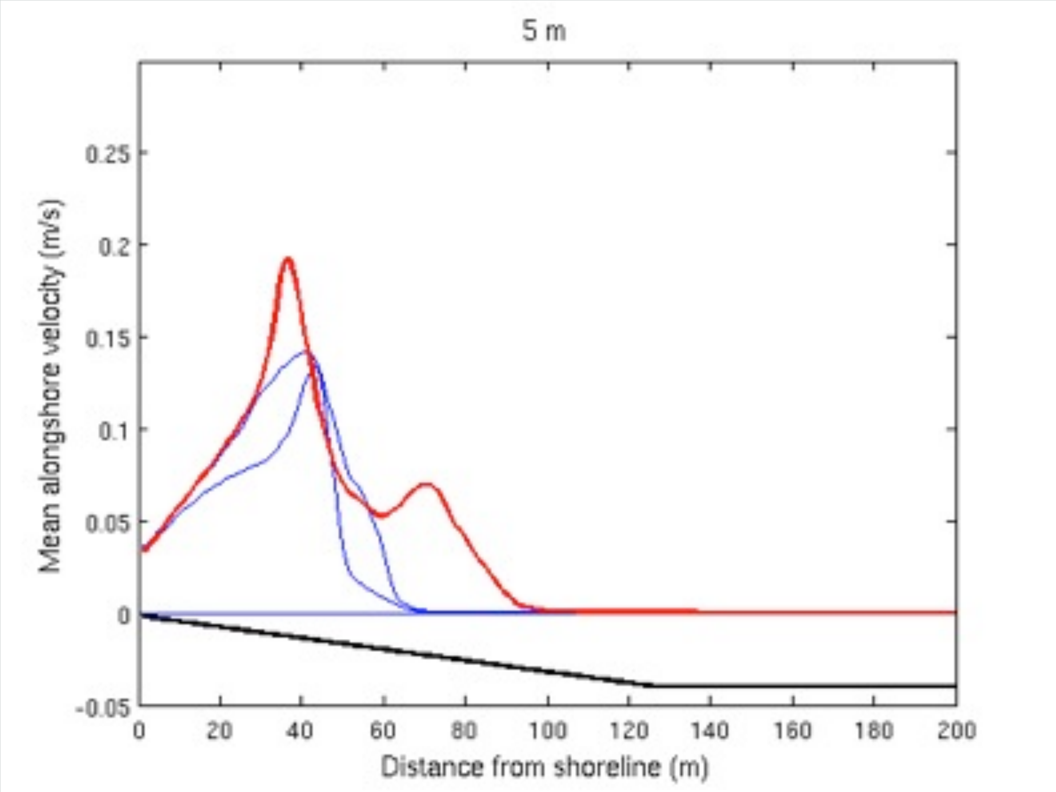

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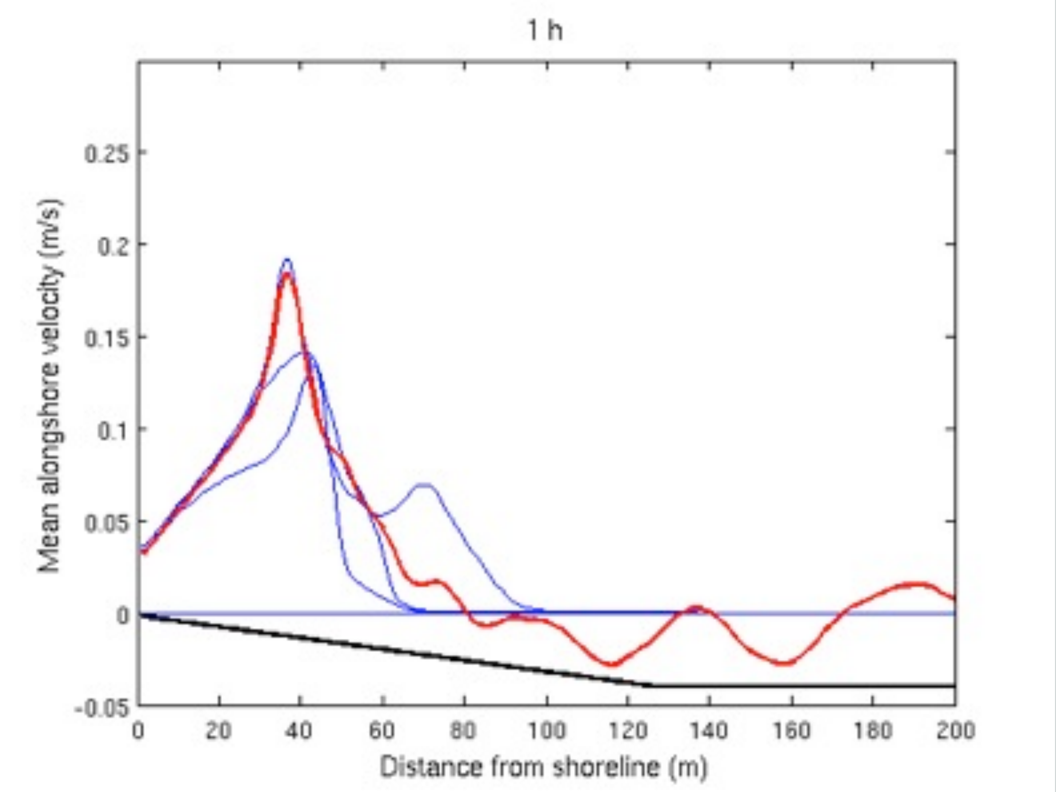

Idealized study: barred beach, packet of waves

- Marked current dislocation

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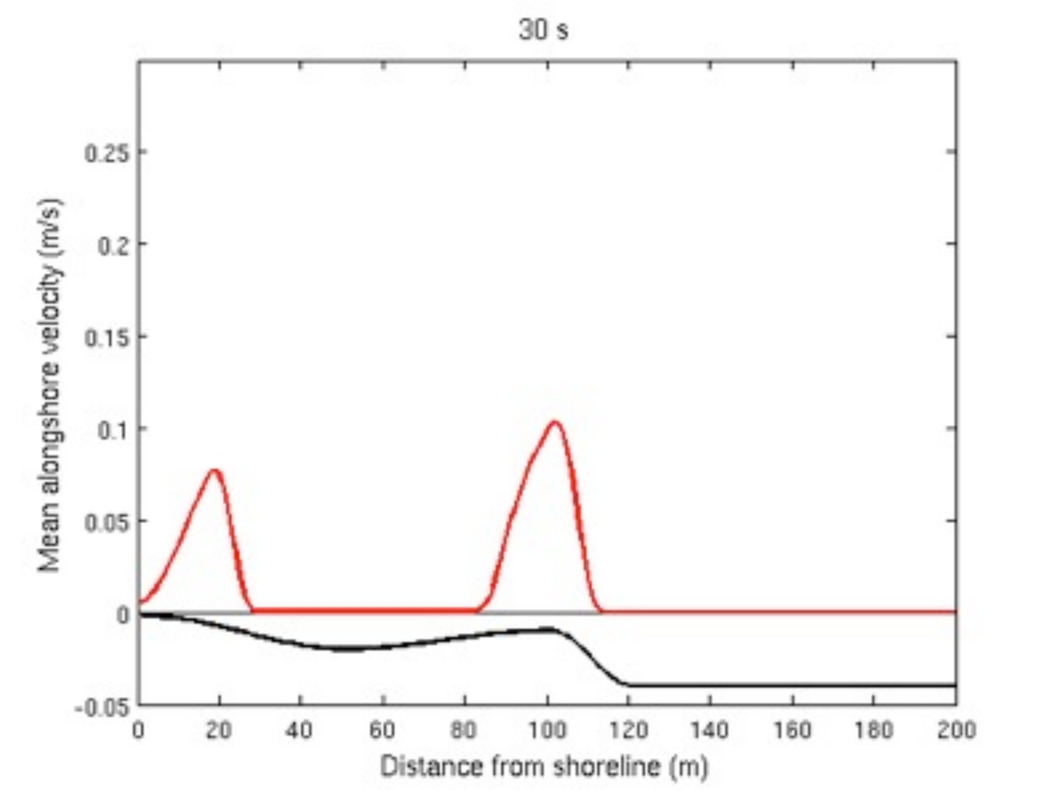

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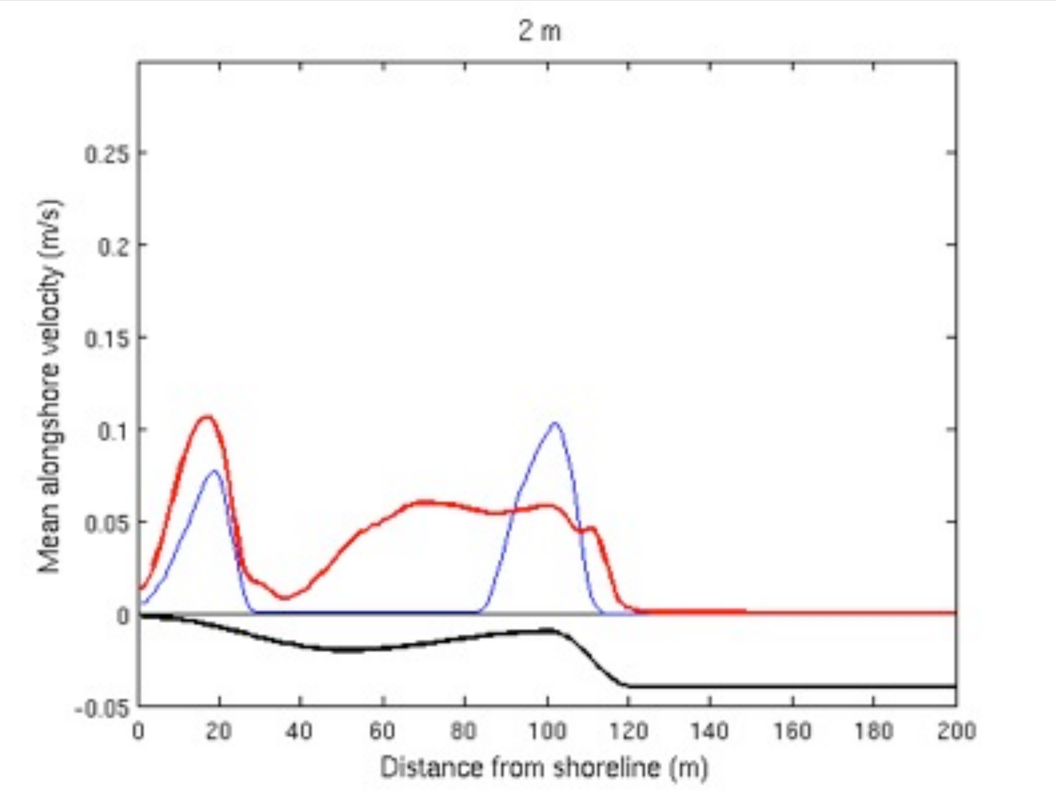

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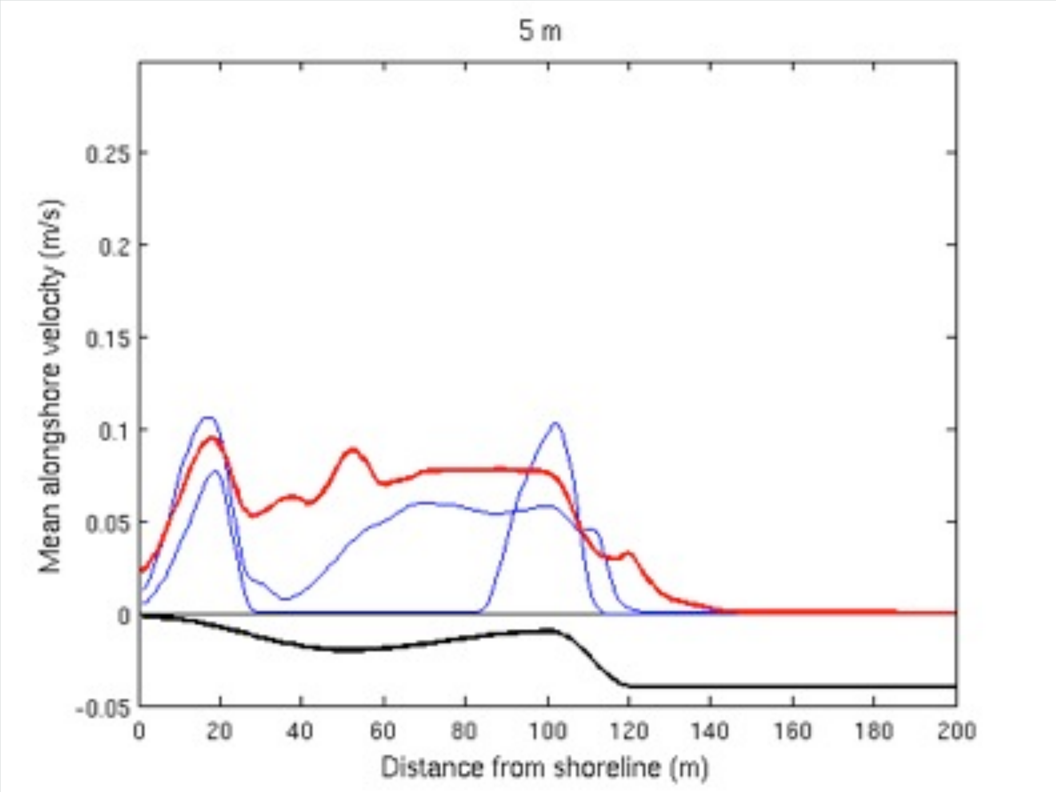

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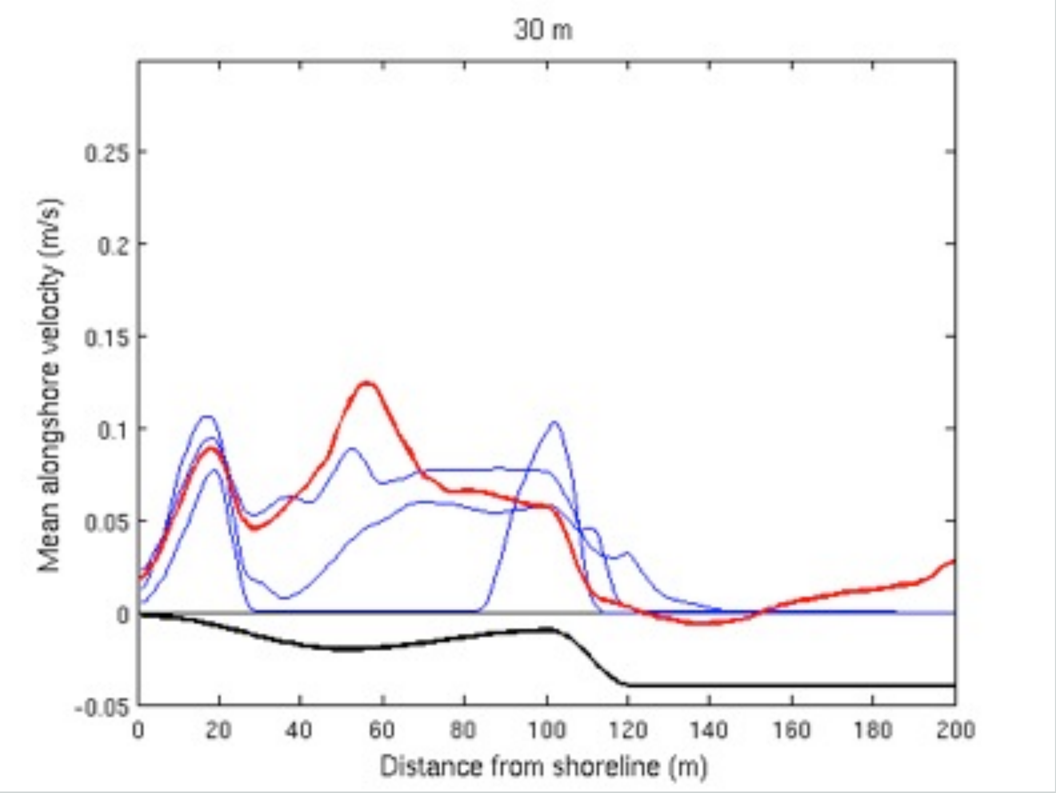

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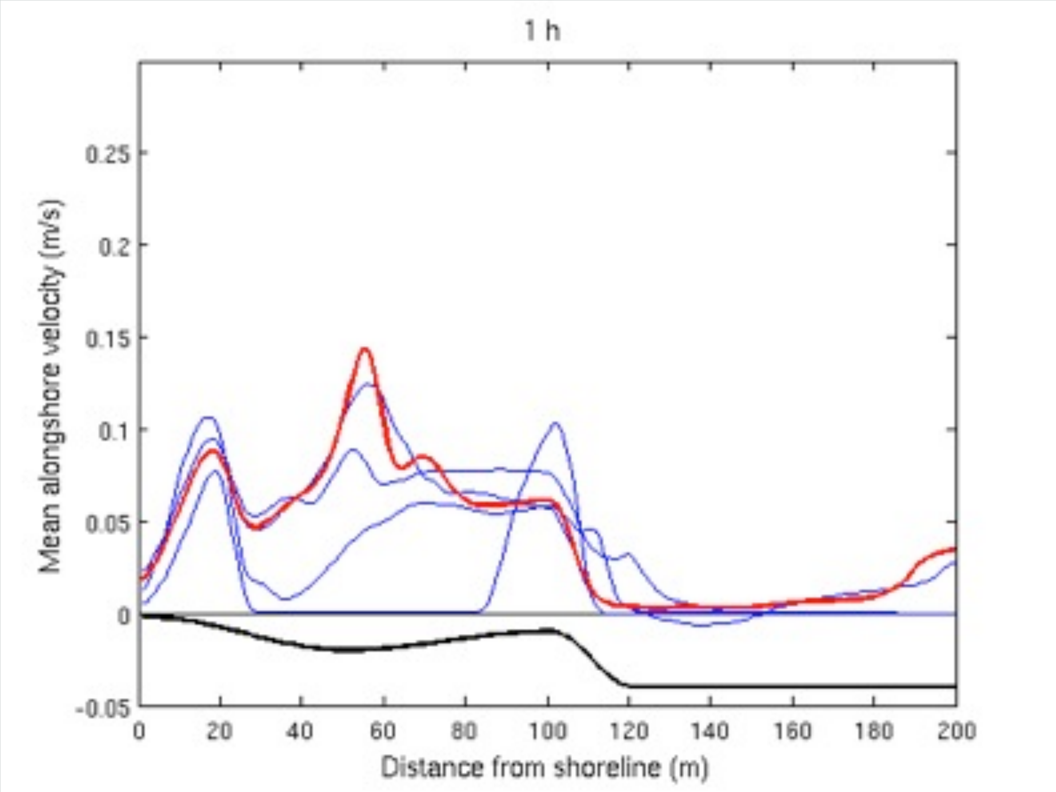
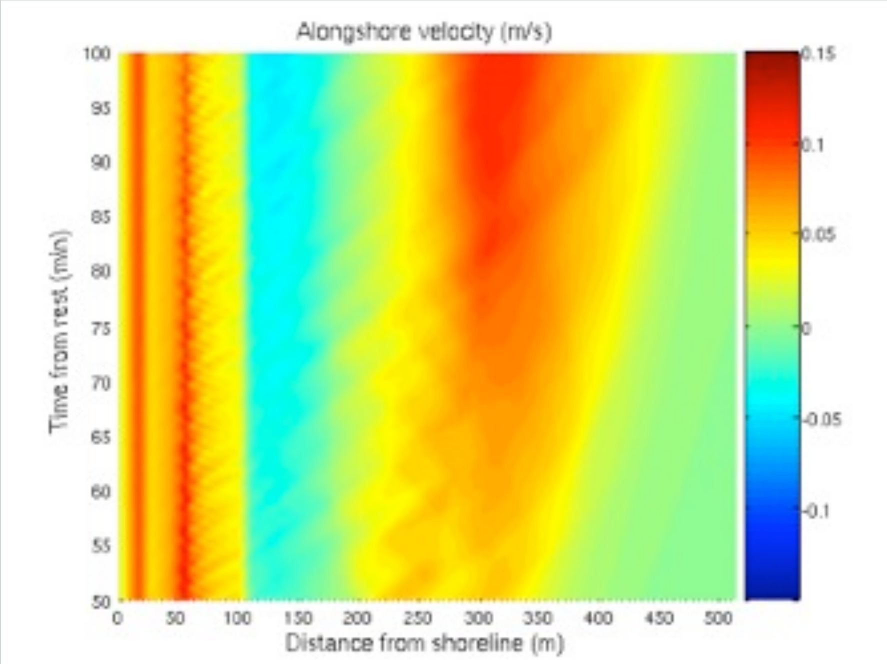
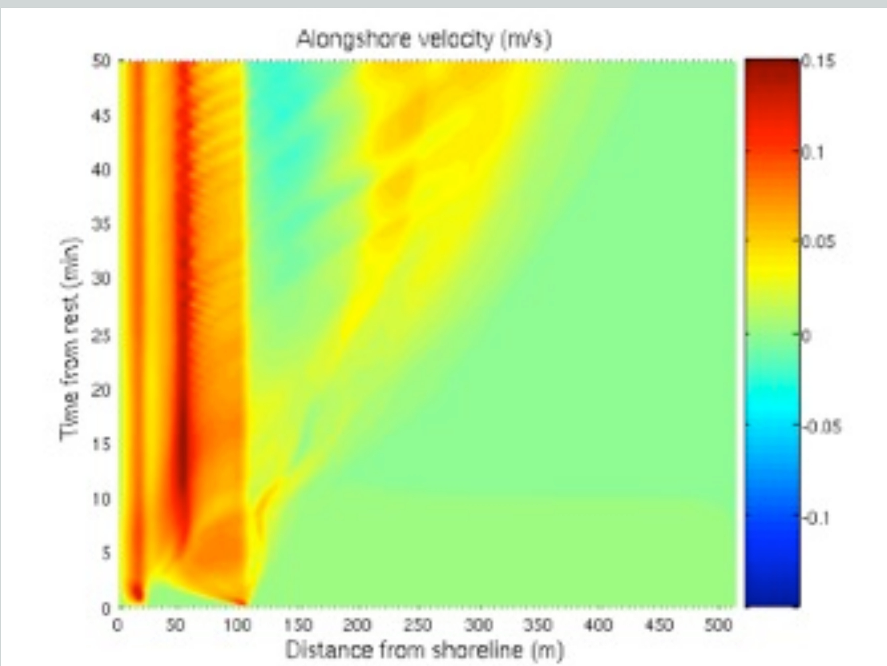
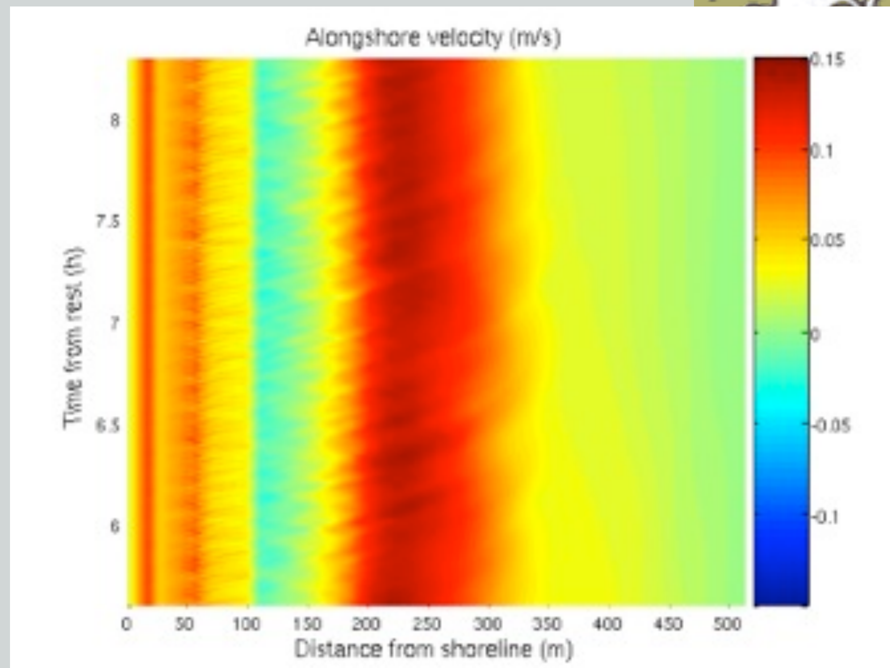
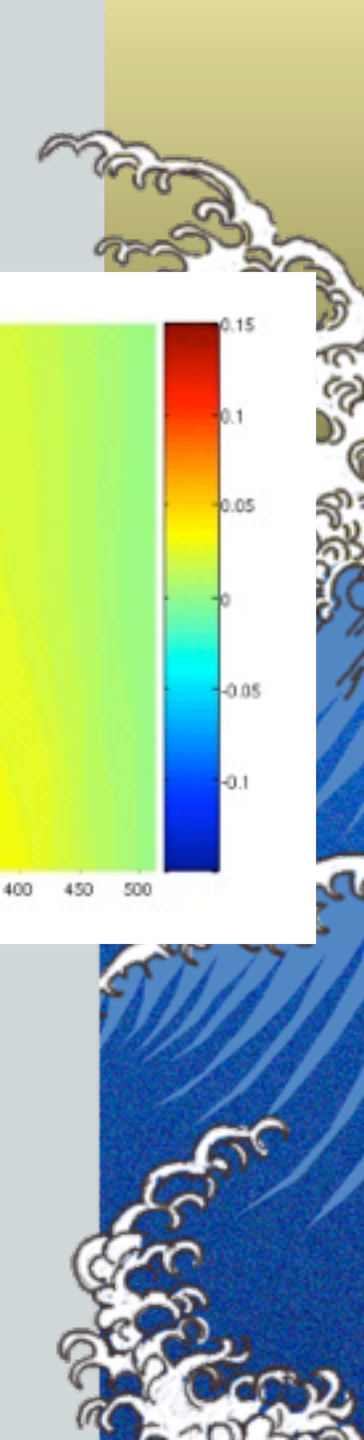


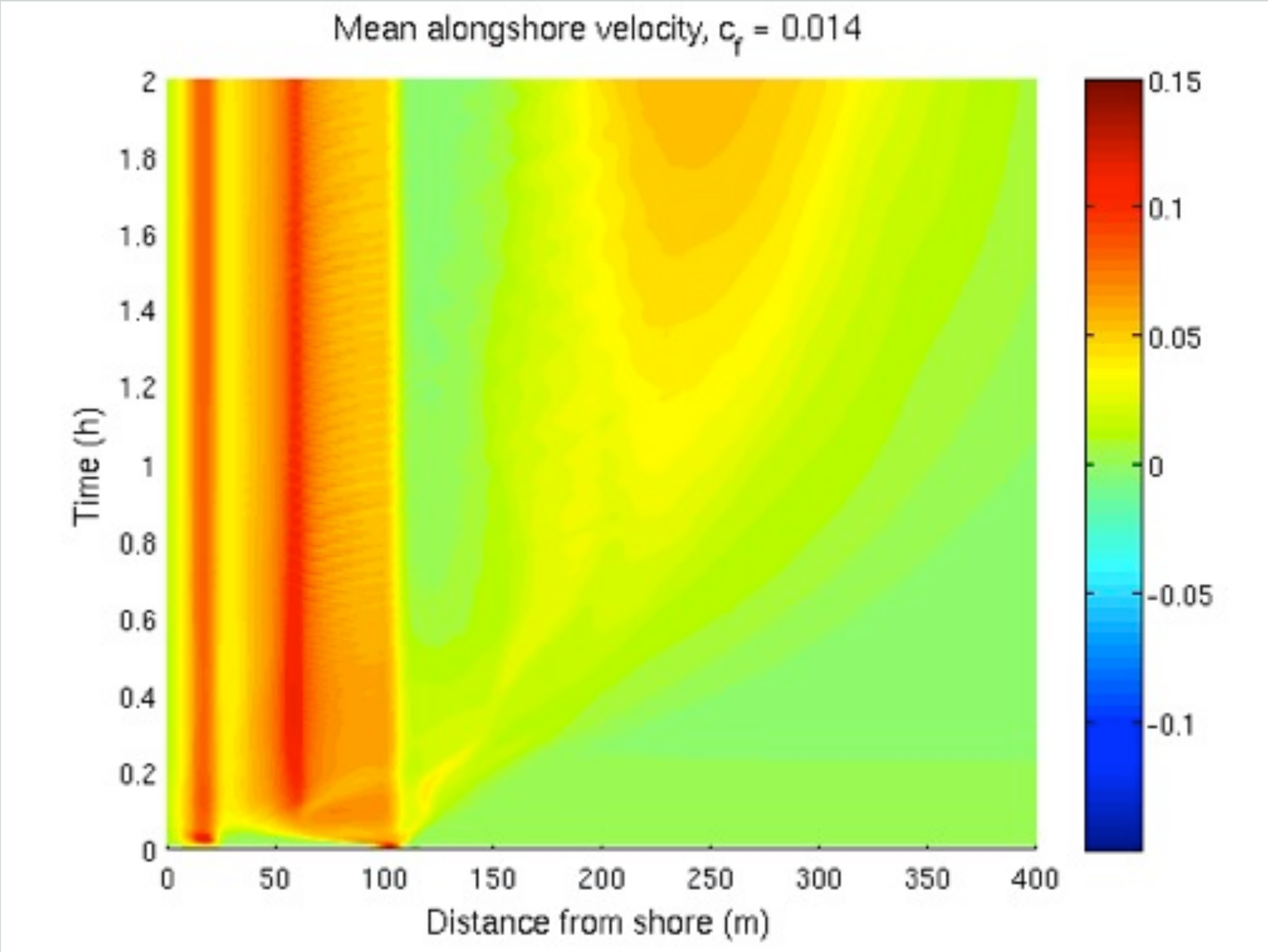

Alongshore velocity (m/s)
Time from rest (min)
Distance from shoreline (m)


Alongshore velocity (m/s)
Time from rest (min)
Distance from shoreline (m)


Alongshore velocity (m/s)
Time from rest (h)
Distance from shoreline (m)



Longer time velocity observations, c_f=0.014

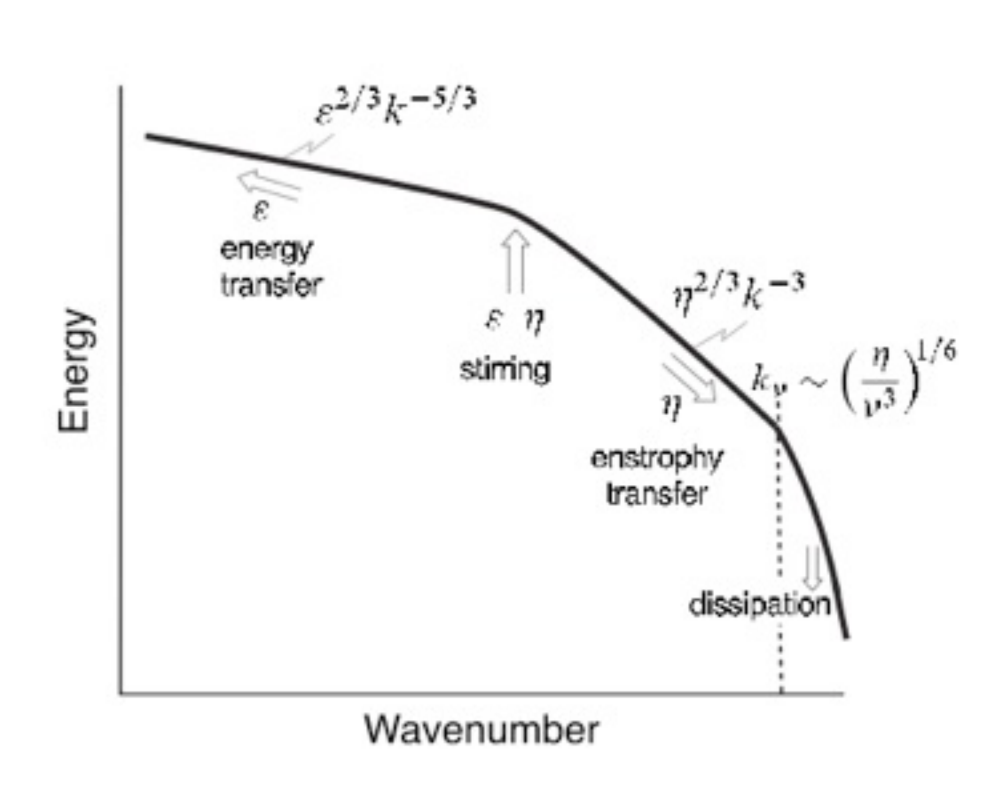


Near-shore current system is non-turbulent

- Shallow water = 2-D fluid with varying fluid depth
- Can we have upward energy cascade with physical parameters typical of the beach (Peregrine 1998, 1999)?

2D Turbulence - phenomenology

- Conservation properties imply “inverse” cascade of energy, “direct” cascade of enstrophy.
- Both cascades “arrested” by dissipative processes.
- At which length scale do these dissipative processes act? *Do they depend on the strength of forcing?*



Vallis, 2006

Near-shore current system is non-turbulent

Grainik et al. study cascade phenomenology of

$$\frac{\partial \xi}{\partial t} + u \cdot \nabla \xi = F_\xi + D_\xi$$

$$D_\xi = \nabla \times \left(-C_d \mid u \mid u\right)$$

ξ is forced at large wavenumber k_f. The upward energy cascade will be arrested at a scale k_a which is *independent* of the strength of forcing.

Near-shore current system is non-turbulent

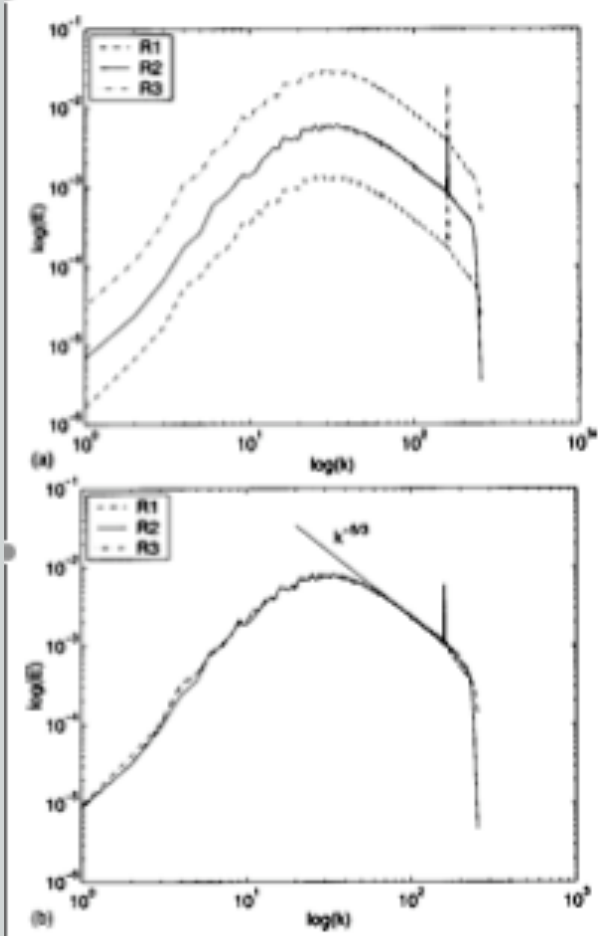


FIG. 1. (a) Steady-state kinetic energy spectra obtained by variation of forcing parameter ϵ (energy flux) while the quadratic drag coefficient is fixed at $C_d=0.6$; $\epsilon=0.1$, 1, 10 for experiments R1, R2, R3. Note that the stopping scale is independent of the energy flux. (b) The same spectra, but now rescaled by an appropriate value of the energy flux, ϵ: $E_{new}(k) = \epsilon^{-2/3}E(k)$.

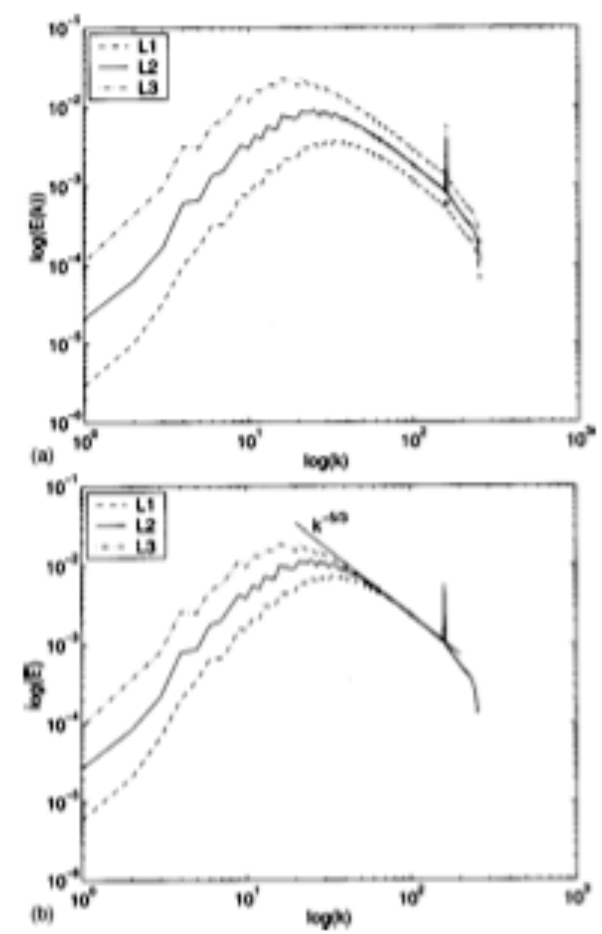


FIG. 2. Same as Fig. 1 but using linear drag to halt the inverse cascade. (a) Energy spectra with linear drag coefficient $r=0.2$ and energy forcing of $\epsilon=0.5$, 1, 2 for runs L1, L2, L3. Note that the stopping scale moves to larger scales as the energy flux increases. (b) Rescaled spectra, $E_{new}(k) = \epsilon^{-2/3}E(k)$.

Near-shore current system is non-turbulent

Grainik et al. estimate

$$k_a \approx 51 C_d$$

$$k_a \approx 51 \frac{c_f}{h}$$

If c_f = 0.01, then

$$k_a h = 0.5$$

But we only model mean motions for which

$$k_a h < 1$$

(on surf zone, mean flow also subject to littoral friction)

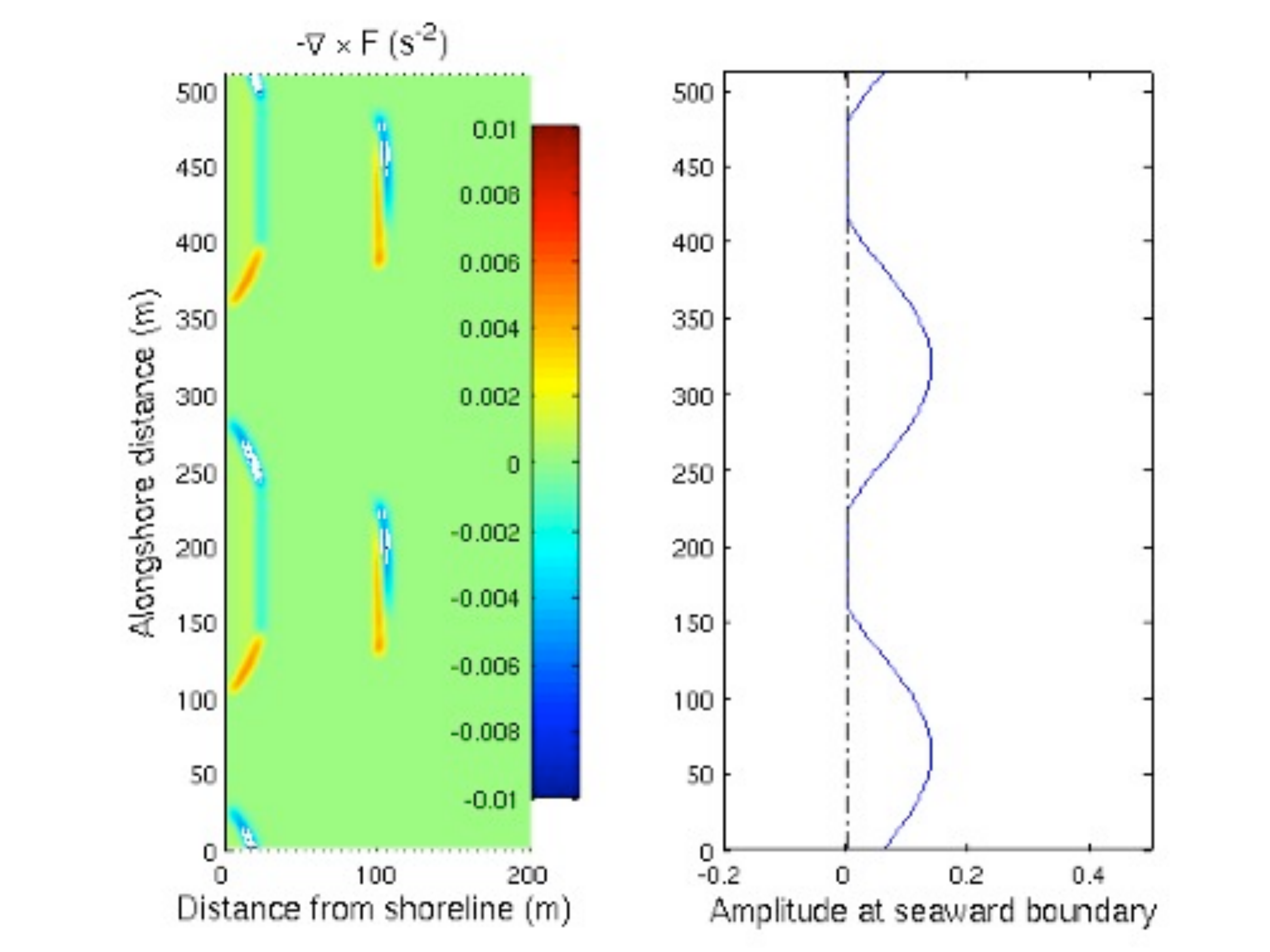
Conclusions and on-going work

- New numerical model for study of near-shore region, with open boundary and parameterized forcing
- Vortex dipole is shown to provide a mechanism for current dislocation
- Surf zone is non-turbulent (2D)
- Long-term goal: When will low-frequency wave energy produce vortex dipoles capable of dislocating current?

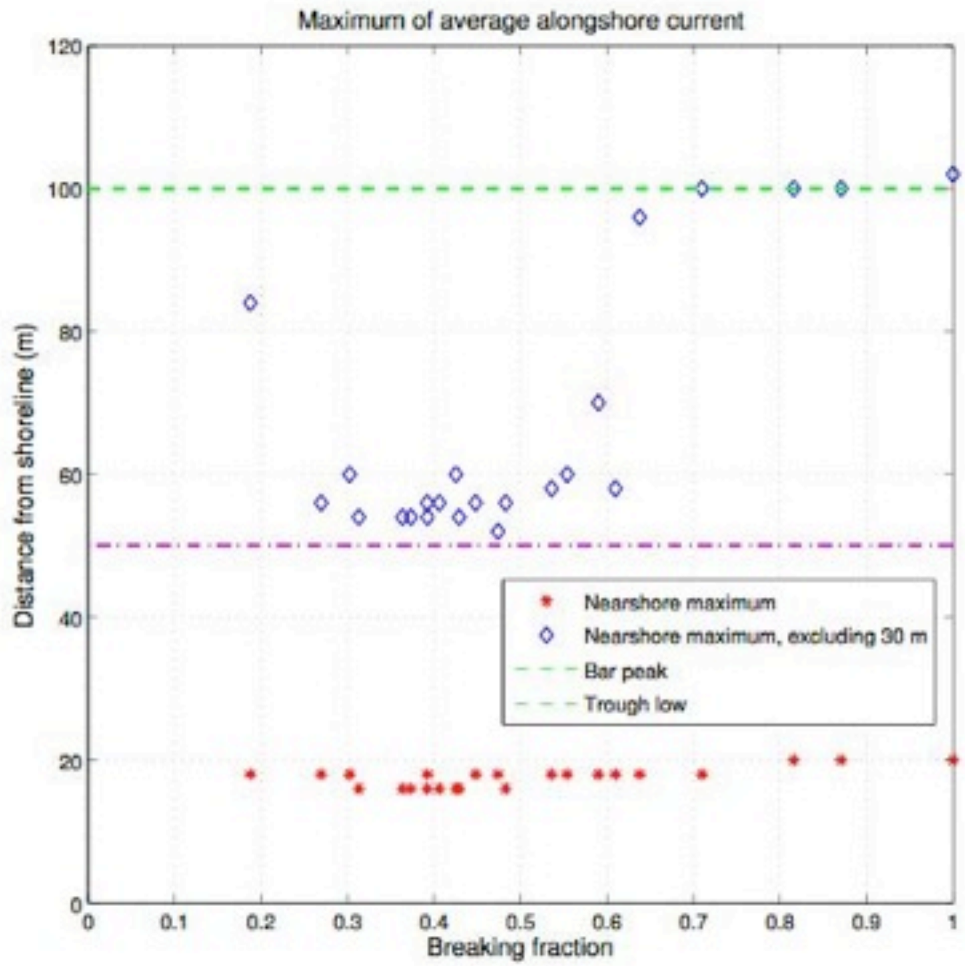
Idealized experiment: sinusoidal forcing

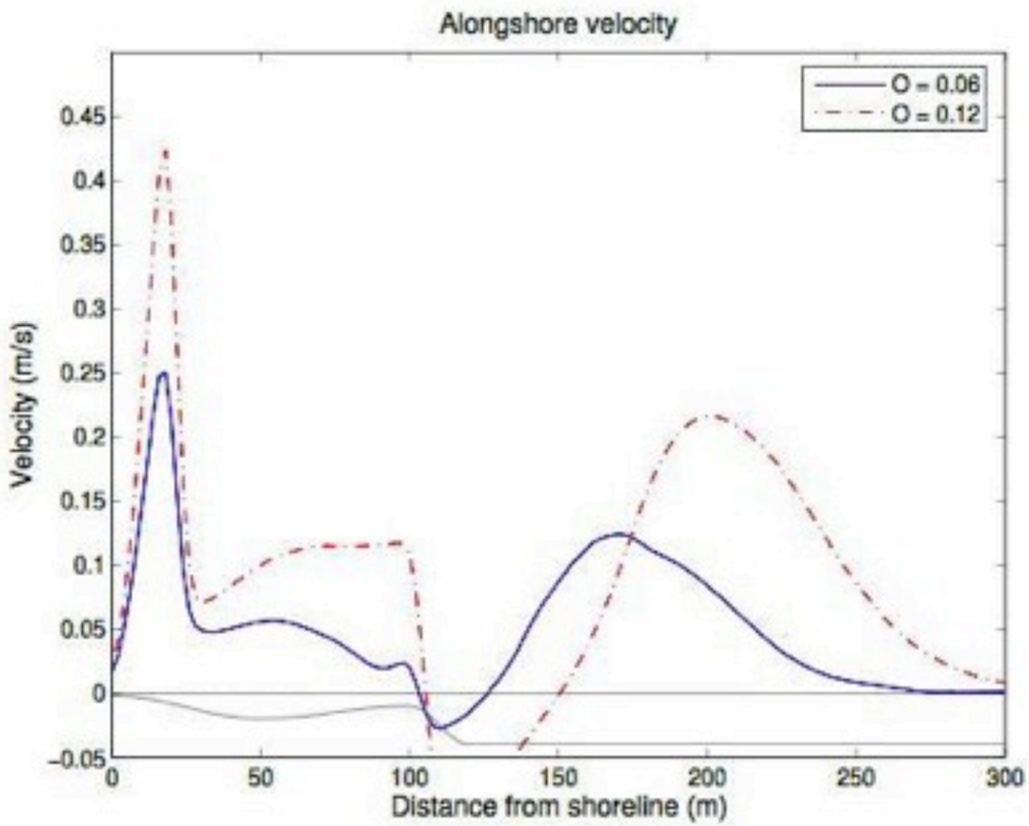
- motivated by Reniers et al.(2002,2004)
- directional spreading and/or frequency spreading cause “groupiness”
- Alongshore and time variation on the order of 100 m

Vortex dipole: sinusoidal forcing



Current maximum vs. f_b




Alongshore velocity
O = 0.06
O = 0.12
Velocity (m/s)
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
−0.05
0
50
100
150
200
250
300
Distance from shoreline (m)