Optical (Modal, differential) Gain

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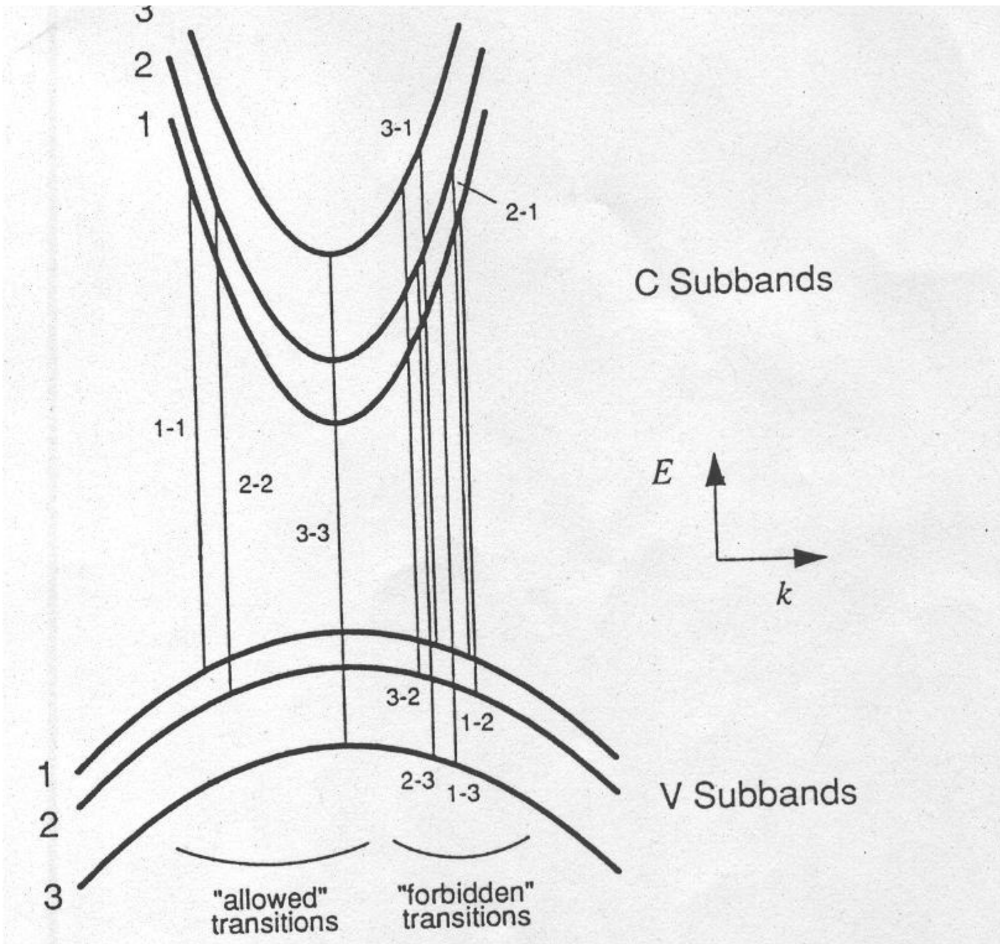
Definition of Gain

- Optical Gain: In terms of the difference between the stimulated emission and absorption rates.
- Modal Gain: which is the material gain adjusted to take into account the poor overlap that always exists between the optical mode and the electron envelope function in the quantum well.

  (I.e: modal gain=material gain* confinement factor)
- Differential gain: The rate at which gain increases as we inject more carriers, dg/dN.

Transitions between the conduction and valence subbands of a quantum well.



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- All transition are drawn with equal transition energy and equal in-plan k vector.
- The allowed transitions have strong transition probabilities.
- The forbidden transitions have zero transition probability in an infinite barrier quantum well and weak probability at best in a finite barrier quantum well.

Fermi's Golden Rule---the transition rate $W_{e\to h}$

$$W_{e\to h} = \frac{2\pi}{\hbar}\left|H'_{eh}\right|^2 \delta(E_e - E_h - \hbar\omega)$$

(1)

Where:

$$H'_{eh} \equiv \left\langle \psi_h \middle| H'(r) \middle| \psi_e \right\rangle = \int_v \psi_h^* H'(r)\psi_e d^3 r$$

(2)

$$H'(r) = \frac{e}{2m_0} A(r)\hat{e}\cdot P$$

(3)

- $H'(r)$ is the time-dependent perturbation to the original Hamiltonian, it is to induce electronic transitions between the conduction and valence bands.
- $E_{e,h}$ are the initial and final energy of the electron.

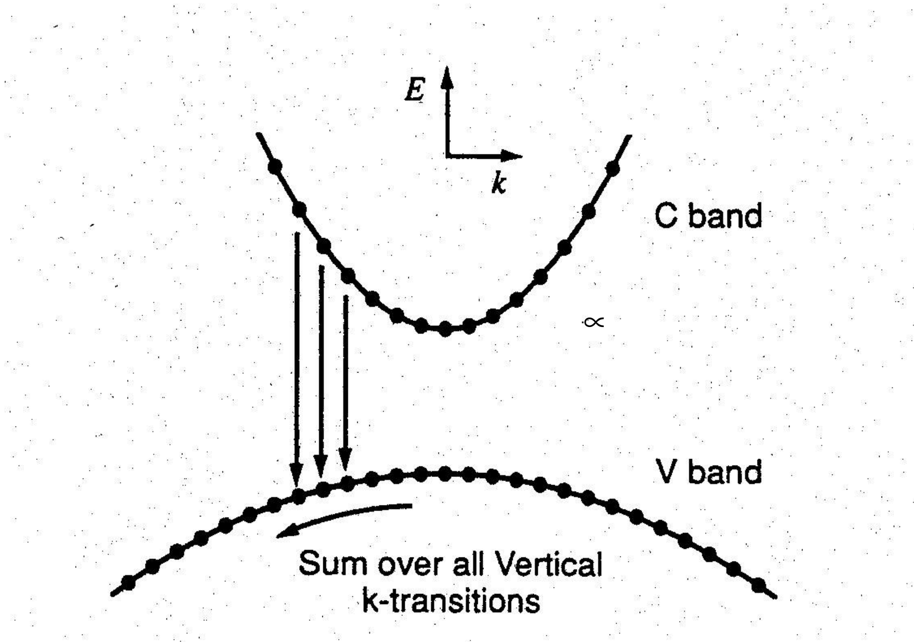
Some description about the Fermi's Golden Rule

- Optical gain in semiconductor is caused by photon-induced transitions of electrons from the conduction band to the valence band.
- Fermi's Golden Rule characterizes electron-photon interactions in the crystal. It gives the transition rate for a single pair of conduction and valence band states.
- Fermi's Golden Rule assumes the electron initially occupies a single state which makes a transition to one of a large number of final states.

Cont'

- Fermi's Golden Rule is applicable to many systems where interaction with photons is of concerns.
- The delta function indicates that the difference between the initial and final energy $(E_e - E_h)$ of the electron must be equal to the energy $\hbar\omega$ of the photon that induced the transition.
- The use of the delta function here implicitly assumes that $W_{e \to h}$ refers to a single transition rate within a continuum of states

The total transition rates



$$W_{c\to v} = \frac{1}{v}\int W_{e\to h} dN_s = \int W_{e\to h} \frac{1}{v}\frac{dN_s}{dk} dk$$

$$= \frac{2\pi}{\hbar}\int \left|H'_{eh}\right|^2 \delta(E_e - E_h - \hbar\omega)\rho(k)dk \qquad (4)$$

Cont'

The downward and upward transition rates are:

$$W_{c\to v} = \frac{2\pi}{\hbar}\left|H'_{eh}\right|^2 \rho_{red} f_c(1-f_v) \qquad (4)$$

$$W_{v\to c} = \frac{2\pi}{\hbar}\left|H'_{eh}\right|^2 \rho_{red} f_v(1-f_c) \qquad (5)$$

$where:$

$$\left|H'_{eh}\right|^2 = \left(\frac{eA_0}{2m_0}\right)^2 \left|M_T\right|^2 \qquad (6)$$

- f_c *and* f_v : the Fermi distribution.
- $|M_T|$: the transition matrix element.
- ρ_{red} : the reduced density of states.
- A_0: the vector potential can be taken as a constant.

Some notes about optical Gain.

- Each downward transition generates a new photon, while upward absorbs one.
- If the number of downward transition per seconds exceeds the number of upward transition, there will be a net generation of photons, and optical gain can be achieved.
- Optical gain in the material is attained when we inject a carrier density beyond N_{tr} such that the quasi-Fermi levels are separated by an energy greater than the band gap.

The simple formula for optical gain.

The optical gain:

$$g \equiv \frac{1}{\Phi}\frac{d\Phi}{dz} \qquad (7)$$

Where Φ is the photon flux (the number of photons per cross section area unit in the unit of time) and z is the direction of the electromagnetic field propagation,

And:

$$\frac{d\Phi}{dz} = W_{c\rightarrow v} - W_{v\rightarrow c} \qquad (8)$$

Expanding the gain formula

The Photon flux:

$$\Phi(\omega) = \frac{1}{\hbar\omega}\left(\frac{c}{\overline{n_g}}\right)\left(\frac{1}{2}\bar{n}^{2}\varepsilon_0\omega^2 A_0^2\right) \qquad (9)$$

$$\overline{n_g} = \overline{n_{eff}} + \omega(d\overline{n_{eff}}/d\omega) \qquad (10)$$

Where:

$\bar{n}$:The index of refraction in the crystal.

$\overline{n_g}$:The group index of refraction.

$\overline{n_{eff}}$:The effective index of the guided mode

Cont'

Replace (4),(5),(6),(8),(9) into (7):

$$g(\hbar\omega) = \left(\frac{1}{\hbar\omega}\right)\frac{\pi e^2 \hbar \overline{n_g}}{\varepsilon_0 c m_0^{\,2} \overline{n}^{2}} \left|M_T\right|^2$$

$$\rho_{red}(E_{eh} - E_g^{'})(f_c - f_v)$$

Total Gain

- The total gain is found by summing over all subband transition pairs.:

$$g(\hbar\omega) = \sum_{n_c} \sum_{n_v} g_{sub}(\hbar\omega, n_c, n_v)$$

- Where: n_c, n_v are the quantum numbers in the conduction and valence subbands.
- Note: Each subband transition will have its own set of envelope function and subband gap.

Results from the gain formula

- The optical gain experienced by an incoming photon is very much dependent on the photon's energy.
- When $f_c(E_e) > f_v(E_h)$, $g(\hbar\omega)$ is positive, and an incoming light wave with photon energy $\hbar\omega$ will be amplified by the material.
- The requirement for gain at a photon energy is:

  $$E_g < \hbar\omega < E_{fc} - E_{fv}$$

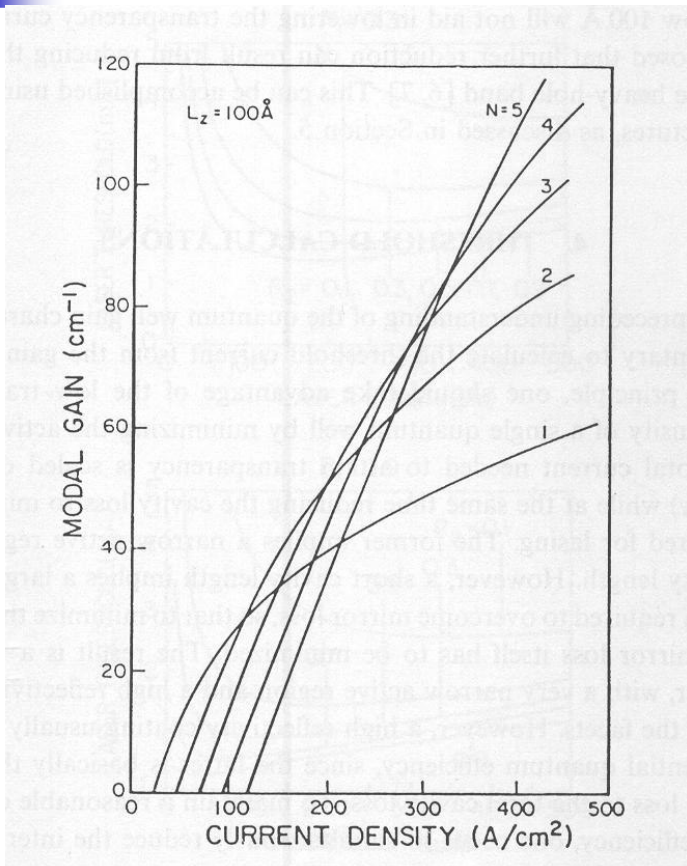
  $E_{fc,fv}$ are the nonequilibrium quasi-Fermi levels in the conduction and valence bands..

Cont'

- The quasi-Fermi level separation must be greater than the bandgap to achieve optical gain in the material.
- Under equilibrium conditions, $E_{fc} = E_{fv}$,

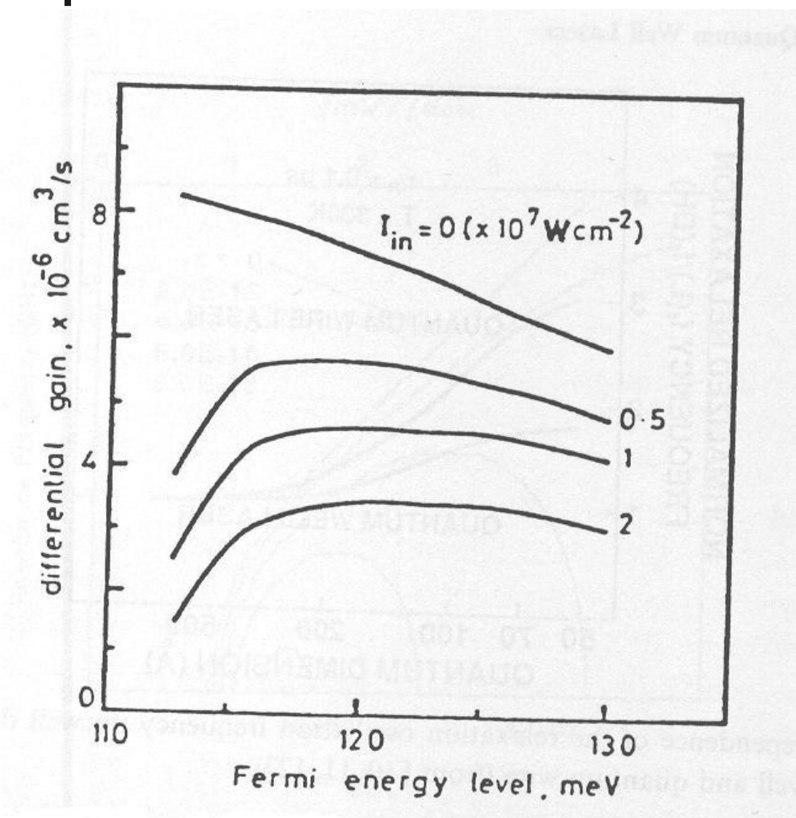
and optical gain is impossible to achieve.

Modal gain



- Mode gain is expressed in terms of the gain coefficient and the gain confinement factor.
- Multiple quantum wells have higher optical gain.

Differential gain



- The differential gain $G^{'}$: dg/dN.
- The differential gain is reduced as the optical intensity is increased.

Effects of differential gain

- $\omega_r \propto \sqrt{G'} \Rightarrow$ high differential gain should lead to high modulation bandwidth.
- The Antiguiding factor or linewidth enhancement factor $\alpha \propto G'^{-1} \Rightarrow$ high differential gain should lead to low frequency chirp (α parameter) and narrow linewidth capabilities

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