

Chapter 23. Nuclear Chemistry

What we will learn:

- Nature of nuclear reactions
- Nuclear stability
- Nuclear radioactivity
- Nuclear transmutation
- Nuclear fission
- Nuclear fusion
- Uses of isotopes
- Biological effects of radiation

Nuclear Reactions

Reactions involving changes in nucleus

Particle	Symbol	Mass		Charge	
		kg	amu	Coulombs	Electron Unit
Proton	${}^1_1\text{H}$ (${}^1_1\text{p}$)	1.67252×10^{-27}	1.007276	1.6022×10^{-19}	1
Neutron	${}^1_0\text{n}$	1.67496×10^{-27}	1.008665	0	0
Electron	${}^0_{-1}\text{e}$ (${}^0_{-1}\beta$)	9.1095×10^{-31}	0.000549	-1.6022×10^{-19}	-1
Positron	${}^0_1\text{e}$ (${}^0_1\beta$)	9.1095×10^{-31}	0.000549	1.6022×10^{-19}	1
α particle	${}^4_2\text{He}$ (${}^4_2\alpha$)	6.64465×10^{-27}	4.001506	3.2044×10^{-19}	2

Atomic mass unit (amu)

It is defined as one twelfth of the rest mass of an unbound neutral atom of carbon-12 in its nuclear and electronic ground state, and has a value of $1.660538921 \times 10^{-27}$ kg

Analogy with elements

Atomic number (Z)

Number of protons in nucleus = number of electrons in a neutral atom

Mass number (A)

Sum of number of protons plus number of neutrons in nucleus



Example

${}^1_1 H$ hydrogen atom having 1 proton in the nucleus

${}^{12}_6 C$ carbon atom having 6 protons and 6 neutrons in the nucleus

Nucleon

Nucleon is one of the particles that makes up the atomic nucleus

Element

A form of matter in which all of the atoms have the same atomic number

Isotopes

Two atoms of the same element with different mass numbers

They have the same number of protons but a different number of neutrons

Examples of isotopes

$^{14}_6\text{C}$ carbon with 6 protons and 8 neutrons

$^{12}_6\text{C}$ carbon with 6 protons and 6 neutrons

$^{238}_{92}\text{U}$ uranium with 92 protons and 146 neutrons

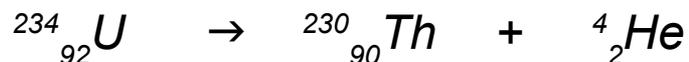
$^{235}_{92}\text{U}$ uranium with 92 protons and 143 neutrons

Examples of H isotopes

Name	Protons	Neutrons	Half-life time	Mass (amu)
${}^1_1\text{H}$	1	0	stable	1.00782
${}^2_1\text{H}$	1	1	stable	2.01410
${}^3_1\text{H}$	1	2	12.32(2) years	3.01604
${}^4_1\text{H}$	1	3	$1.39(10)\times 10^{-22}$ s	4.02781
${}^5_1\text{H}$	1	4	9.1×10^{-22} s	5.03531
${}^6_1\text{H}$	1	5	$2.90(70)\times 10^{-22}$ s	6.04494
${}^7_1\text{H}$	1	6	$2.3(6)\times 10^{-23}$ s	7.05275

Balancing nuclear equations

Total of Z and A must be same for products as for reactants
U(uranium) Th(thorium)

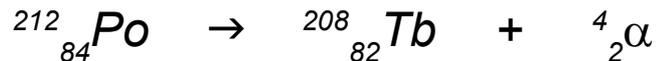


Problem

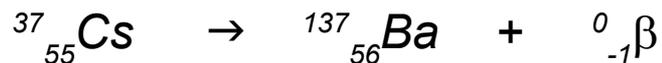
Balance the following nuclear equations Po(polonium) Tb(terbium)



The mass difference is $212 - 84 = 4$, it means X is α particle



The mass difference is 0, it means X is β particle with an atomic number -1



Comparison of chemical and nuclear reactions

Chemical reaction	Nuclear reaction
Atoms are rearranged by the breaking and forming chemical bonds	Elements are converted from one to another
Only electrons in atomic or molecular orbitals are involved	Protons, neutrons, electrons and other particles may be involved
Relative small energy is formed or consumed	A very big amount of energy is formed
Rates of reactions are influenced by temperature, pressure, or concentrations	Rates of reactions are generally not affected

Nuclear density

The calculation of a nucleous density

Radius of a nucleus

$$r = 5 \times 10^{-13} \text{ cm}$$

Volume of a nucleus

$$V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (5 \times 10^{-13} \text{ cm})^3$$

Mass of a nucleus $1 \times 10^{-22} \text{ g}$ (30 protons and 30 neutrons)

$$d = m/V$$

$$= 2 \times 10^{14} \text{ g/cm}^3 \quad (\text{this is a very big density})$$

The highest density of an element is 22.6 g/cm^3

Nuclear stability

General facts and rules

- Neutrons stabilize the nucleus
- More protons require more neutrons to stabilize the nucleus
- Up to $Z = 20$, most stable nuclei have (neutron/proton) ratio = 1
- larger $Z \rightarrow$ larger n/p ratio (2.5 for Bi-209)
- All isotopes with $Z > 83$ are radioactive - undergo spontaneous decay to give a stable isotope
- Magic numbers - 2, 8, 20, 50, 82, 126 protons or neutrons
- Even number of p and n - generally more stable
- Belt of stability nuclei outside are radioactive

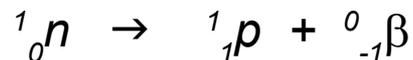
Belt of stability

The stable nuclei are located in an area of the belt

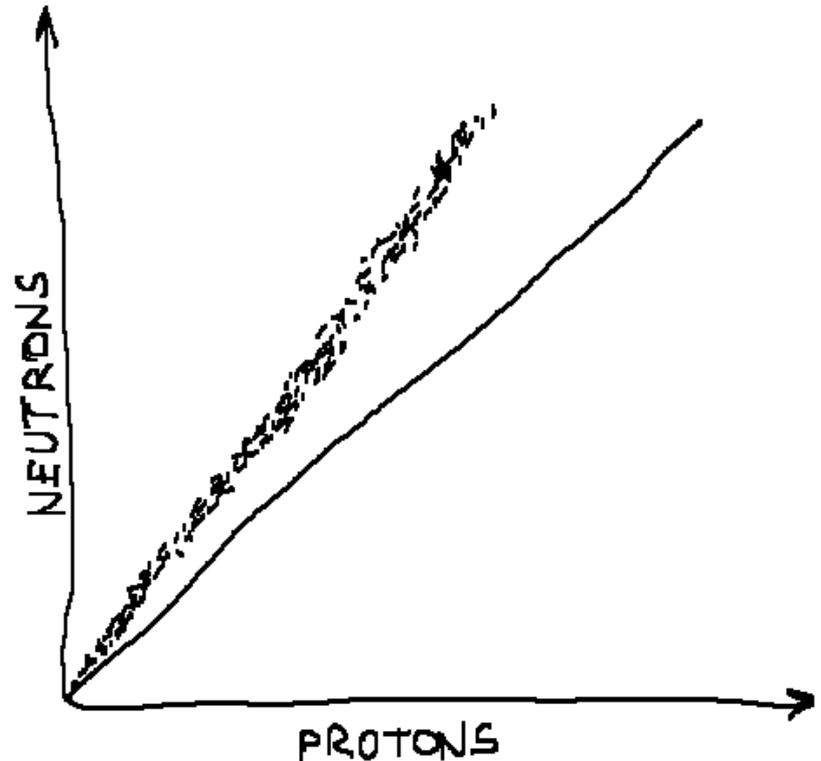
The most radioactive nuclei are outside this belt

The ratio of $(n/p) = 1$ stability

When a nucleus is above the belt and it has the (n/p) ration bigger than 1 it undergous the following process (called $-\beta$ -particle emission)



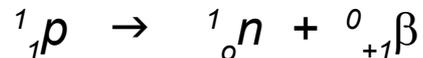
which increases the number of protons in the nucleus making the ratio (n/p) closer to 1



Examples



When a nucleus is below the belt and it has the (n/p) ratio smaller than 1 it undergoes the following process (called + β -particle emission)



or undergoes electron capture, which decreases the number of protons, and hence moves up toward the belt of stability

Examples



Nuclear binding energy

- Energy required to break up a nucleus into protons and neutrons
- Mass defect - difference in mass of an atom and the sum of the masses of its protons, neutrons, and electrons
- $E = (\Delta m)c^2$

m = mass difference (kg)

c = speed of light (3×10^8 m/s)

E = energy liberated (Joules)

Mass defect

Difference in mass of an atom and the sum of the masses of its protons, neutrons, and electrons

Example

$^{19}_9\text{F}$ isotope has an atomic mass 18.9984 amu. Mass of ^1_1H atom (1.007825 amu), mass of the neutron (1.008665 amu).

The mass of 9 ^1_1H atoms (9 protons and 9 electrons) is

$$9 \times 1.007825 \text{ amu} = 9.070425 \text{ amu}$$

and the mass of 10 neutrons is

$$10 \times 1.008665 \text{ amu} = 10.08665 \text{ amu}$$

Therefore, the atomic mass of a $^{19}_9\text{F}$ calculated from known numbers of electrons, protons and neutrons is

$$9.070425 \text{ amu} + 10.08665 \text{ amu} = 19.15705 \text{ amu}$$

which is larger than 18.9984 amu (the measured mass of $^{19}_9\text{F}$) by 0.1587 amu.

The difference between the mass of an atom and the sum of masses of individual particles is called mass defect.

Based on the mass-energy equivalence relationship we can calculate the amount of energy

$$\Delta E = (\Delta m)c^2$$

$$\Delta E = \text{energy of the product} - \text{energy of the reactant}$$

$$\Delta m = \text{mass of the product} - \text{mass of the reactant}$$

$$\Delta m = -0.1587 \text{ amu}$$

$$\Delta E = (-0.1587 \text{ amu}) (3.0 \times 10^8 \text{ m/s})^2$$

$$= -1.43 \times 10^{16} \text{ amu m}^2 / \text{s}^2$$

$$1 \text{ kg} = 6.022 \times 10^{26} \text{ amu}$$

$$1 \text{ J} = 1 \text{ kg m}^2 / \text{s}^2$$

$$\Delta E = -2.37 \times 10^{-11} \text{ J}$$

This is the amount of energy released when one fluorine nucleus is formed from 9 protons and 10 neutrons. Therefore in the formation of 1 mole of fluorine nuclei this energy need to be multiplied by the Avogadro number

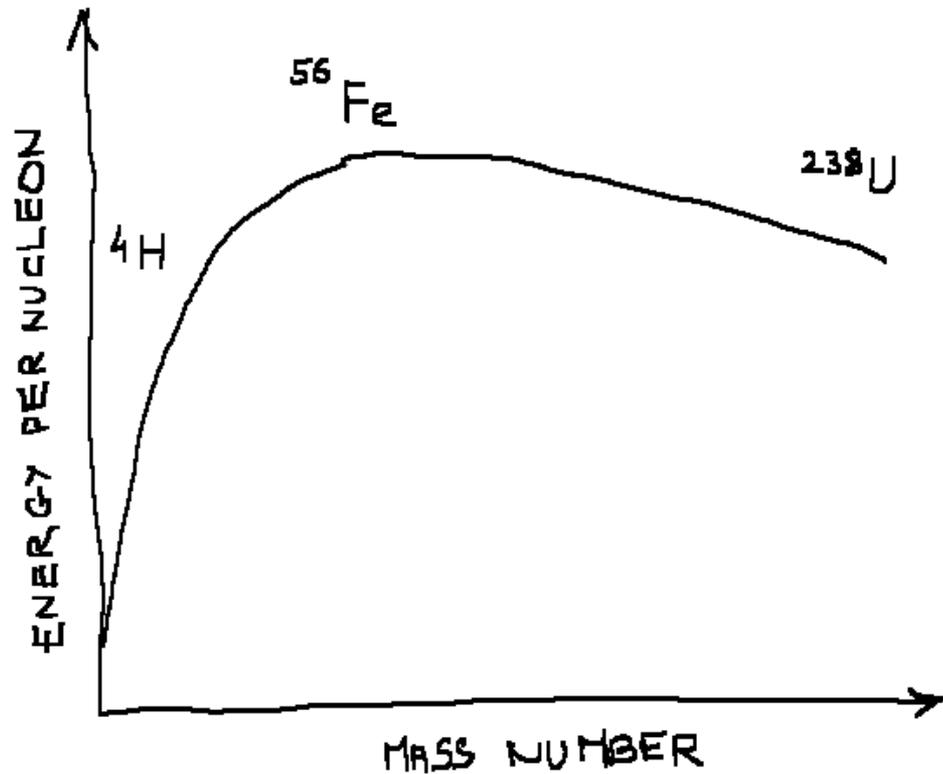
$$\Delta E = (-2.37 \times 10^{-11}) \times (6.022 \times 10^{23} / \text{mol})$$

$$\Delta E = -1.43 \times 10^{10} \text{ kJ / mol} \quad (\text{which is a very big number})$$

The average chemical reaction releases about 200 kJ / mol energy.

Nuclear binding energy per nucleon

$$= \frac{\text{nuclear binding energy}}{\text{number of nucleons}}$$



Problem

The mass of a ${}^7_3\text{Li}$ nucleus is 7.016005 amu. Given that the mass of a proton is 1.007276 amu and that of a neutron is 1.008665 amu, calculate the mass defect, the binding energy per nucleon, and the binding energy per mole

$$\text{mass of protons} = 3 (1.007276 \text{ amu}) = 3.021828 \text{ amu}$$

$$\text{mass of neutrons} = 4 (1.008665 \text{ amu}) = 4.034660 \text{ amu}$$

$$\text{Total mass} = 7.056488 \text{ amu}$$

$$\text{Actual mass } {}^7_3\text{Li nucleus} = 7.016005$$

Mass defect

$$7.056488 - 7.016005 = 0.040483 \text{ amu}$$

$$(0.040483 \text{ amu}) / (6.023 \times 10^{26} \text{ amu/kg}) = 6.721 \times 10^{-29} \text{ kg}$$

Binding energy

$$E = (\Delta m)c^2$$

$$- (6.721 \times 10^{-29} \text{ kg}) (2.998 \times 10^8 \text{ m/s})^2 = 6.04 \times 10^{-12} \text{ J/nucleus}$$

$$-(6.04 \times 10^{-12} \text{ J}) / (\text{nucleus}) \times (\text{nucleus}) / (7 \text{ nucleons}) = 8.628 \times 10^{-13} \text{ J/nucleon}$$

Binding energy per mole

$$\begin{aligned} & - (6.04 \times 10^{-12} \text{ J/nucleon}) \times 6.02 \times 10^{23} \text{ nuclei/mole} \\ & = - 3.64 \times 10^{12} \text{ J/mol} \end{aligned}$$

Problem

The mass of a $^{127}_{53}\text{I}$ nucleus is 126.9004 amu. Given that the mass of a proton is 1.007276 amu and that of a neutron is 1.008665 amu, calculate the mass defect, and the binding energy per nucleon

There are 53 protons and 74 neutrons the mass of 53 ^1_1H atoms is

$$53 \times 1.007276 \text{ amu} = 53.41473 \text{ amu}$$

The mass of 74 neutrons is

$$74 \times 1.008665 \text{ amu} = 74.64121 \text{ amu}$$

Therefore the predicted mass for $^{127}_{53}\text{I}$ is $53.41473 + 74.64121 = 128.05594$ amu

The mass defect

$$\Delta m = 126.9004 - 128.05594$$

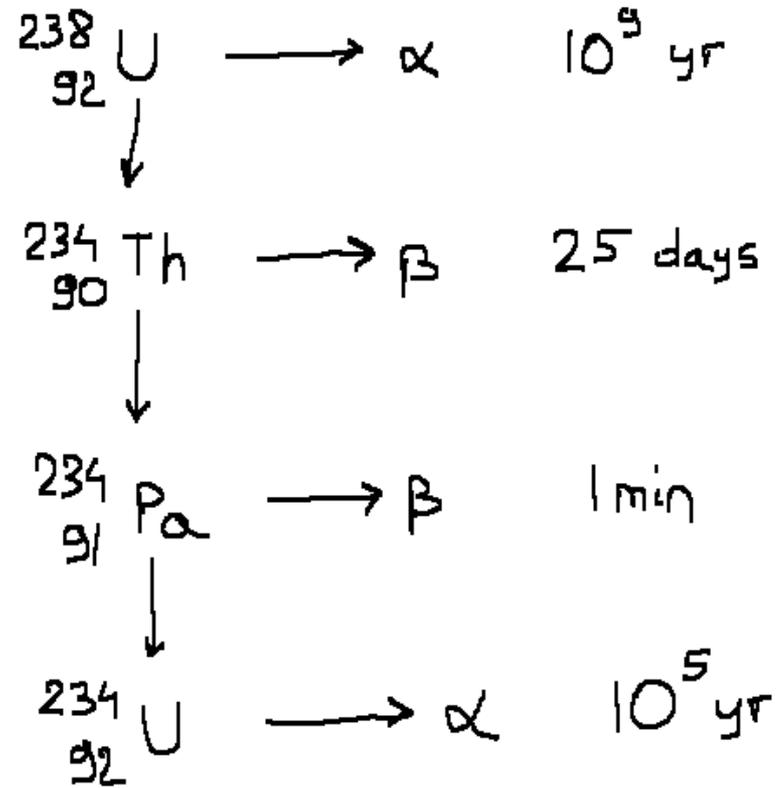
$$= -1.1555 \text{ amu}$$

$$\begin{aligned}\Delta E &= (\Delta m)c^2 \\ &= (-1.1555 \text{ amu}) (3.0 \times 10^8 \text{ m/s})^2 \\ &= -1.04 \times 10^{17} \text{ amu m}^2 / \text{s}^2 \\ &= -1.73 \times 10^{-10} \text{ J}\end{aligned}$$

Thus the nuclear binding energy is $-1.73 \times 10^{-10} \text{ J}$. The nuclear binding energy per nucleon is

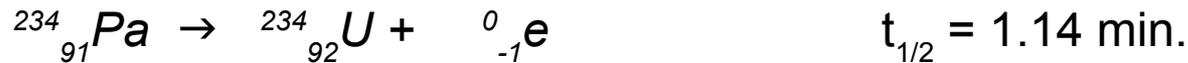
$$= (-1.73 \times 10^{-10} \text{ J}) / (127 \text{ nucleons}) = 1.36 \times 10^{-12} \text{ J/nucleon}$$

The decay series of naturally occurring uranium 238 which involves 14 steps



Naturally occurring isotopes

- U(Uranium), Th (Thorium), Pa(Protactinium), Pb(Lead)
- Uranium Series



Net decay



Half-life and rates of decay

All radioactive decays obey first-order kinetics

$$\text{rate of decay} = \lambda N$$

λ - the first order rate constant

N - the number of radioactive nuclei present at time t

$$\ln(N_t / N_0) = -\lambda t$$

The corresponding half-time of the reaction is

$$t_{1/2} = 0.693 / \lambda$$

The value of $t_{1/2}$ depends on a particular nucleus



Radioactive dating

The half-life of radioactive isotopes have been used as “atomic clock” to determine the ages of certain objects

Carbon dating $^{14}_6\text{C}$

$^{14}_6\text{C}$ is produced in atmosphere and this isotope decays according to the equation



CO_2 in living plants has fixed amount of $^{14}_6\text{C}$ taken from the atmosphere. When plant dies, $^{14}_6\text{C}$ decay begins, and the concentration of this isotope decreases with time. If we know a current concentration of $^{14}_6\text{C}$ in a dead plant with the concentration of this isotope in a living plant, we can use the formula

$$\ln(N_t / N_0) = -\lambda t$$

to estimate the time of plant dead

Dating using $^{238}_{92}\text{U}$

$^{238}_{92}\text{U}$ is a naturally occurring mineral which is used to estimate a time of rocks in the earth. The decay of this isotope is very long



The time is estimated from the ratio of concentrations of $^{238}_{92}\text{U}$ and $^{206}_{82}\text{Pb}$ in a sample.

If only half of a mole of uranium undergone decay, the mass ratio of

$$^{206}_{82}\text{Pb} / ^{238}_{92}\text{U} = 206 \text{ g} / 238 \text{ g} = 0.866$$

Ratios lower than 0.866 mean that the rocks are less than 4.51×10^9 year old.

Dating using $^{40}_{19}\text{K}$

Dating using $^{40}_{19}\text{K}$ is based on the reaction of electron capture



The concentration of gaseous argon is used to measure the age of an object. This technique is used in geochemistry. A mineral is melted and the concentration of argon is measured using a mass spectrometer.

Nuclear transmutations

Bombardment of nuclei in particle accelerator - may form new isotopes

Historically, the first reaction of nuclear transmutation was performed by Rutherford in 1919



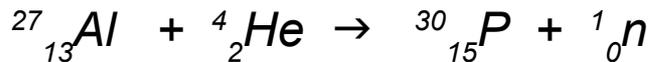
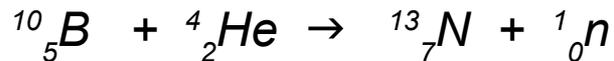
where oxygen-17 was produced with the emission of a proton. This reaction opened a possibility for making new elements

Problem

Write the balanced equation for the nuclear reaction ${}^{56}_{26}\text{Fe}(d,\alpha){}^{54}_{25}\text{Mn}$, where the d symbol represents the deuterium nucleus ${}^2_1\text{H}$



Nucleosynthesis



Transuranium elements

Elements with atomic numbers greater than 92, made in particle accelerators

Z	Symbol	Synthetic Method	Half-life
93	Np	${}^{238}_{92}\text{U} + {}^1_0\text{n} \rightarrow {}^{239}_{93}\text{Np} + {}^0_{-1}\text{e}$	2.35 da
94	Pu	${}^{239}_{93}\text{Np} \rightarrow {}^{239}_{94}\text{Pu} + {}^0_{-1}\text{e}$	86.4 yr
95	Am	${}^{239}_{94}\text{Pu} \rightarrow {}^1_0\text{n} \rightarrow {}^{240}_{95}\text{Am} + {}^0_{-1}\text{e}$	458 yr
96	Cm	${}^{239}_{94}\text{Pu} + {}^4_2\text{He} \rightarrow {}^{242}_{96}\text{Cm} + {}^1_0\text{n}$	4.5 hr
99	Md	${}^{253}_{99}\text{Es} + {}^4_2\text{He} \rightarrow {}^{256}_{101}\text{Md} + {}^1_0\text{n}$	1.5 hr

Nuclear Fission

Process in which heavy nuclei are split into smaller nuclei and neutrons



The heavy nucleus is less stable than its products, and this process releases a large amount of energy

	Nuclear binding energy
${}_{92}^{238}\text{U}$	$2.82 \times 10^{-10} \text{ J}$
${}_{38}^{90}\text{Sr}$	$1.23 \times 10^{-10} \text{ J}$
${}_{54}^{143}\text{Xe}$	$1.92 \times 10^{-10} \text{ J}$

The difference in binding energy of the reactants and products per nucleus

$$= (1.23 \times 10^{-10} + 1.92 \times 10^{-10})\text{J} - (2.82 \times 10^{-10})\text{J} = 3.3 \times 10^{-11} \text{ J}$$

The difference in binding energy of the reactants and products for 1 mole

$$= (3.3 \times 10^{-11})\text{J} \times (6.02 \times 10^{23}) / \text{mol} = 2.0 \times 10^{13} \text{ J/mol}$$

For comparison, 1 ton of coal is only 5×10^7 J

During this reaction neutrons are produced and next are originally captured in the process making the reaction self-sustaining

Chain reaction

Self-sustaining sequence of nuclear fission reactions

Critical mass

Minimum amount of fissionable material needed for chain reaction

Atomic Bomb

The first application of nuclear fission was in development of the atomic bomb.

A small atomic bomb is equivalent to 20,000 ton of TNT (trinitrotoluene)

1 ton TNT releases 4×10^9 J of energy

20,000 ton TNT releases 8×10^{13} J of energy

1 mole (235 g) of uranium-235 releases 2×10^{13} J of energy, therefore the mass of the isotope present in a small atomic bomb is

$$(235 \text{ g}) (8 \times 10^{13} \text{ J}) / (2 \times 10^{13} \text{ J}) = 1 \text{ kg}$$

The critical mass if formed in a bomb using a conventional explosive like TNT

Uranium-235 was used in Hiroshima

Plutonium-239 was used in Nagasaki

Nuclear Reactor

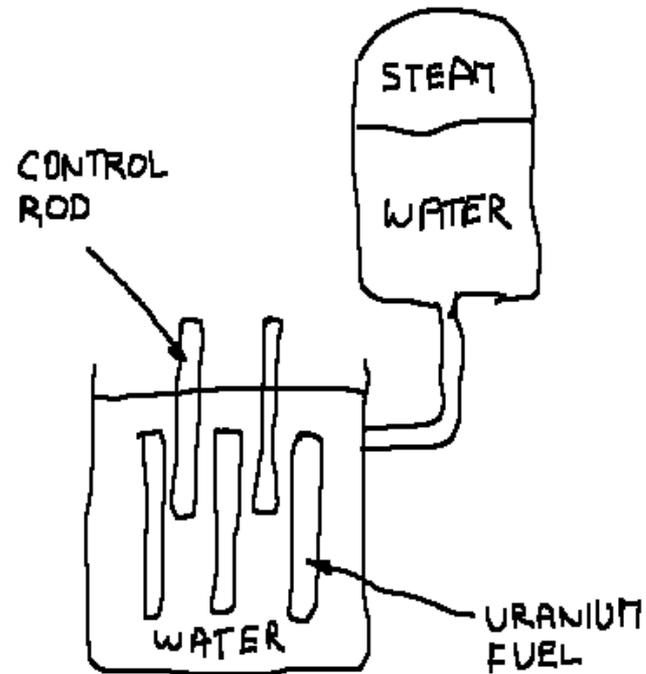
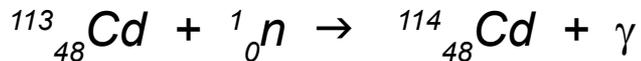
Light water reactors (H₂O)

Most of the nuclear reactors in US are *light water reactors* (H₂O)

Water is used to transport heat of the nuclear reaction outside the reaction and to generate electricity

Uranium-235 is used as a reactor fuel

The nuclear reaction is controlled by cadmium or boron rods which capture neutrons



Heavy water reactors (D₂O)

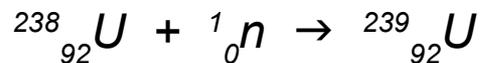
Heavy water absorbs neutrons less efficiently than light water, and therefore heavy water reactors do not require enriched uranium, which is needed in light water reactors

Heavy water is produced from light water by fractional distillation or electrolysis of ordinary water which includes of very small amount of heavy water.

Canada is currently the only nation producing heavy water for nuclear reactors

Breeder reactors

The reactor using uranium fuel, from which it produces more fissionable materials than it uses



Therefore nonfissionable uranium-238 is converted into fissionable plutonium-239

Nuclear Fusion

Combining of small nuclei into large ones

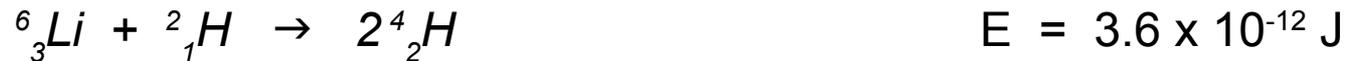
Produces even more energy than fission

Nuclear fusion occurs constantly in the sun. The sun is made up mostly of hydrogen and helium. At the temperature 15 million C the following reactions are believed to take place



Fusion reactors

The fusion reaction requires a very high temperature to proceed



Those reactions require a temperature of about 100 million C. At this temperature molecules can not exist losing electrons, and forming a new state of matter which is called plasma

Plasma

The state of matter formed at very high temperature (million of degrees Celsius) from a gaseous mixture of positive ions and electrons

Technically plasma can be formed inside a very strong magnetic field keeping the particles together

Hydrogen bomb

Thermonuclear bomb containing solid lithium deuteride (LiD) which can be packed very tightly

The detonation of a hydrogen bomb occurs in two stages

- fission reaction
- fusion reaction

The required high temperature for fusion reaction is achieved from an atomic bomb



There is no critical mass of the fusion bomb

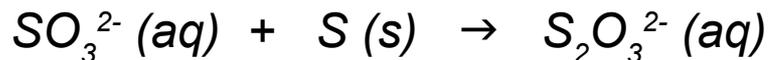
Uses of isotopes

Structure determinations (helps locate atoms via radioactive emission)

Mechanism studies (trace steps via radioactivity)

Example

The formula of thiosulfate ion is $S_2O_3^{2-}$. The ion is prepared from the reaction

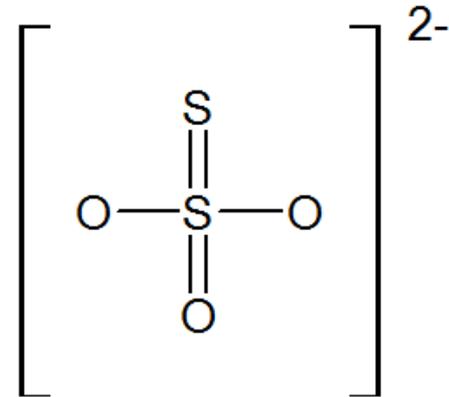
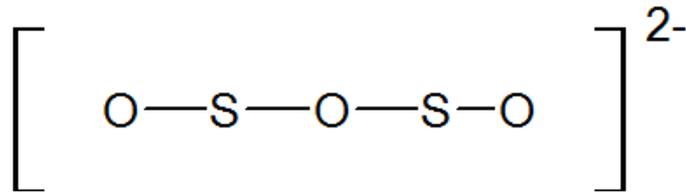


When the ion is treated with an acid, the reaction is reversed



and the elemental sulfur is precipitated. Chemists were uncertain whether the two sulfur atoms occupied equivalence position in the ion

Two possible structures were proposed



The first reaction started with elemental sulfur enriched with the radioactive sulfur-35. After precipitation the label sulfur is found in the solid elemental sulfur, and not in the solution, which indicates that nonlinear structure is valid, where sulfur atoms occupy non equivalent positions

Medicine

- Diagnosis (measure radioactivity)
 - Sodium-24 - traces blood flow
 - Iodine-125 goes to thyroid and gives way to view activity
 - Technetium-99 - traces organs such as heart
- Therapy (destroying tumor sites)
 - Cobalt-60 - γ radiation directed at tumor sites
 - Iodine-131 destroys overactive thyroid

Biological effects of radiation

The fundamental unit of radioactivity is *curie* (Ci)

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ nuclear disintegrations (results of radioactive decay) per second}$$

This decay rate is equivalence to that of 1 g of radium. A millicurie (mCi) is one-thousand of a curie.

Example

$$10 \text{ mCi (carbon-14)} = (10 \times 10^{-3}) (3.7 \times 10^{10}) = 3.7 \times 10^8$$

disintegrations per second

The unit of the absorbed dose of radiation is the *rad* (radiation absorbed dose)

$$1 \text{ rad} = \text{amount of radiation that results in the absorption of } 1 \times 10^{-2} \text{ J energy per kilogram of irradiated material}$$

The biological effect of radiation also depends on the part of the body and the type of radiation

RBE - relative biological effectiveness

= 1 for beta and gamma

= 10 for alpha

Generally

α particles usually have the least penetration power

β particles usually have stronger penetration power

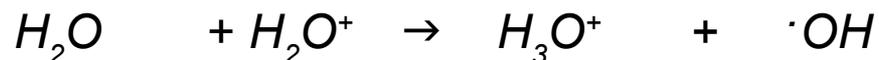
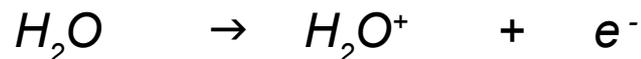
γ rays usually have the strongest penetration power

Radiation can remove electrons from atom and molecules forming ions and radicals

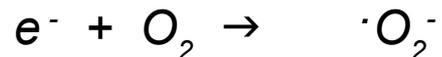
Radicals

A molecular fragment having one or more unpaired electrons

These are short-lived species and they are very reactive



The electron can also react with molecular oxygen



forming the superoxide ion $\cdot O_2^-$ which reacts with organic compounds of enzyme and DNA molecules, leading to cancer

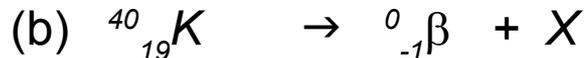
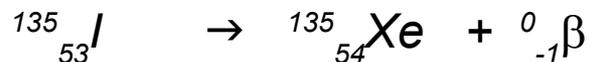
Problem

Complete the following nuclear equations and identify X



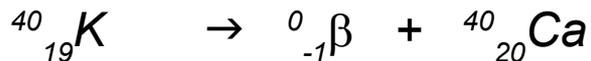
The sum of masses must be conserved, $A = 0$

The atomic number must be conserved, $Z = -1$



The sum of masses must be conserved, $A = 40$

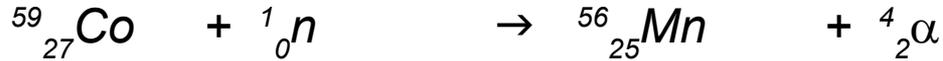
The atomic number must be conserved, $Z = 20$





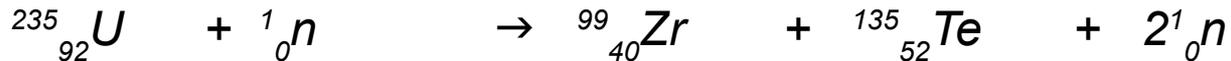
The sum of masses must be conserved, $A = 4$

The atomic number must be conserved, $Z = 2$



The sum of masses must be conserved, $A = 4$

The atomic number must be conserved, $Z = 0$



Problem

Estimations show that the total energy output of the sun is 5×10^{26} J/s. What is the corresponding mass loss in kg of the sun.

We use the $\Delta E = (\Delta m)c^2$ equation

$$1 \text{ J} = 1 \text{ kg m}^2 / \text{s}^2$$

$$\Delta m = \Delta E / c^2 = (5 \times 10^{26} \text{ kg m}^2 / \text{s}^2) / (3 \times 10^8 \text{ m/s})^2$$

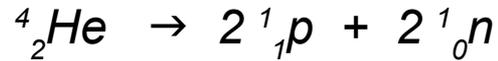
$$\Delta m = 6 \times 10^9 \text{ kg}$$

Problem

Calculate the nuclear binding energy (in J) and the binding energy per nucleon of the following isotopes

a) ${}^4_2\text{He}$ (4.0026 amu)

The binding energy is the energy required for the process



There are two protons and 2 neutrons in the helium nucleus. The mass of 2 protons is

$$2 (1.007825 \text{ amu}) = 2.015650 \text{ amu}$$

and the mass of 2 neutrons

$$2 (1.008665 \text{ amu}) = 2.017330 \text{ amu}$$

Therefore the predicted mass of ${}^4_2\text{He}$ is

$$m({}^4_2\text{He}) = 2.015650 + 2.017330 \text{ amu} = 4.032980 \text{ amu}$$

The mass defect is

$$\Delta m = 4.032980 - 4.0026 = 0.0304 \text{ amu}$$

The energy change is

$$\begin{aligned}\Delta E &= (\Delta m)c^2 \\ &= (0.0304 \text{ amu}) (3 \times 10^8 \text{ m/s})^2 \\ &= 2.74 \times 10^{15} \text{ amu m}^2 / \text{s}^2\end{aligned}$$

Conversion the energy into J

$$\begin{aligned}1 \text{ J} &= 1 \text{ kg m}^2 / \text{s}^2 \\ &= (2.74 \times 10^{15} \text{ amu m}^2 / \text{s}^2) / (6.022 \times 10^{23} \text{ amu} / \text{g}) (1 \text{ kg} / 1000 \text{ g}) \\ &= 4.55 \times 10^{-12} \text{ J}\end{aligned}$$

This is the nuclear binding energy required to break up one helium-4 nucleus into 2 protons and 2 neutrons

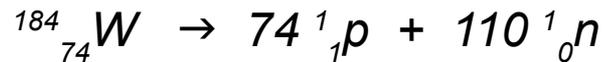
Nuclear binding energy per nucleon

$$= \frac{\text{nuclear binding energy}}{\text{number of nucleons}}$$

$$= (4.55 \times 10^{-12} \text{ J}) / (4 \text{ nucleon}) = 1.14 \times 10^{-12} \text{ J/nucleon}$$

b) ${}^{184}_{74}\text{W}$ (183.9510 amu)

The binding energy is the energy required for the process



There are 74 protons and 110 neutrons in the wolfram nucleus. The mass of 74 protons is

$$74 (1.007825 \text{ amu}) = 74.57905 \text{ amu}$$

and the mass of 110 neutrons is

$$110 (1.008665 \text{ amu}) = 110.9532 \text{ amu}$$

Therefore the predicted mass of $^{184}_{74}\text{W}$ is

$$m(^{184}_{74}\text{W}) = 74.57905 + 110.9532 \text{ amu} = 185.5323 \text{ amu}$$

The mass defect is

$$\Delta m = 185.5323 - 185.9510 = 1.5813 \text{ amu}$$

The energy change is

$$\begin{aligned}\Delta E &= (\Delta m)c^2 \\ &= (1.5813 \text{ amu}) (3 \times 10^8 \text{ m/s})^2 \\ &= 1.42 \times 10^{17} \text{ amu m}^2 / \text{s}^2\end{aligned}$$

Conversion the energy into J

$$1 \text{ J} = 1 \text{ kg m}^2 / \text{s}^2$$

$$= (1.42 \times 10^{17} \text{ amu m}^2 / \text{s}^2) / (6.022 \times 10^{23} \text{ amu} / \text{g}) (1 \text{ kg} / 1000 \text{ g})$$

$$= 2.36 \times 10^{-10} \text{ J}$$

This is the nuclear binding energy required to break up one wolfram-184 nucleus into 74 protons and 110 neutrons

Nuclear binding energy per nucleon

$$= \frac{\textit{nuclear binding energy}}{\textit{number of nucleons}}$$

$$= (2.36 \times 10^{-10} \text{ J}) / (184 \text{ nucleons}) = 1.28 \times 10^{-12} \text{ J/nucleon}$$

Problem

A freshly isolated sample of $^{90}_{39}\text{Y}$ was found to have an activity of 9.8×10^5 disintegrations per minute at 1:00 p.m. on December 3, 2003. At 2:15 p.m. on December 17 2003, its activity was redetermined and found to be 2.6×10^4 disintegrations per minute. Calculate the half-life time of $^{90}_{39}\text{Y}$.

The radioactive decay process has first-order rate law, therefore

$$t_{1/2} = 0.693 / \lambda$$

where λ is the rate constant

$$\ln(N_t / N_o) = -\lambda t$$

The time interval is:

$$\begin{aligned} (2:15 \text{ p.m. } 12/17/2003) - (1:00 \text{ p.m. } 12/3/2003) &= 14\text{d} + 1\text{hr} + 15\text{min} \\ &= 20,235 \text{ min} \end{aligned}$$

$$\ln [(2.6 \times 10^4 \text{ dis/min}) / (9.8 \times 10^5 \text{ dis/min})] = -\lambda (20,235 \text{ min})$$

$$\lambda = 1.8 \times 10^{-4} / \text{min}$$

$$t_{1/2} = 0.693 / (1.8 \times 10^{-4} \text{ min}) = 3.9 \times 10^3 \text{ min}$$

Problem

What is the activity (rate), in millicuries, of a 0.5 g sample of $^{237}_{93}\text{Np}$. The isotope decays by α -particle emission and has a half-life of 2.2×10^6 yr. Write a balanced nuclear equation for the decay of $^{237}_{93}\text{Np}$

One millicurie represents 3.7×10^7 disintegrations/s. The rate of decay of this isotope is given by the rate law

$$\text{rate} = \lambda N$$

where N is the number of atoms in the sample.

$$t_{1/2} = 0.693 / \lambda$$

$$\lambda = 0.693 / t_{1/2}$$

$$= 0.693 / (2.2 \times 10^6 \text{ yr}) (1 \text{ yr} / 365 \text{ d}) (1 \text{ d} / 24 \text{ h}) (1 \text{ h} / 3600 \text{ s})$$

$$= 9.99 \times 10^{-15} / \text{s}$$

The number of atoms (N) in 0.5 g sample of neptunium-237 is:

$$0.5 \text{ g} / (237 \text{ g}) (6.022 \times 10^{23}) = 1.27 \times 10^{21} \text{ atoms}$$

The rate of decay

$$= 1.27 \times 10^7 \text{ dis/s}$$

The activity of the sample (in millicuries) is the rate of decay / (1 millicurie)

The activity is

$$= (1.27 \times 10^7 \text{ dis/s}) (1 \text{ millicurie} / 3.7 \times 10^7 \text{ dis/s})$$

= 0.343 millicuries

(b) The decay equation is:



Problem

Strontium-90 is one of the products of fission of uranium-235. This strontium isotope is radioactive, with a half-life of 28.1 yr. Calculate how long (in yr) it will take for 1.0 g of the isotope to be reduced to 0.2 g by decay.

The radioactive decay process has a first-order rate law, therefore

$$\ln (N_t / N_o) = -\lambda t$$

where λ is the rate constant, and a half-life time is

$$t_{1/2} = 0.693 / \lambda$$

Therefore

$$\lambda = 0.693 / t_{1/2}$$

The half-life time in seconds

$$t_{1/2} = (28.1 \text{ yr}) (365 \text{ dy / yr}) (24 \text{ h / dy}) (3600 \text{ s / h}) = 8.86 \times 10^8 \text{ s}$$

$$\begin{aligned}\lambda &= 0.693 / t_{1/2} \\ &= 7.82 \times 10^{-10} / \text{s}\end{aligned}$$

$$\ln (N_t / N_o) = -\lambda t$$

$$t = -\ln (N_t / N_o) / \lambda$$

$$t = -\ln(0.2/1.0) / 7.82 \times 10^{-10} / \text{s}$$

$$t = 2.06 \times 10^9 \text{ s}$$

$$= (2.06 \times 10^9 \text{ s}) / [(365 \text{ dy} / \text{yr}) (24 \text{ h} / \text{dy}) (3600 \text{ s} / \text{h})]$$

$$= 65.2 \text{ yr}$$

Problem

Cobalt-60 is used in radiation therapy. It has a half-life time of 5.26 years.

(a) Calculate the rate constant for radioactive decay

(b) What fraction of a certain sample will remain after 12 years

The radioactive decay process has first-order rate law, therefore

$$\ln (N_t / N_o) = -\lambda t$$

where λ is the rate constant, and a half-life time is

$$t_{1/2} = 0.693 / \lambda$$

Therefore

$$\begin{aligned}\lambda &= 0.693 / t_{1/2} \\ &= 0.693 / (5.26 \text{ yr}) = 0.132 \text{ yr}^{-1}\end{aligned}$$

The fraction of a certain sample that will remain after 12 years is

$$N_t / N_o$$

where $t = 12 \text{ yr}$

Rearrange the equation

$$\ln (N_t / N_o) = -\lambda t$$

$$N_t / N_o = e^{-\lambda t}$$

$$= e^{-(0.132 \text{ yr})(12 / \text{yr})}$$

$$= 0.205$$