Strategic Behavior
Fall, 2022.
Solution to Problem Set 2.
Problem 1.3 in textbook

Choose any $\alpha \in [0,1]$. If player 1 sets $s_1 = \alpha$, then player 2 can do no better than set $s_2 = 1-\alpha$ (asking for more would get player 2 zero and asking for less would not give him higher payoff). Conversely, if player 2 sets $s_2 = 1-\alpha$, player 1 can do no better than setting $s_1 = 1-\alpha$. Thus neither player has unilateral incentive to deviate from $(s_1 = \alpha, s_2 = 1-\alpha)$. This is true for every α in $[0,1]$. Therefore the set of Nash Equilibria is the set

$$\{(s_1, s_2) : s_1 = \alpha, s_2 = 1-\alpha, 0 \leq \alpha \leq 1\}.$$

Problem 1.4 in textbook

Follow hint. If player i sets output q_i and the total output of all other $(n-1)$ firms is Q_{-i}, then the price at which the goods are sold is $p = a - (q_i + Q_{-i})$. The profit π_i of firm i is then given by

$$\pi_i = [a - (q_i + Q_{-i})]q_i - cq_i$$

To find the best response or reaction of firm i to any Q_{-i}, differentiate π_i with respect to q_i and set it equal to zero:

$$a - 2q_i - Q_{-i} - c = 0$$

i.e.,

$$q_i = \frac{a - Q_{-i} - c}{2}$$

As all firms are symmetric you can guess that in Nash Equilibrium they will produce identical quantity, say q^*. So in the equation for the best response or reaction of firm i, you can set $q_i = q^*, Q_{-i} = (n-1)q^*$.

$$q^* = \frac{a - (n-1)q^* - c}{2}$$

which yields the Nash equilibrium output of each firm:

$$q^* = \frac{a-c}{n+1}$$

The total industry output is $nq^* = \frac{n}{n+1}(a-c)$. As $n \to \infty$, each firm's output tends to zero but the industry output tends to $(a-c)$ because $\frac{n}{n+1} \to 1$. The market price then goes to $a - (a-c) = c$ i.e., price converges to marginal cost and profits go to zero, the perfectly competitive outcome.

Problem 1.7 in textbook

Worked out in class. The unique Nash Equilibrium is $p_1^* = p_2^* = c$. At these prices, both firms earn zero profit. To see that this is a Nash equilibrium, note that given $p_1^* = c$, firm 2 cannot earn strictly positive profit; if firm 2 sets $p_2 > c$, it will sell zero and if it sets $p_2 < c$ it can sell a lot but only earn negative profit (price $<$ per unit cost). Thus, $p_2^* = c$ is a best response to $p_1^* = c$. And vice-versa.

To see that this is the only Nash Equilibrium, observe that:

(a) if $p_1^* > p_2^* > c$ or $p_2^* > p_1^* > c$, the higher priced firm makes zero profit and will always do better by deviating to slightly below the lower price and taking over the market (still selling at price above per unit cost which yields strictly positive profit).

(ii) if $p_1^* = p_2^* > c$, both firms share the market equally, but either firm can do better by unilaterally deviating to a slightly lower price where it takes over the entire market

(iii) if $p_1^* > p_2^* = c$ or $p_2^* > p_1^* = c$; here the lower priced firm sells at price equal to unit cost (earning zero profit) and can do better by unilaterally deviating to a slightly higher price which is still below the rival's price; the deviating firm will still have full market share and now earn strictly positive profit as the price would be above per unit cost.

1. Consider the tragedy of the commons with n identical farmers discussed in Section 1.2.D. Suppose that $n = 2$ and that

$$\upsilon(G) = \overline{G} - G$$

where $\overline{G}$ is the maximum number of goats that can be grazed on the green ($G_{\max}$ in the textbook) and G is the total number of goats. Assume $\overline{G} > c$ where c is the cost of purchasing a goat. Derive the number of goats on the commons in the Nash equilibrium & compare it to the joint profit maximizing (cooperative) solution.

Given g_2 set by player 2, player 1's profit if it sets g_1 is

$$\begin{aligned}\pi_1(g_1, g_2) &= g_1 \upsilon(g_1 + g_2) - cg_1 \\ &= g_1[\overline{G} - (g_1 + g_2)] - cg_1\end{aligned}$$

To find the best response of player 1, differentiate π_1 with respect to g_1 and set it equal to zero:

$$\overline{G} - 2g_1 - g_2 - c = 0$$

which yields:

$$g_1 = \frac{\overline{G} - g_2 - c}{2}$$

As this is a symmetric game, you can guess that in Nash Equilibrium, $g_1^* = g_2^* = g^*$ and so

$$g^* = \frac{\overline{G} - g^* - c}{2}$$

which yields the Nash Equilibrium number of goats for each player:

$$g^* = \frac{\overline{G} - c}{3}$$

Thus, the total number of goats in Nash Equilibrium is $G^* = 2g^*$ i.e.,

$$G^* = \frac{2}{3}(\overline{G} - c)$$

Compare this to the joint profit maximizing solution where the total profit is

$$\begin{aligned} & (g_1 + g_2)v(g_1 + g_2) - c(g_1 + g_2) \\ = \; & Gv(G) - cG \\ = \; & G(\overline{G} - G) - cG \end{aligned}$$

which only depends on total number of goats G. Taking the derivative with respect to G and setting it equal to zero, we have

$$\overline{G} - 2G - c = 0$$

so that the joint profit maximizing number of goats :

$$G^J = \frac{1}{2}(\overline{G} - c)$$

Observe that $G^* > G^J$ (tragedy of the commons).