Strategic Behavior
Fall, 2022.
Solution to Problem Set 3.

Problem 1.11
No player plays a strictly dominated strategy with positive probability.
Strategies T and M for player 1 and L and R for player 2 survive iterated elimination of dominated strategies.

	L	R
T	$2,0$	$4,2$
M	$3,4$	$2,3$

Suppose player 2 plays L and R with probability p and $1-p, 0 \leq p \leq 1$.Then, player 1's expected payoff from strategy T is $2p+4(1-p)$ and that from strategy M is $3p+2(1-p)$. Player 1 will randomize between these two strategies only if they yield identical expected payoffs which requires:

$$2p + 4(1-p) = 3p + 2(1-p)$$

which yields $p = \frac{2}{3}$.
Suppose player 1 plays T and M with probability q and $1-q, 0 \leq q \leq 1$.Player 2's expected payoff from strategy L is $4(1-q)$ and that from strategy R is $2q+3(1-q)$. Player 2 will randomize between these two strategies only if they yield identical expected payoffs which requires:

$$4(1-q) = 2q + 3(1-q)$$

which yields $q = \frac{1}{3}$.
This yields the mixed strategy NE where player 2 sets $p = \frac{2}{3}$ and player 1 sets $q = \frac{1}{3}$.

Problem 1.12.
Proceed similarly to above. Mixed Strategy NE: Player 2 plays L with probability $\frac{3}{4}$ and R with probability $\frac{1}{4}$. Player 1 plays T with probability $\frac{2}{3}$ and B with probability $\frac{1}{3}$.

1. Consider the second stage. For any $p_1 > c$, firm 2 will slightly undercut p_1 so that firm 1 will make zero profit (as it sells zero). If $p_1 = c$, both firm 2 is indifferent between $p_2 = c$ and $p_2 > c$ (gets zero profit in both cases) and thus, firm 1 gets zero profit. So, working backwards, firm 1 knows it cannot make strictly positive profit regardless of what p_1 it chooses. So, it is indifferent between all prices $p_1 \geq c$. There is a continuum of solutions by backward induction; firm 1 gets zero profit in all solutions. One solution is $p_1 = c$ with firm 2 setting $p_2 = c$ on the equilibrium path. A very different one is firm 1 setting a very high price and allowing firm 2 to charge the monopoly price and make monopoly profit.

2. Observe that if company Y spends exactly the same amount as Company X then it will get an expected revenue of $\frac{1}{2}(1million) = 500,000$. It is then easy to check that it is optimal for Y to spend 1 dollar more than company X on advertising as long as the latter amount is strictly less than 999,999. Thus, any advertising expense strictly lower than 999,999 yields negative net expected profit to company X. If company X spends 999,999 company Y would be indifferent between spending 1 million and spending zero. If company X spends 1 m, company Y would optimally spend zero. One backward induction solution is one where X spends 1 million, company Y spends zero on the equilibrium path (need to specify company Y's equilibrium strategy here: spend a dollar more than company X if X spends an amount strictly below 999,999 and spend zero if X spends 999,999 or 1 m). Another solution is company X spends 999,999 and company Y spends zero on the equilibrium path (company Y's equilibrium strategy here: spend a dollar more than company X if X spends an amount strictly below 1m, spend zero if X spends 1 m).