**Strategic Behavior**
Fall, 2022.
Solution to Problem Set 4

Exercise 2.11.

Yes. Note that the one shot game has two Nash equilibria (in pure strategies): (T,L) and (M,C).

Consider the following history dependent strategies for the twice repeated game:

Player 1 plays B in period 1. In period 2, player 1 plays T if (B,R) was played in period 1, and plays M otherwise.

Player 2 plays R in period 1. In period 2, player 2 plays L if (B,R) was played in period 1, and plays C otherwise.

Consider the subgame in period 2 reached after (B,R) was played in period 1. The prescribed strategies suggest that player 1 play T and player 2 play L which is a Nash equilibrium in that subgame. Consider any subgame in period 2 reached after (B,R) was not played in period 1. The prescribed strategies suggest that player 1 play M and player 2 play C which is a Nash equilibrium in that subgame.

Now, consider the reduced form game in period 1 taking into account the Nash equilibrium payoffs in period 2 resulting from the prescribed strategies. The total payoffs are as follows:

| | $L$ | $C$ | $R$ |
|---|---|---|---|
| $T$ | $4,3$ | $1,2$ | $6,2$ |
| $M$ | $3,3$ | $2,4$ | $4,3$ |
| $B$ | $2,4$ | $1,3$ | $7,5$ |

It is easy to check that player 1 will find it optimal to follow the prescribed strategy of choosing B in period 1, if it believes that player 2 will follow its strategy and choose R in period 1, and vice-versa.

Thus, the prescribed strategies constitute a subgame perfect Nash equilibrium and players attain a payoff of (4,4) in period 1 & a payoff of (3,1) in period 2.

2. If both firms enter the market, they engage in Cournot competition

where firms set output $(q_1, q_2)$ simultaneously. The profits are given by:

$$\begin{aligned}\pi_1 &= (12 - (q_1 + q_2))q_1 - F \\ \pi_1 &= (12 - (q_1 + q_2))q_2 - F\end{aligned}$$

Differentiating $\pi_1$ with respect to $q_1$ and setting it equal to zero we obtain the best response of firm 1 to any $q_2$ :

$$q_1 = \frac{12 - q_2}{2}$$

and in a symmetric NE, $q_1 = q_2 = q^* = 4$. Each firm's equilibrium profit is then $(12 - 8)4 - F = 16 - F$

If only one firm enters the market it is a monopolist whose profit is given by:

$$(12 - q)q - F$$

Taking the derivative with respect to $q$ and setting equal to zero, we obtain the optimal monopoly output equal to 6 and the monopoly profit is then $36 - F$.

This yields the reduced form payoffs in the first stage simultaneous entry game:

| | $Enter$ | $Not$ $Enter$ |
|---|---|---|
| $Enter$ | $16 - F, 16 - F$ | $36 - F, 0$ |
| $Not$ $Enter$ | $0, 36 - F$ | $0, 0$ |

In a Nash equilibrium of this reduced form game:
(a) no firm enters if $F \geq 36$
(b) one firm enters if $16 \leq F \leq 36$
(c) both firms enter if $F \leq 16$

2. Consider the following simultaneous move game:

| $1 \downarrow, 2 \rightarrow$ | $L$ | $R$ |
|---|---|---|
| $T$ | $0, 0$ | $5, -1$ |
| $B$ | $-1, 5$ | $3, 3$ |

(a) (T,L) is the unique Nash equilibrium of this game.

(b) Suppose the game is repeatedly played an infinite number of times and the players maximize the present value of the stream of payoffs discounting future payoff with discount factor $\delta \in (0, 1)$. For what range of values of $\delta$ is there a subgame perfect equilibrium where both players earn payoff $(3, 3)$ in every period? Outline the strategies underlying this equilibrium clearly.

Trigger Strategies:
Player 1: choose B in period 1. In every period $t > 1$,
- choose $B$ if $(B, R)$ has been played in all previous periods
- otherwise, choose $T$

Player 1: choose $R$ in period 1. In every period $t > 1$,
- choose $R$ if $(B, R)$ has been played in all previous periods
- otherwise, choose $L$.

Consider any subgame beginning in period $t$. There are two possibilities:

(a) $t > 1$ and $(B, R)$ was not played in a previous period
(b) $t = 1$ OR $t >!$ and $(B, R)$ was played in all previous periods

In a subgame of type (a), if player 1 believes player 2 plays according to the trigger strategy outlined above, then he believes player 2 will play $L$ (in every period of this subgame) in which case it is optimal for him to play $T$ ever after; likewise, it is optimal for player to play $L$ ever after in this subgame. So, sticking to the trigger strategy is a Nash Equilibrium in this subgame.

In a subgame of type (b), if player 1 believes player 2 plays according to the trigger strategy outlined above, then he believes player 2 will play $R$; if player

1 sticks to the trigger strategy and plays $B$, player 2 will play $R$ again and so on. In other words, if player 2 sticks to the trigger strategy, his discounted sum of payoff in this subgame is:

$$\begin{aligned} & 3 + 3\delta + 3\delta^2 + ... \\ = & \ 3(1 + \delta + \delta^2 + .....) \\ = & \ \frac{3}{1-\delta} \end{aligned}$$

On the other hand if player 1 deviates from the trigger strategy and plays $T$ in this subgame, then given that player 2 plays the trigger strategy and therefore chooses $R$ in this period, player 1 can get 5 for one period but if he does that, then next period onwards player 2's strategy says he will choose $L$ forever which also means player will choose $T$ next period onwards. Thus, the deviation by player 1 in this subgame will yield payoff:

$$\begin{aligned} & 5 + 0\delta + 0\delta^2 + ... \\ = & \ 5 \end{aligned}$$

So it is optimal for player 1 to stick to the trigger strategy (not deviate) if

$$\frac{3}{1-\delta} \geq 5$$

i.e., $1 - \delta \leq 3/5$ or $\delta \geq 2/5$.

As the game is symmetric, same is true for player 2.

Thus, for $\delta \geq 2/5$, there a subgame perfect equilibrium where both players play the trigger strategies outlined above. In the actual play of these strategies, players choose $(B, R)$ every period i.e., earn payoff $(3, 3)$ in every period.

(c) How does your answer in (b) change if the game is repeated up to a finite number of times? Explain

If the game is repeated a finite number of times, then the unique subgame perfect equilibrium outcome is that players play $(T, L)$ in every period. In the last period, players play a one period game (their actions cannot matter for future payoff as this is the terminal period); thus, the only Nash equilibrium at this stage is $(T, L)$ and this is regardless of what happened before. In the period before last, players now understand that the last period's outcome is independent of what transpires this period which effectively makes the period before last a one shot game; once again players play $(T, L)$ regardless of what happened before. Using backward induction, one can see that players play $(T, L)$ in every period.