Strategic Behavior
Sample Midterm Exam

Total points: 30
Time: 75 minutes.

This exam consists three compulsory questions. Each question carries 10 points. The answers should be written clearly and legibly in the space provided next to the question. The process by which an answer is derived must be clearly indicated. Good luck.

NAME:

STUDENT-ID:

1. Consider the following normal form game with two players:

1 ↓ / 2 ⟶	L	C	R
T	$2,2$	$1,1$	$0,1$
M	$1,1$	$5,5$	$0,0$
B	$1,0$	$0,0$	$-1,-1$

What are the strategies that survive iterated elimination of strictly dominated strategies? What is (are) the Nash equilibrium (equilibria) of this game?

2. Consider the following problem of "tragedy of the commons" with 2 identical farmers. Both farmers simultaneously determine the number of goats to acquire which then graze on the commons. Let g_1, g_2 denote the number of goats acquired by farmer 1 and 2, respectively. The total number of goats grazing on the commons is then

$$G = g_1 + g_2.$$

Let $v(G)$ denote the revenue *per goat* when G goats graze on the commons. Assume

$$v(G) = 20 - G.$$

Each farmer incurs a cost = 2 for purchasing a goat. The payoff of each farmer is given by his profit:

$$\begin{aligned} \pi_1(g_1, g_2) &= (20 - G)g_1 - 2g_1 \\ \pi_2(g_1, g_2) &= (20 - G)g_2 - 2g_2. \end{aligned}$$

Derive the Nash equilibrium of this game.

3. Two firms are bidding in an auction for the right to mine a mineral deposit. Firm 1 can make a profit of \$2 *million* by mining the deposit. Firm 2 has an inferior mining technology which allows it to make a profit of only \$1 *million* by mining the deposit. The bidding game goes as follows. First, firm 1 announces its bid b_1. This is observed by firm 2 which then announces its bid b_2.The mineral deposit is awarded to the highest bidder and if both bids are equal, the deposit is divided evenly between both bidders (each firm getting half of the profit it would make if it got the right over the entire deposit). Thus, the payoff of firm 1 is given by

$$\begin{aligned}\pi_1(b_1, b_2) &= 0, if\ b_2 > b_1 \\ &= 2million - b_1, if\ b_2 < b_1 \\ &= 1million - b_1, if\ b_2 = b_1.\end{aligned}$$

and that for firm 2 is given by

$$\begin{aligned}\pi_2(b_1, b_2) &= 0, if\ b_2 < b_1 \\ &= 1million - b_2, if\ b_2 > b_1 \\ &= 0.5million - b_2, if\ b_2 = b_1.\end{aligned}$$

Assume that bids are in whole dollars. Derive the backward induction solution to this two stage game.