Dynamic Games.

Most economic interactions:

agents choose actions over time where some information over actions chosen in previous periods is available.

If previous actions unobserved or unknown to players that move later, the game is effectively a simultaneous move game.

Otherwise, its a true dynamic game.

Extensive forms capture the dynamic structure of moves.

Every extensive form game can be reduced to a normal form game where players simultaneously choose *strategies*.

Why not just use the theory of games developed for normal form games - for example, the concept of NE and apply it to the reduced normal form associated with the extensive form of a dynamic game?

Problem: Some NE strategies may not be *credible* under the sequential structure captured by the extensive form (information suppressed in the normal form).

Need refinement of Nash equilibrium.

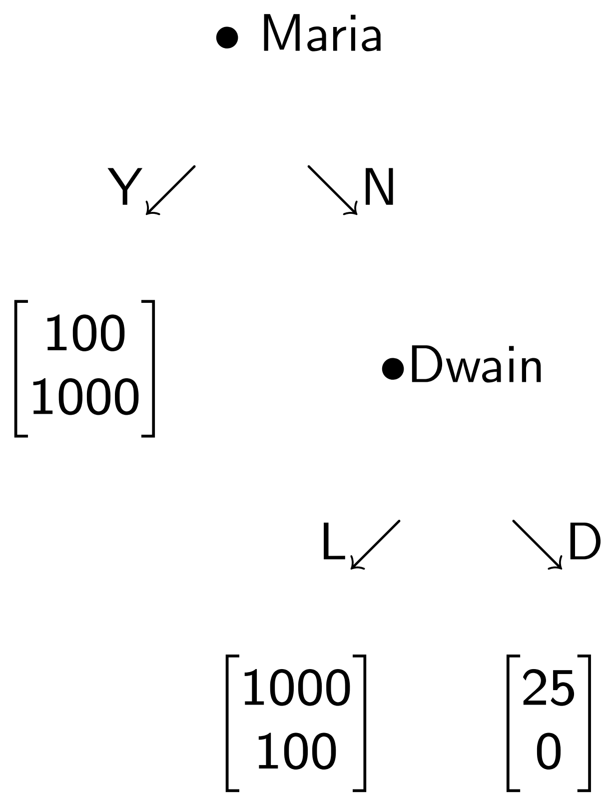
Dynamic Finite Games of Complete and Perfect Information

Players not only have common knowledge of payoffs, but also know every choice made by players whose moves precede them.

Every information set consists of a single node.

Finite number of nodes.

Example.



Strategy Sets:

Maria: {Y,N}

Dwain: {L if N, D if N}

Normal form:

	L if N	D if N
Y	100, 1000	100, 1000
N	1000, 100	25, 0

Two pure strategy NE:

NE1: (N, L if N)

NE2: (Y, D if N).

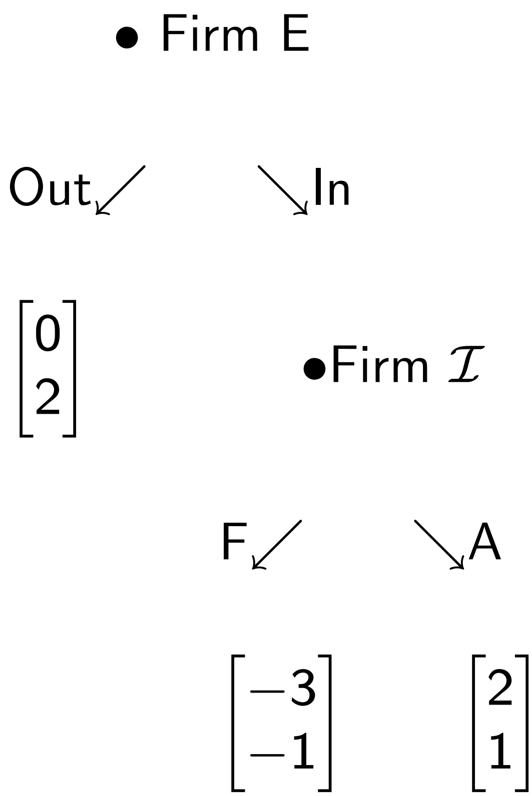
NE2 is not credible because if we look at the extensive form we immediately know that Dwain would never choose D if Maria chose N.

NE2: based on Dwain playing a strategy where he threatens to play an action (D) that he would never play if he actually had to choose an action in the real play of the game - his strategy is a bluff.

The reason why the outcome in NE2 remains a Nash equilibrium in the normal form is because in the actual play of this NE, Dwain will never have to actually choose between L and D - what he says he will do in that node does not affect his payoff - his bluff will never be called.

The normal form of the game does not allow us to see this credibility problem (the strategies can be re-labelled as A,B,C, D and the underlying story is not visible) - one needs the extensive form to discover it.

Example.



Strategy Sets:

Firm E: {Out,In}

Firm $\mathcal{I}$: {F if In, A if In}

Normal form:

$$\begin{bmatrix} & \text{F if In} & \text{A if In} \\ \text{Out} & 0,2 & 0,2 \\ \text{In} & -3,-1 & 2,1 \end{bmatrix}$$

Two pure strategy NE:

NE1: (In, A if In)

NE2: (Out, F if In).

NE2 is not credible.

In this Nash equilibrium, what firm I's strategy says it will do at the unreached node can actually ensure that firm E, taking firm I's strategy as given, wants to play "out" (even though, given firm E's strategy, firm I's strategy choice does not really make a difference to firm I's payoff).

* *Principle of Sequential Rationality*: A player's strategy should specify optimal actions at every point in the game tree.

At each decision node in the tree, a player should choose an action that is optimal *from that point on*, given the strategies of other players.

Sequential rationality violated by NE2 in both examples above.

NE1 is sequentially rational in both examples: credible.

Work backwards from the last stage of the game i.e., decision nodes whose only successor nodes are terminal nodes).

Solve for optimal behavior at each such decision node.

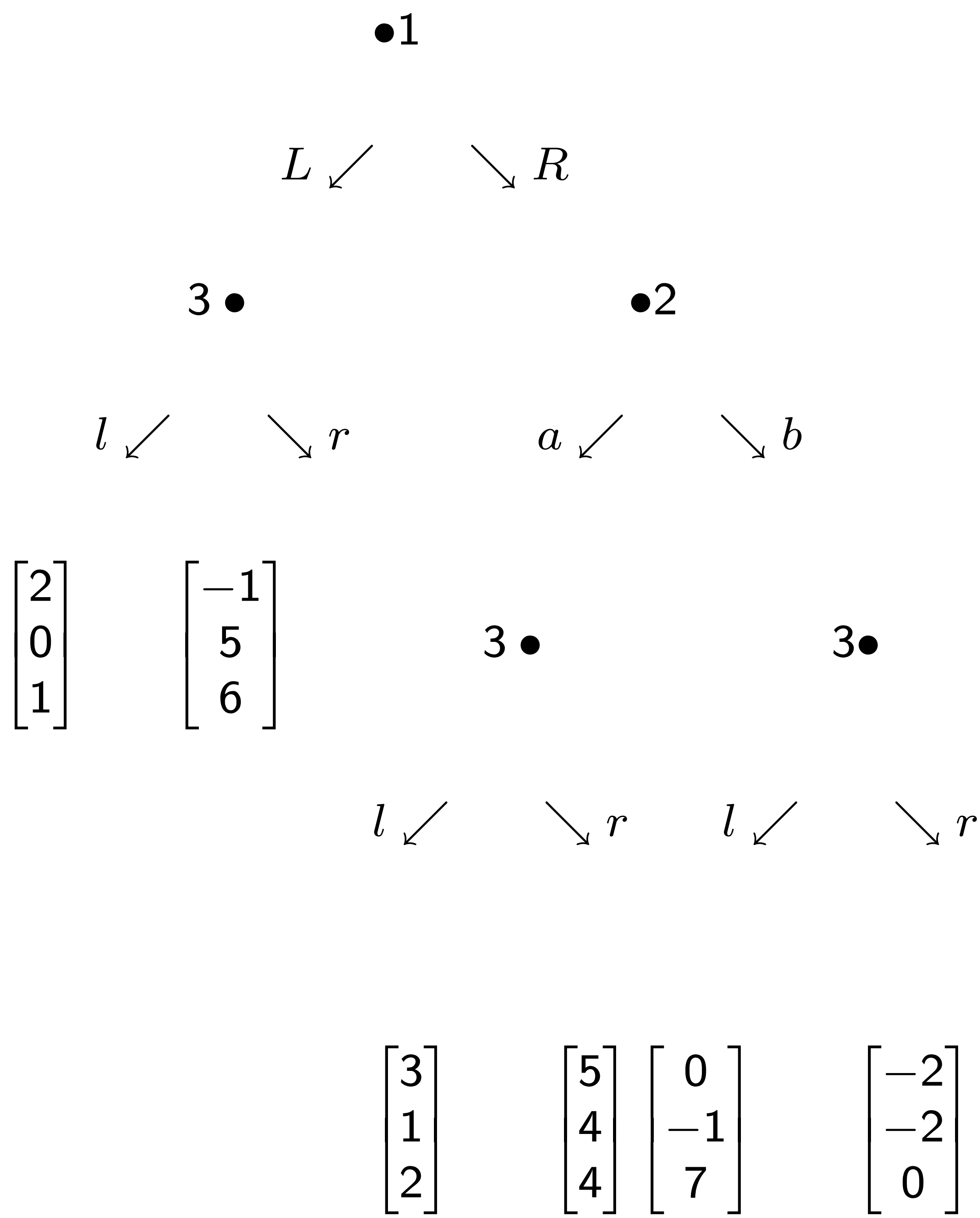
Then go to previous stage (i.e., nodes preceding the above) and figure out optimal action of decision maker

(while fixing continuation play in the successor nodes to the optimal actions derived previously)

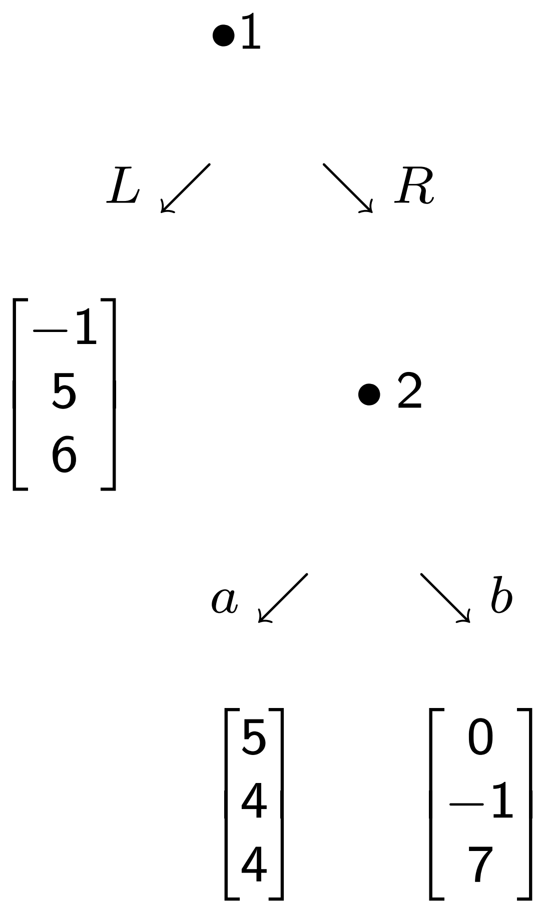
& so on until one reaches the beginning of the game.

This is called *backward induction*.

Example.



Solving for optimal action in last stage leads to the following reduced form two-stage game :



Reduced form first stage game:

$$\bullet 1$$

$$L \swarrow \qquad \searrow R$$

$$\begin{bmatrix} -1 \\ 5 \\ 6 \end{bmatrix} \qquad \begin{bmatrix} 5 \\ 4 \\ 4 \end{bmatrix}$$

Solution by backward induction:

$[R, a\ if\ R, (r\ if\ L, r\ if\ R$ and $a, l\ if\ R$ and $b]$.

Can check that this is a NE.

There are two other NE that are not sequentially rational.

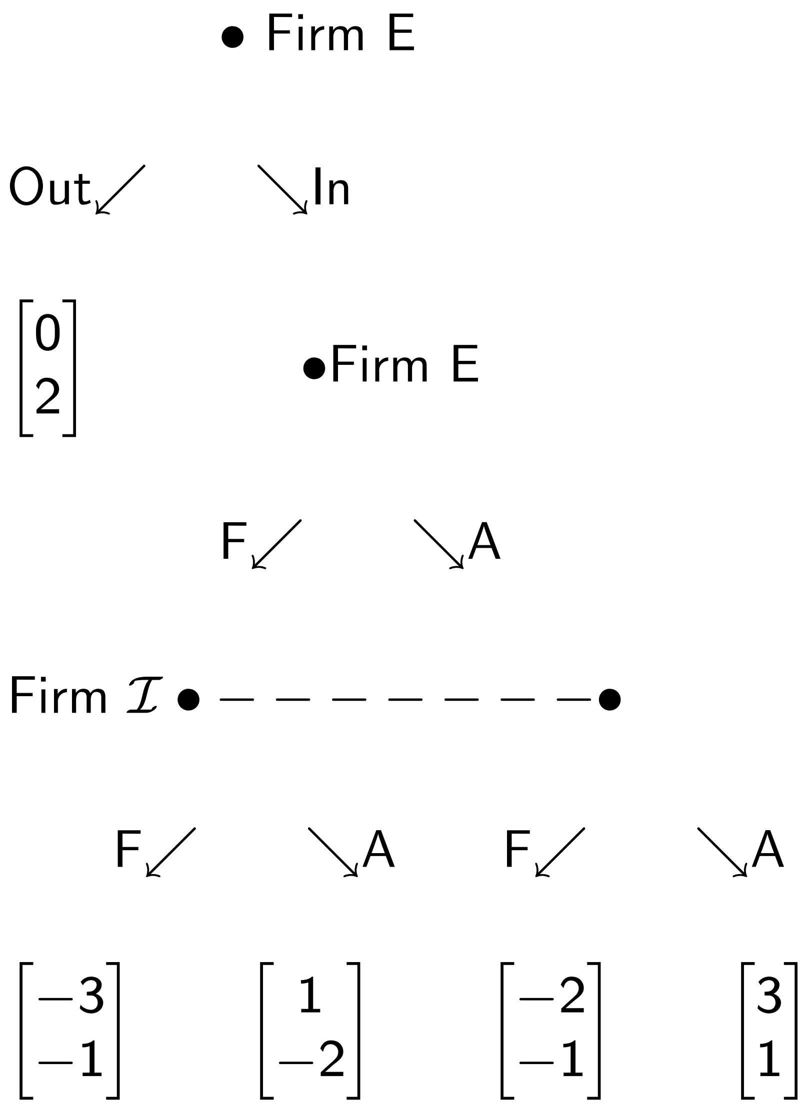
Games of Complete but possibly Imperfect Information.

How to apply sequential rationality if decision nodes are not necessarily singletons i.e., players do not necessarily observe all predecessor moves.

Example:

Consider the entrant-incumbent game with one modification.

If entrant form decides to play "In" (i.e., enter), entrant and incumbent play a simultaneous move game where they both choose whether to fight or accommodate.



• Firm E
Out
In
[0, 2]
•Firm E
F
A
Firm $\mathcal{I}$
F
A
F
A
[−3, −1]
[1, −2]
[−2, −1]
[3, 1]

Normal form:

Row player: E

Column Player: $\mathcal{I}$

	A if In	F if In
Out,A if In	$0, 2$	$0, 2$
Out,F if In	$0, 2$	$0, 2$
In, A if In	$3, 1$	$-2, -1$
In, F if In	$1, -2$	$-3, -1$

Three pure strategy NE in the normal form game:

NE1: [(Out, A if In), (F if In)]

NE2: [(Out, F if In),(F if In)]

NE3: [(In,A if In),(A if In)]

Extensive form equivalent to:

$\bullet$ Firm E

Out $\swarrow$ $\searrow$ In

$$\begin{bmatrix} 0 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} & A & F \\ A & 3,1 & -2,-1 \\ F & 1,-2 & -3,-1 \end{bmatrix}$$

In the simultaneous move (sub)game that follows "In" (captured in the matrix), unique NE: (A,A).

Thus, firm E should expect that if it enters, they will both play (A,A).

So, firm E should choose 'In".

Only NE3 is a reasonable prediction of the game.

Subgame Perfect Nash Equilibrium. [Selten, 1965].

Definition. A subgame of an extensive form game is a subset of the game having the following properties:

(i) It begins with an information set containing a single decision node, contains all the decision nodes that are successors (both immediate and later) of this node, and contains only these nodes;

(ii) If a decision node x is in the subgame, then every other node contained in the information set containing x is also in the subgame (no broken information sets).

The entire game is also a subgame.

A subgame is an extensive form game in its own right and one can apply all of the equilibrium/solution concepts - including Nash equilibrium to it.

* A strategy profile σ in extensive form game Γ_E is said to induce a Nash equilibrium in a particular subgame of Γ_E if the moves specified by σ for information sets within the subgame constitute a Nash equilibrium when this subgame is considered in isolation.

Definition. *A profile of strategies $\sigma = (\sigma_1, ..., \sigma_I)$ in an I-player extensive form game Γ_E is a subgame perfect Nash equilibrium (SPNE) if it induces a Nash equilibrium in every subgame of Γ_E.*

By definition, every SPNE is a NE but the converse is not true.

SPNE: the most widely used refinement of NE in economic applications.* If the only subgame of a game is the game as a whole, then every NE is subgame perfect.

Generalized backward induction to solve for SPNE in more general *finite* dynamic games (not necessarily perfect information):

1. Look at the final subgames at the end of the game tree (no further nested subgame) and solve for NE.

2. Select one NE for each of them and replace the final subgames in the game tree by terminal payoffs equal to the NE payoffs of the players (in the relevant final subgames).

This is called the *reduced game*.

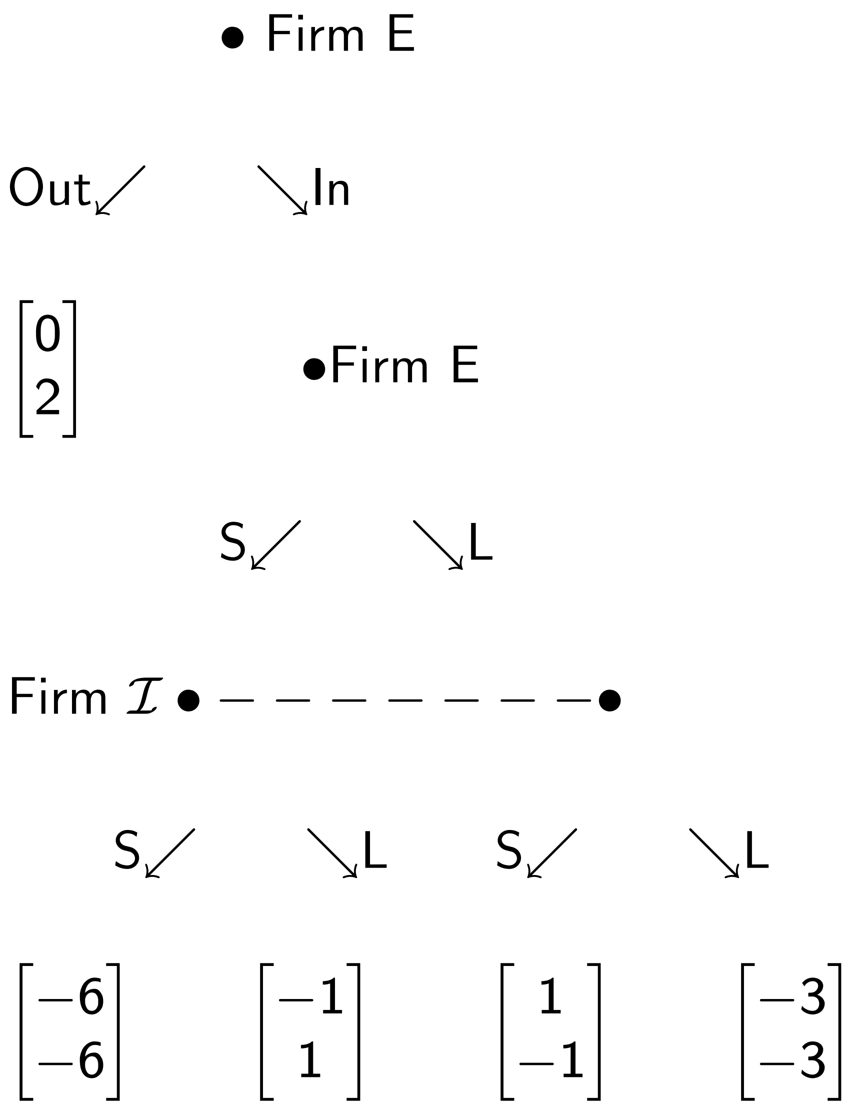
3. Now, repeat this for the reduced game & continue doing this until the moves at all information sets of the original game have been determined.

The strategies that specify the collection of moves obtained through this process constitute a SPNE.

If multiple Nash equilibria are never encountered in this generalized backward induction process, then this profile of strategies is the unique SPNE.

If multiple NE are encountered, the full set of SPNE is identifying the procedure for each possible equilibrium that could occur at the subgames.

Example.



- Firm E

$$\text{Out} \swarrow \qquad\qquad \searrow \text{In}$$

$$\begin{bmatrix} 0 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} & S & L \\ S & -6, -6 & -1, 1 \\ L & 1, -1 & -3, -3 \end{bmatrix}$$

Two NE in the last subgame: $(S, L), (L, S)$.

Two reduced games:

1) • Firm E

Out $\swarrow$ $\searrow$ In

$$\begin{bmatrix} 0 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

2) • Firm E

Out $\swarrow$ $\searrow$ In

$$\begin{bmatrix} 0 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Two SPNE:

{(In, L if In), S if In}

{(Out, S if In), L if In}.