Microeconomic Theory I.
Fall, 2022.
Problem Set 3.

Due: Tuesday, September 26, In class.

1. Exercise 3.E.5

2. For the utility functions given below, derive the Hicksian compensated demand and expenditure functions (assume $p \gg 0, u > u(0)$):

(a) $u(x_1, x_2) = [\alpha(x_1)^\rho + \beta(x_2)^\rho]^{\frac{1}{\rho}}, -\infty < \rho < 1, \alpha > 0, \beta > 0.$
(b) $u(x_1, x_2) = {x_1}^\alpha x_2^\beta, \alpha > 0, \beta > 0.$
(c) $u(x_1, x_2) = \min\{ax_1, bx_2\}, a > 0, b > 0$
(d) $u(x_1, x_2) = \max\{x_1, x_2\}$
(e) $u(x_1, x_2) = x_1 + (x_2)^\alpha, 0 < \alpha < 1$
(f) $u(x_1, x_2) = (x_1)^2 + (x_2)^2$
(g) $u(x_1, x_2) = \alpha x_1 + \beta x_2, \alpha > 0, \beta > 0.$

3. For $L = 2, p \gg 0$ and $u > u(0)$, consider the expenditure function:

$$e(p_1, p_2, u) = u(ap_1 + bp_2), a > 0, b > 0.$$

(a) Derive the Hicksian demand functions.
(b) Derive the indirect utility function.
(c) Derive the Walrasian demand function.

4. For $L = 2, p \gg 0$ and $w > 0$,consider the indirect utility function:

$$v(p_1, p_2, w) = \alpha \ln \frac{\alpha}{p_1} + (1 - \alpha) \ln \frac{(1 - \alpha)}{p_2} + \ln w, 0 < \alpha < 1.$$

(a) Derive the Walrasian demand function.
(b) Derive the expenditure function.
(c) Derive the Hicksian demand function.

5. Exercise 3.I.3

6. Consider a consumer whose preferences defined on $\mathbb{R}^2_+$ are represented by the following quasi-linear utility function:

$$u(x_1, x_2) = x_1 + \sqrt{x}_2$$

Given wealth $w > 0$ large enough and the price of good 1 $p_1 = 1$, show that the compensating and equivalent variations associated with a change in the price of good 2 are identical.

7. Consider a consumer whose preferences defined on $\mathbb{R}^2_+$ are represented by the following utility function:

$$u(x_1, x_2) = x_1 x_2$$

Given wealth $w = 10$ and price of good 1 $p_1 = 1$, consider a decline in the price of good 2 from $p_2^0 = 4$ to $p_2^1 = 2$. Evaluate the change in consumer welfare as measured by (i) Compensating Variation, (ii) Equivalent Variation and (iiii) Change in Consumer Surplus.

8. Exercise 2.F.13 (a)- (c).
NOTE: Correction to Exercise 2.F.13 (b): The last line should be as follows: (*) For any $x \in x(p, w)$ and $x' \in x(p', w')$, if $p \cdot x' < w$, then $p' \cdot x > w'$.