

Microeconomic Theory I.
Fall, 2022.
Problem Set 4.

Due: Tuesday, October 4.

1. Exercise 5.B.3
2. Show that for a single output technology with production function f , the production set Y is strictly convex if f is strictly concave.
3. Show that a production technology Y must exhibit nonincreasing returns to scale if it is convex and contains the possibility of inaction.
4. Exercise 5.C.9
5. Exercise 5.C.10
6. For single output technology with production function f , show that given strictly positive input prices, the average cost of production is nondecreasing (nonincreasing) in output if f exhibits nonincreasing (nondecreasing) returns to scale. What is the implication in the case of constant returns to scale?
7. Show how the statement of part (ix) of Proposition 5.C.2 modified if f is strictly concave.
8. A firm produces a single output using three inputs. The production function is given by:

$$f(x_1, x_2, x_3) = \min\{x_1, 2x_2\} + \min\{x_2, x_3\}, x_i \geq 0,$$

where x_i is the quantity of input i used in the production process. Solve the cost minimization problem and derive the cost function (given input prices $(w_1, w_2, w_3) \gg 0$ and output requirement $q > 0$).

9. Exercise 5.F.1
10. Consider a firm that produces output using two inputs 1 and 2. Let q denote the quantity of output, while z_1 and z_2 denote the quantities of the two inputs 1 and 2 respectively. The firm's technology is given by the production set :

$$Y = \{(-z_1, -z_2, q) : z_1 \geq q^2, z_2 \geq 2q, q \geq 0\}.$$

The firm is a price taker in the input markets; the price of input i is $w_i \geq 0, i = 1, 2$.

- (a) Is the production technology convex? What can you say about the returns to scale?
 - (b) Given the above input prices, derive the cost function of the firm.
 - (c) Suppose the firm is a price taking competitive firm in the output market. Is there a solution to the profit maximization problem for every $p \geq 0$? Is the solution unique? Derive the supply of the competitive firm
11. Consider a price taking firm that produces *two goods as output* (goods 1 and 2) *using only one good as input* (good 3). The production set is given by:

$$Y = \{(q_1, q_2, -z) : q_i \geq 0, i = 1, 2, z \geq (q_1)^2 + (q_2)^2\}.$$

Given prices $p_1, p_2 \gg 0$ of the two goods produced as output and price $w > 0$ of the input, solve the profit maximization problem of the firm outlining each step very carefully and derive the supply correspondence.