

Microeconomic Theory I.
Fall, 2022.
Problem Set 7.

Due: Tuesday, November 29, in class.

Exercises: 12.B.2 (a,b), 12.B.6, 12.C.3, 12.C.7, 12.C.9 (a,b).

1. Consider a homogenous good Bertrand duopoly where firms produce at constant unit cost and market demand function is $x(p)$. $x(p)$ is continuous and nonincreasing; further, it is strictly decreasing on $\{p : x(p) > 0\}$. The unit cost of firm $j = 1, 2$, is c_j where

$$0 \leq c_1 < c_2$$

and $x(c_2) > 0$. Prove that in any Nash equilibrium outcome, firm 1 cannot charge a price strictly above c_2 and further, firm 2 must have zero market share.

2. Consider a symmetric Cournot oligopoly in a homogenous good market with $J \geq 1$ firms where the cost function of each firm is given by:

$$C(q) = \frac{q^2}{2}$$

and the market demand is given by

$$\begin{aligned} x(p) &= a - p, p \leq a, a > 0, \\ &= 0, p > a. \end{aligned}$$

Derive the Cournot-Nash equilibrium and the associated price and firm profit.

Derive the deadweight welfare loss.

What happens to the industry outcome as $J \rightarrow \infty$?

3. Consider a homogenous good Cournot oligopoly with $J > 1$ firms where each firm's production cost function is given by $C_j(q_j)$; $C_j(\cdot)$ is strictly increasing and differentiable on \mathbb{R}_+ . The inverse market demand is given by $p(q)$ where $p(\cdot)$ is continuous and non-decreasing on \mathbb{R}_+ ; further, $p(\cdot)$ is continuously differentiable and $p' < 0$ on $\{q : p(q) > 0\}$ and $p(0) > \min_j C'_j(0)$. Show that the Nash equilibrium industry outcome is not socially efficient.