Microeconomic Theory I.
Fall, 2023.
Problem Set 1.

Due: Tuesday, September 5, in class.

1. Show that lexicographic preferences (as defined in Example 3.C.l) are complete, transitive, strongly monotone and strictly convex.

2. Consider the proof of Proposition 3.C.l under the additional assumption that the preference relation is monotone. Prove that $x \succsim 0$ for all $x \in \mathbb{R}^L_+$.

3. Prove the following: if $\succsim$ is monotone, then it satisfies local non-satiation.

4. Let u be a utility function representing a preference relation $\succsim$. Show that $\succsim$ is convex if, and only if, u is quasi-concave.

5. Show that (strict) concavity of a utility function on $\mathbb{R}^L_+$ implies (strict) quasi-concavity. Give an example of a utility function that is quasi-concave, but not concave.

6. Show that continuity and concavity are not ordinal properties of utility functions (i.e., may not necessarily be preserved under strictly increasing transformations).

(Hint: Simple examples are sufficient to show this)

7. Are the following utility functions are quasi-concave on $\mathbb{R}^2_+$? Are the represented preferences monotone? Strongly Monotone?

(a) $u(x_1, x_2) = [\alpha(x_1)^\rho + \beta(x_2)^\rho]^{\frac{1}{\rho}}, -\infty < \rho < 1, \alpha > 0, \beta > 0.$
(b) $u(x_1, x_2) = {x_1}^\alpha x_2^\beta, \alpha > 0, \beta > 0$
(c) $u(x_1, x_2) = \min\{ax_1, bx_2\}, a > 0, b > 0$
(d) $u(x_1, x_2) = \max\{ax_1, bx_2\}, a > 0, b > 0$
(e) $u(x_1, x_2) = x_1 + (x_2)^\alpha, 0 < \alpha < 1$
(f) $u(x_1, x_2) = (x_1)^2 + (x_2)^2$
(g) $u(x_1, x_2) = \alpha x_1 + \beta x_2, \alpha > 0, \beta > 0.$

For the utility functions that you find to be quasi-concave, please comment on whether or not they are strictly quasi-concave?