Microeconomic Theory I.
Fall, 2023.
Problem Set 2.

Due: Tuesday, September 19, In class.

1. For each of these utility functions, derive the Walrasian demand at prices $(p_1, p_2) \gg 0$ and wealth $w > 0$.

(a) $u(x_1, x_2) = [\alpha(x_1)^\rho + \beta(x_2)^\rho]^{\frac{1}{\rho}}, -\infty < \rho < 1, \alpha > 0, \beta > 0.$

(b) $u(x_1, x_2) = {x_1}^\alpha x_2^\beta, \alpha > 0, \beta > 0$

(c) $u(x_1, x_2) = \min\{ax_1, bx_2\}, a > 0, b > 0$

(d) $u(x_1, x_2) = \max\{ax_1, bx_2\}, a > 0, b > 0$

(e) $u(x_1, x_2) = x_1 + (x_2)^\alpha, 0 < \alpha < 1$

(f) $u(x_1, x_2) = (x_1)^2 + (x_2)^2$

(g) $u(x_1, x_2) = \alpha x_1 + \beta x_2, \alpha > 0, \beta > 0.$

2. (a) Suppose that the utility function $u(x)$ is homogenous of degree one. Show that for any $p, w \gg 0, \alpha > 0$,

$$x(p, \alpha w) = \{x \in \mathbb{R}^L_+ : x = \alpha x',\ x' \in x(p, w)\}$$

and

$$v(p, \alpha w) = \alpha v(p, w).$$

If, further, u is strictly quasi-concave, show that the Walrasian demand function is of the form:

$$x(p, w) = w f(p)$$

where f only depends on prices. What does it imply about the wealth elasticity of demand?

(b) A function is said to be homothetic if it is a strictly increasing transformation of a function that is homogenous of degree one. Using (a), argue that if an agent's preferences can be represented by a homothetic and strictly quasi-concave utility function, then all goods are normal. (A good is normal if quantity consumed always increases with wealth).

3. Exercise 3.C.6 and part (d) of exercise 3.D.5 in the textbook.