Microeconomic Theory I.
Fall, 2023.
Problem Set 5.

Due: Thursday, November 9, in class.

Exercises 6.C.2(a), 6.C.1, 6.C.6 (a), 6.C.8, 6.C.11, 6.C.13, 6.C.16, 6.C.18, 6.C.20, 6.D.2 .

Other problems:

1. Consider an agent whose preferences on the space of monetary lotteries on $\mathbb{R}$ have an expected utility representation. Show that the following are equivalent:

(a) the agent is strictly risk averse

(b) the agent has a strictly concave (Bernoulli) utility function

(c) the certainty equivalent of any non-degenerate lottery is strictly lower than the expected payoff from the lottery

(d) the probability premium associated with any additive risk is always strictly positive.

You can use the fact that

$$u(\frac{1}{2}x + \frac{1}{2}y) > \frac{1}{2}u(x) + \frac{1}{2}u(y) \text{ fir all } x, y, x \neq y,$$

is equivalent to strict concavity of u.

2. Assuming that preferences can be represented by a utility function of the expected utility form, consider a decision maker who has the option of dividing her wealth $w > 0$ between two investment assets - both of which are risky - with (non-degenerate) random returns (per dollar invested) denoted by R_1 and R_2. Assume that R_1 and R_2 are independent and identically distributed. Also, assume that the Bernoulli utility function u is differentiable with $u' > 0$. Show that the agent will divide her wealth between both assets (diversify her portfolio) if she is strictly risk-averse (strictly concave Bernoulli utility).

3. Consider a strictly risk averse expected utility maximizer with Bernoulli utility function u on the space of monetary outcomes taken to be equal to $\mathbb{R}_+$.In particular, assume u is twice differentiable on $\mathbb{R}_{++}$ with $u' > 0, u'' < 0$. For initial wealth $w > 0$, consider a lottery that assigns probability $\frac{1}{2}$ to the outcome $w - \alpha$ and probability $\frac{1}{2}$ to the outcome $w + \alpha$ where $\alpha \in (0, w)$.

(i) Rigorously examine how the certainty equivalent of this lottery compares to w.

(ii) Examine rigorously the effect of an increase in α on the certainty equivalent of this lottery.