Asymmetric Information and Adverse Selection.

Based on Akerlof (1970).

A Simple Labor Market Model.

Many identical firms.

Firms risk neutral (Maximize expected profit).

Firms price taking.

Price of output =1.

Production uses only labor.

Constant marginal productivity of labor: θ

Productivity θ differs across workers.

$\theta \in [\underline{\theta}, \overline{\theta}]$.

$$0 \leq \underline{\theta} < \overline{\theta} < \infty$$

Distribution of workers' productivity: distribution function $F(\theta)$.

$F(\theta) =$ proportion of workers with productivity of at most θ.

Assume: F is non-degenerate (at least two types of productivity in the population of workers).

Finite number (measure) of workers.

A worker can work at home or at a firm.

If a worker of type (i.e., productivity) θ works at home she earns $r(\theta)$.

[We refer to $r(\theta)$ as "reservation wage" or opportunity cost of accepting employment of a worker of type θ].

A worker of type θ accepts employment if and only if her wage $\geq r(\theta)$.

Benchmark: Full information (productivity of each worker is publicly observable).

Competitive equilibrium.

A distinct equilibrium wage $w^*(\theta)$ for each θ.

Price taking agents in the labor market.

wage = marginal product of labor.

$$w^*(\theta) = \theta$$

Firms earn zero profit (constant returns technology).

Set of workers that are employed in the firms in equilibrium are those with productivity in the set:

$$\{\theta : r(\theta) \leq \theta\}.$$

This equilibrium is Pareto optimal: every worker who is more productive at home then in the firms is employed at home and the rest work in the firms.

No other allocation can increase total surplus generated.

Asymmetric Information.

Workers' productivity not observed by firms.

Competitive equilibrium:

One wage w for all worker types.

Set of worker types willing to work in the firms at wage rate w :

$$\Theta(w) = \{\theta : r(\theta) \leq w\}.$$

so that the realized average productivity of workers employed at wage w is:

$$E[\theta \mid \theta \in \Theta(w)].$$

If a firm believes that the expected productivity of workers is μ, then its demand for labor at wage w is

$$\begin{aligned} 0, \text{ if } w &> \mu \\ [0, \infty), \text{ if } w &= \mu \\ \infty, \text{ if } w &< \mu. \end{aligned}$$

For market clearing with positive employment in firms we need:

$$w = \mu$$

Rational expectations: expectations are fulfilled (expected productivity = average productivity of workers that work in firms):

$$\mu = E[\theta \mid \theta \in \Theta(w)].$$

So, equilibrium wage w^* satisfies the equation :

$$w^* = E\{\theta \mid r(\theta) \leq w^*\}.$$

In this equilibrium, the set of workers finding employment:

$$\Theta^*(w^*) = \{\theta : r(\theta) \leq w^*\}.$$

Note:

If no worker accepts employment the average productivity $E[\theta \mid \theta \in \Theta(w)]$ is not well defined as $\Theta(w)$ is a set of measure zero. In this case we take $\mu = E[\theta]$, the unconditional expectation.

* Competitive equilibrium may be Pareto inefficient.

Suppose $r(\theta) = r, \forall \theta \in [\underline{\theta}, \overline{\theta}]$.

In that case, the set of workers willing to work in in firms

$$\begin{aligned} \Theta(w) &= [\underline{\theta}, \overline{\theta}], \text{ if } w \geq r \\ &= \phi, \text{if } w < r. \end{aligned}$$

So, the expected productivity of workers in a competitive equilibrium is $E(\theta)$, the unconditional expectation of random variable θ.

This does not depend on the wage.

If $E(\theta) \geq r$, then in equilibrium

$$w^* = E(\theta)$$

and all workers are employed in firms.

If $E(\theta) < r$,no worker is employed in firms.

There is possibility of Pareto inefficiency in both situations.

For example, if

$$\underline{\theta} < r < E(\theta)$$

then workers whose productivity $\theta \in [\underline{\theta}, r)$ are employed in firms (attracted by the high wage that exceeds their productivity) but actually would be more socially productive at home.

On the other hand, if

$$E(\theta) < r < \overline{\theta}$$

then workers of type $\theta \in (r, \overline{\theta}]$ are employed at home though they would be more productive at work.

Both problems arise because employers cannot distinguish types of workers and price discriminate; they pay according to the population average.

This inefficiency is a consequence of asymmetric information.

Note: if it is socially optimal for all workers to work in the firms i.e., $r < \underline{\theta}$ or for all workers to work at home $\overline{\theta} < r$, then competitive equilibrium is Pareto optimal.

But a potentially more serious problem for the market arises when $r(\theta)$ varies with θ.

In that case, the type of workers willing to work at a certain wage and their average productivity can vary with the wage.

Adverse Selection:

Adverse selection occurs when an informed individual's trading decision depends on her unobservable characteristics (type) in such a way that it adversely affects the uninformed agents in the market.

Here: relatively less productive workers are willing to accept employment in firms at any wage.

This can lead to unraveling of the market.

Consider a situation where it is socially optimal for all workers to work in the firms:

$$r(\theta) \leq \theta, \forall \theta \in [\underline{\theta}, \overline{\theta}].$$

Assume: $r(\theta)$ is a strictly increasing function.

More productive workers are more productive both at home and in the firms.

Reservation wage of workers increasing in their productivity.

"*The valuations of buyers and sellers in the market are correlated.*"

This is what generates adverse selection.

Assume: $F(.)$ has a density function $f(\theta) > 0$ on $[\underline{\theta}, \overline{\theta}]$.

As noted, in equilibrium:

$$w^* = E[\theta \mid r(\theta) \leq w^*]$$

Let θ^* denote the highest productivity worker employed in the firms in equilibrium.

Then

$$r(\theta^*) = w^*$$

so that

$$\begin{aligned} r(\theta^*) &= E[\theta \mid r(\theta) \leq r(\theta^*)] \\ &= E[\theta \mid \underline{\theta} \leq \theta \leq \theta^*] \end{aligned}$$

First note that if

$$r(\overline{\theta}) > E(\theta)$$

then,

$$\theta^* < \overline{\theta}.$$

Highest productivity workers do not find employment in the firms.

For best quality workers to employment in firms, the wage would have to be high enough - but in that case all workers of lower productivity would also find it optimal to be employed.

The average productivity of all workers is not good enough for firms to offer a wage that attracts the best workers.

So, in equilibrium, only lower productivity workers ($\theta \leq \theta^*$) are employed by firms.

Bad workers drive good workers out of the market. (Gresham's Law).

How bad can it get?

Suppose $\underline{\theta} = 0, \overline{\theta} = 2$ and $r(\theta) = \alpha\theta$, $\forall \theta \in [0, 2]$.

Also assume:

$$\frac{1}{2} < \alpha < 1.$$

Suppose F is the uniform distribution on $[0, 2]$.

Then,

$$\begin{aligned} E\{\theta \quad | \quad & r(\theta) \leq w\} \\ &= E\{\theta \mid 0 \leq \theta \leq \frac{w}{\alpha}\} \\ &= \frac{w}{2\alpha}, \text{ for } w \leq 2\alpha. \\ &= 1, \text{ for } w > 2\alpha. \end{aligned}$$

Note that in equilibrium, as $E\{\theta \mid r(\theta) \leq w\} \leq 1$, the equilibrium wage $w \leq 1 < 2\alpha$.

Further, at every $w \in (0, 2\alpha)$,

$$E[\theta \mid r(\theta) \leq w] = \frac{w}{2\alpha} < w$$

so that no such wage can be an equilibrium.

The unique market equilibrium

$$w^* = 0$$

where only workers of productivity zero may work (but they are a set of measure zero).

Essentially, no workers are hired by firms even though social optimality requires that all workers be hired in firms.

Other possibilities:

Multiple equilibria that are Pareto ranked.

Equilibria with higher wage - bigger range of θ employed, workers earn greater surplus.

(Firms always earn zero profit).

Signaling.

Spence (1973,1974).

What mechanisms can allow firms to distinguish between workers?

Agents with private information can choose actions that enrich the information structure of uninformed agents - more particularly, allow the latter to make rational inference about the "types" of informed agents.

Direct revelation of types through a "test" or certification by experts may not always be feasible.

Need for sophisticated signaling mechanisms.

Modify the labor market model to a strategic model.

2 firms

1 worker

Worker of two possible types with constant marginal productivity: θ_H, θ_L

$$\theta_H > \theta_L > 0$$

Priors:

$$\begin{aligned} \Pr[\theta &= \theta_H] = \lambda \in (0,1) \\ \Pr[\theta &= \theta_L] = 1 - \lambda \in (0,1) \end{aligned}$$

Before entering job market a worker can get some education.

Amount of education received is publicly observable.

Assume: Education does not affect a worker's productivity.

Education here is a pure information signaling device.

Cost of obtaining education level e for a type θ worker: $c(e, \theta)$.

Assume:$c(e, \theta)$ is a twice continuously differentiable function with

$$\begin{aligned} c(0, \theta) &= 0, \\ c_e(e, \theta) &> 0, c_{ee}(e, \theta) > 0, c_\theta(e, \theta) < 0, \forall e > 0 \\ c_{e\theta}(e, \theta) &< 0. \end{aligned}$$

This implies: Both total and marginal cost of education are lower for high productivity workers.

Let $u(w, e \mid \theta)$ denote utility of type θ worker who chooses education level e and receives wage w.

Let

$$u(w, e \mid \theta) = w - c(e, \theta).$$

Note: Single Crossing Condition. The indifference curves of type θ_H and type θ_L workers in the (w, e) space cross only once and at the point of intersection the indifference curve of the θ_L type worker is steeper.

[Indifference curve

$$w - c(e, \theta) = K$$

with slope:

$$\frac{dw}{de} = c_e(e, \theta)$$

and this is decreasing in θ as $c_{e\theta} < 0$.]

As before, a worker of type θ can earn $r(\theta)$ by working at home.

Assume:

$$r(\theta_H) = r(\theta_L) = 0$$

Note: if there is no signaling, firms first make wage offers and then the worker chooses whether to accept and which one, then the unique equilibrium in that case is that both firms offer wage (Bertrand competition)

$$w^* = E(\theta).$$

The market equilibrium is Pareto efficient.

We use this to illustrate the peculiar inefficiencies that signaling generates.

Signaling Game:

1. Nature determines type of worker using a probability distribution that assumes value θ_H with probability λ and θ_L with probability $1 - \lambda$. This move is not observed by firms.

2. Worker chooses education level (signal) conditional on her realized type: $e(\theta_H), e(\theta_L)$.

3. Both firms observe the chosen education level e of the worker and then choose wages.

4. Worker decides whether to accept one of the two offers & if so, which one.

Strategy of firm i: $w_i(e)$.

Perfect Bayesian Equilibrium: A set of strategies and a belief function $\mu(e) \in [0, 1]$ giving the firms' common probability assessment that the worker is of type θ_H after observing education level e where:

(i) The worker's strategy is optimal given the firms' strategies

(ii) The belief function $\mu(e)$ is derived from the worker's stategy using Bayes' rule wherever possible

(iii) The firms' wage offers following each e constitute a Nash equilibrium of the simultaneous move wage offer game in which the probability that the worker is of type θ_H is $\mu(e)$.

[This PBE is equivalent to a Sequential Eqm].

Work backwards.

The last stage of worker's decision is easy - worker will accept the higher wage offer and will randomize across the two firms (say, with equal probability) if they both make the same wage offer.

Now we go to third stage.

Firms have observed e and attached posterior probability $\mu(e)$ to the event that the worker is of type θ_H.

So for both firms, the (conditional) expected productivity of worker (conditional on e):

$$\mu(e)\theta_H + (1 - \mu(e))\theta_{L.}$$

Therefore, Bertrand like competition leads to a unique NE in this simultaneous move wage offer game:

$$w_1(e) = w_2(e) = w(e) = \mu(e)\theta_H + (1 - \mu(e))\theta_{L.}$$

Note $w(e) \in [\theta_L, \theta_H]$.

Separating Equilibria:

Let $e^*(\theta)$ denote equilibrium strategy of worker and $w^*(e)$ the firms'equilibrium wage schedule.

* In any separating PBE, each worker type receives a wage equal to her productivity level i.e.,

$$\begin{aligned} w^*(e^*(\theta_L)) &= \theta_L \\ w^*(e^*(\theta_H)) &= \theta_H. \end{aligned}$$

Why? Because beliefs of firms' must be consistent with equilibrium strategies.

* In any separating PBE, a low-ability worker chooses zero education i.e., $e^*(\theta_L) = 0$.

Why? Low ability worker can never lose on the wage by choosing zero education, the worst he can be thought of is that he is low ability and that still gets him the same wage as he could get in any separating equilibrium; in addition he saves on the cost of education.

Constructing a separating PBE:

Set

$$e^*(\theta_L) = 0, w^*(0) = w^*(e^*(\theta_L)) = \theta_L.$$

A low ability worker then earns utility θ_L.

Now, consider the wage θ_H that must be received by the high ability worker.

As $\theta_H > \theta_L$, the low ability worker would love to pretend to be a high ability worker.

To take away this incentive, we need to set the education level required to signal high ability type sufficiently high.

Let $\widetilde{e} > 0$ be defined by

$$\theta_L = \theta_H - c(\widetilde{e}, \theta_L).$$

As long as $e^*(\theta_H) \geq \widetilde{e}$, the low ability worker has no incentive imitate the high ability worker.

As $c(\widetilde{e}, \theta_L) > c(\widetilde{e}, \theta_H)$,

$$\theta_H - c(\widetilde{e}, \theta_H) > \theta_L$$

so that a high ability worker has no incentive to imitate a low ability worker if $e^*(\theta_H) = \widetilde{e}$ or even slightly higher.

Set

$$e^*(\theta_H) = \widetilde{e}, w^*(\widetilde{e}) = w^*(e^*(\theta_H)) = \theta_H.$$

We now need to ensure that the way $w^*(e)$ function behaves outside the points $\{0, \widetilde{e}\}$, makes it optimal for both types of workers to not deviate from their assigned education levels.

Of course, at each e, $w^*(e)$ must equal the expected productivity of worker with that education level given the beliefs $\mu(e)$ of the firms.

PBE: no restriction on how beliefs can be assigned off the equilibrium path in the signaling game.

So, we have a wide degree of freedom in choosing $w^*(e)$ on $(0, \widetilde{e})$.

For example, consider beliefs:

$$\mu^*(e) = 0, \text{if } e < \widetilde{e}.$$
$$\mu^*(e) = 1, \text{if } e \geq \widetilde{e}.$$

and the wage schedule:

$$w^*(e) = \theta_L, \text{if } e < \widetilde{e}.$$
$$w^*(e) = \theta_H, \text{if } e \geq \widetilde{e}.$$

Easy to check that neither type worker wants to deviate.

Can sustain same PBE with a variety of off-equilibrium beliefs.

Other separating PBE where $e^*(\theta_H) > \widetilde{e}$.

Separating PBE with lower value of $e^*(\theta_H)$ Pareto dominates separating PBE with higher value of $e^*(\theta_H)$.

Same wages, same output, more education cost.

Separating PBE with $e^*(\theta_H) = \widetilde{e}$ Pareto dominates all other separating PBE.

Compare separating PBE with signaling to outcome when there is no possibility of signaling (and no education cost either).

In latter case, both types of worker employed at wage $E(\theta) = \lambda\theta_H + (1-\lambda)\theta_L$.

If we allow for signaling, in the separating PBE, worker of type t employed at wage θ_t.

Low type worker worse off.

High type worker may or may not be better off depending on whether:

$$E(\theta) \lesseqqgtr \theta_H - c(e^*(\theta_H), H)$$

Observe: as λ changes, the set of separating PBE is unaffected.

As $\lambda \to 1$,the cost of education (signaling cost) in any separating PBE remains high, and nearly all workers are getting educated to signal type even though there are very few low types.

Pooling Equilibria:

$$e^*(\theta_H) = e^*(\theta_L) = e^*.$$

Belief on the equilibrium path must be:

$$\mu^*(e^*) = \lambda$$

so that

$$w^*(e^*) = \lambda\theta_H + (1 - \lambda)\theta_L = E(\theta).$$

Let e' be defined by:

$$\theta_L = E(\theta) - c(e', L)$$

* Any $e \in [0, e']$ can be sustained as $w^*(e) = \theta_L$,if $e < e'$ in a pooling PBE.

Beliefs:

$$\begin{aligned} \mu^*(e) &= 0, \text{if } e < e'. \\ \mu^*(e) &= \lambda, \text{if } e = e'. \\ \mu^*(e) &= 1, \text{if } e > e'. \end{aligned}$$

and the wage schedule:

$$\begin{aligned} w^*(e) &= \theta_L, \text{if } e < e'. \\ w^*(e) &= E(\theta), \text{if } e = e' \\ w^*(e) &= \theta_H, \text{if } e > e'. \end{aligned}$$

Pooling equilibria with higher e^* Pareto dominated by one with lower e^*.

Pooling equilibrium with $e^* = 0$ generates same outcome as a competitive outcome with no possibility of signaling.

If the outcome without signaling is Pareto inefficient, then signaling (despite its costs) can generate Pareto improvement.

Consider a situation where

$$r(\theta_L) = r(\theta_H) = r > 0$$

where

$$\theta_L < E(\theta) < r < \theta_H.$$

If signaling is not allowed, then since wage cannot exceed $E(\theta)$, we have $w < r$ & so neither type is employed.

This is Pareto inefficient as high type worker produces more at work than at home ($r < \theta_H$).

Now allow signaling through education.

Separating PBE:

As before, in any separating PBE,

$$e^*(\theta_L) = 0, w^*(0) = w^*(e^*(\theta_L)) = \theta_L$$

and since $\theta_L < r$, a low type worker must work at home (turning down offers from both firms) in such an equilibrium and receive utility r in equilibrium.

Also, in any separating PBE, high type worker must choose $e^*(\theta_H) > 0$ & be accepting employment in a firm to receive wage θ_H and utility

$$\theta_H - c(e^*(\theta_H), \theta_H).$$

Let $\widehat{e}, \overline{e}$ be defined by:

$$u(\theta_H, \overline{e} \mid \theta_H) = \theta_H - c(\overline{e}, \theta_H) = r$$

$$u(\theta_H, \widehat{e} \mid \theta_L) = \theta_H - c(\widehat{e}, \theta_L) = r$$

Can check:

$$\widehat{e} < \overline{e}$$

* Any $e \in [\widehat{e}, \overline{e}]$ can be sustained as $e^*(\theta_H)$ in a separating PBE.

If $e^*(\theta_H) < \overline{e}$, separating PBE strictly Pareto dominates the outcome with no signaling.

Asymmetric information develops between parties after contracting.

Contract has to be designed anticipating the difficulties caused by this.

Important class of such problems:

Principal- Agent Problems:

One individual (principal) hires another individual (agent) to take some actions for him.

Two kinds of informational problems can arise post-contract:

(i) Those arising from actions of agent being hidden from or unobservable by principal (moral hazard)

(ii) Those arising from some information possessed or acquired by the agent being hidden the principal.

A Model of Moral Hazard.

Principal: Owner of a firm.

Agent: Manager.

Owner hires manager for a single project.

Project's profit π depends on:

- effort $e \in \{e_L, e_H\}$ exerted by manager.

- random shock.

In particular, profit π varies on the interval $[\underline{\pi}, \overline{\pi}]$ and depends on e according to a conditional density function $f(\pi \mid e)$ where:

$$f(\pi \mid e) > 0 \text{ for all } e \text{ and } \pi \in [\underline{\pi}, \overline{\pi}].$$
$$f(\pi \mid e) = 0 \text{ for any } e \text{ and } \pi \notin [\underline{\pi}, \overline{\pi}].$$

Any potential realization of π (in $[\underline{\pi}, \overline{\pi}]$) can arise from any given effort choice by the manager.

So, observing π does not reveal effort level chosen.

Let $F(\pi \mid e)$ be the conditional distribution function associated with f.

Assume: The conditional distribution of profit given effort e_H has a strict first order stochastic dominance over that corresponding to effort level e_L.

$$F(\pi \mid e_H) \leq F(\pi \mid e_L), \forall \pi \in [\underline{\pi}, \overline{\pi}]$$
$$F(\pi \mid e_H) < F(\pi \mid e_L), \forall \pi \in \Pi \subset [\underline{\pi}, \overline{\pi}]$$

where Π is an open subset of $[\underline{\pi}, \overline{\pi}]$.

Thus:

$$E[\pi \mid e_H] > E[\pi \mid e_L].$$

Manager: expected utility maximizer with Bernoulli utility function:

$$u(w, e) = v(w) - g(e)$$

where v is twice continuously differentiable in w on $\mathbb{R}_+$ with

$$v' > 0, v'' \leq 0$$

and

$$g(e_H) > g(e_L).$$

- prefers more income to less

- weakly risk-averse over income lotteries

- dislikes high effort.

Reservation utility of agent: $\overline{u} > 0$.

Manager accepts a contract if and only if it gives him an expected utility of at least $\overline{u}$.

Owner: risk neutral and maximizes expected profit net of payment made to manager.

Assume: If manager does not accept offer, then owner gets zero

Assume: owner finds it profitable to make the manger an offer that he will accept

(i.e., its not the case that the owner makes an offer knowing that it will be rejected).

First Best (Full information):

Effort is Observable.

Contract specifies the manager's effort $e \in \{e_H, e_L\}$ and wage payment as a function of observed profits $w(\pi)$.

The first best optimal contract:

$$\max_{e \in \{e_H, e_L\}, w(\pi)} \int_{\underline{\pi}}^{\overline{\pi}} (\pi - w(\pi)) f(\pi \mid e) d\pi$$

$$s.t. \int_{\underline{\pi}}^{\overline{\pi}} v(w(\pi)) f(\pi \mid e) d\pi - g(e) \geq \overline{u}$$

Solve it in two stages:

1. For any e, what is the optimal $w(\pi)$ so that the manager still accepts the contract?

2. What is the optimal e?

First, consider (1).

Given e, the problem

$$\max_{w(\pi)} \int_{\underline{\pi}}^{\overline{\pi}} (\pi - w(\pi)) f(\pi \mid e) d\pi$$

$$s.t. \int_{\underline{\pi}}^{\overline{\pi}} v(w(\pi)) f(\pi \mid e) d\pi - g(e) \geq \overline{u}$$

reduces to

$$\min_{w(\pi(} \int_{\underline{\pi}}^{\overline{\pi}} w(\pi) f(\pi \mid e) d\pi$$

$$s.t. \int_{\underline{\pi}}^{\overline{\pi}} v(w(\pi)) f(\pi \mid e) d\pi - g(e) \geq \overline{u}$$

The constraint is always binding (otherwise the manager's wages can always be lowered while giving him his reservation utility).

$$\mathcal{L} = \int_{\underline{\pi}}^{\overline{\pi}} w(\pi) f(\pi \mid e) d\pi$$

$$+\gamma[\int_{\underline{\pi}}^{\overline{\pi}} v(w(\pi)) f(\pi \mid e) d\pi - g(e) - \overline{u}]$$

$$= \int_{\underline{\pi}}^{\overline{\pi}} (w(\pi) + \gamma v(w(\pi))] f(\pi \mid e) d\pi - g(e) - \overline{u}$$

Taking the first order condition with respect to the manager's wage at each level of π :

$$[-1 + \gamma v'(w(\pi))] f(\pi \mid e) = 0$$

which implies:

$$v'(w(\pi)) = \frac{1}{\gamma}$$

If $v" < 0$, then this implies that for all $\pi \in [\underline{\pi}, \overline{\pi}]$,

$$w(\pi) = v'^{-1}(\frac{1}{\gamma}), \text{ a constant}$$

so that the unique optimal compensation scheme is one that is constant valued for all π i.e., a fixed wage scheme.

The risk neutral owner should fully insure the risk averse manager against any risk in his income stream.

The fixed wage $w^*(e)$ satisfies:

$$v(w^*(e)) - g(e) = \overline{u}.$$

As $g(e_H) > g(e_L)$, the manager' wage is higher if the contract gets him to choose e_H rather than e_L.

If manager is risk neutral:

Suppose $v(w) = w$.

Any compensation scheme $w(\pi)$ that gives the manager an expected wage payment equal to $\overline{u} + g(e)$ is optimal.

This includes the fixed wage scheme $w^*(e)$ described above.

As for optimal choice of e :

$$\max_{e \in \{e_H, e_L\}} \int_{\underline{\pi}}^{\overline{\pi}} \pi f(\pi \mid e) d\pi - v^{-1}(\overline{u} + g(e)).$$

The optimal fixed wage scheme follows.

The above is the unique first best scheme if $v" < 0$.

It is *a* first best scheme if $v" = 0$ i.e., manager is risk neutral.

Second Best: Optimal Contract When Effort is Not Observable.

Contract cannot specify an effort level.

Use $w(\pi)$ to indirectly induce the right kind of effort.

Case of Risk-Neutral Manager.

Assume $v(w) = w$.

If effort was observable, firm owner would want the manager to exert effort e^* that solves:

$$\max_{e \in \{e_H, e_L\}} \int_{\underline{\pi}}^{\overline{\pi}} \pi f(\pi \mid e) d\pi - (\overline{u} + g(e))$$

and owner's net profit in this first best world is:

$$\int_{\underline{\pi}}^{\overline{\pi}} \pi f(\pi \mid e^*) d\pi - (\overline{u} + g(e^*))$$

while the manager receives an utility of $\overline{u}$.

We now claim that even when effort is not observable, the owner can offer a contract that gives the owner the same payoff (and the manager the same utility) as under full information.

This is the optimal contract.

To see this contract, first consider any compensation schedule of the form:

$$w(\pi) = \pi - \alpha$$

where α is a constant.

Essentially, the owner "sells the project" to the manager making him bear all the risk (fluctuation in profit) in return for a lump sum payment of α (independent of profit).

If the manager accepts this contract, he will effectively maximize expected profit & choose effort so as to

$$\max_{e \in \{e_H, e_L\}} [\int_{\underline{\pi}}^{\overline{\pi}} w(\pi) f(\pi \mid e) d\pi - g(e)]$$

which is equivalent to

$$\max_{e \in \{e_H, e_L\}} [\int_{\underline{\pi}}^{\overline{\pi}} \pi f(\pi \mid e) d\pi - \alpha - g(e)]$$

which implies that the manager chooses $e = e^*$, the first best optimal effort level.

The manager will accept this contract as long as

$$[\int_{\underline{\pi}}^{\overline{\pi}} \pi f(\pi \mid e^*)d\pi - \alpha - g(e^*)] \geq \overline{u}$$

So the optimal contract sets:

$$\alpha^* = [\int_{\underline{\pi}}^{\overline{\pi}} \pi f(\pi \mid e^*)d\pi - g(e^*) - \overline{u}]$$

This is also the payoff to the owner under this contract - which is exactly his first best payoff.

When both principal and agent are risk neutral, the problem of risk sharing disappears.

Manager can be given incentive to bear the full marginal return from his effort.

Efficient.

This kind of a scheme becomes problematic if :

(1) Agent is Risk averse: manager requires additional risk premium to take the entire fluctuation in profit on himself.

(2) Agent has limited liability: does not have assets to bear hugely negative profit realizations.

Risk-averse Manager.

To provide incentive for high effort, we need the manager's compensation to vary with profit - we need the manager to bear some risk.

But then, compensating him for the risk is costly.

So, there is a trade-off.

Leads to inefficiency.

Consider any fixed effort level e.

Suppose the owner wants to design an optimal incentive scheme so as induce the manager to select e.

$$\min_{w(\pi)} [\int_{\underline{\pi}}^{\overline{\pi}} w(\pi) f(\pi \mid e) d\pi$$

subject to

$$\int_{\underline{\pi}}^{\overline{\pi}} v(w(\pi)) f(\pi \mid e) d\pi - g(e) \geq \overline{u}$$

[INDIVIDUAL RATIONALITY]

$$e \text{ solves } \max_{\overline{e}} [\int_{\underline{\pi}}^{\overline{\pi}} v(w(\pi)) f(\pi \mid \overline{e}) d\pi - g(\overline{e})]$$

[INCENTIVE COMPATIBILITY

If $e = e_L$, then the optimal scheme for the owner is to offer the same fixed wage it would offer if he wanted agent to choose e_L in the first best case viz., $w_e^* = v^{-1}(\overline{u} + g(e_L))$.

With this scheme, agent chooses e_L even if effort is not observable as it has lower disutility.

This is an optimal way of getting the agent to exert this effort, as the owner can never do better than in the first best world.

If $e = e_H$, then the situation is more complicated.

The optimization problem:

$$\min_{w(\pi)} [\int_{\underline{\pi}}^{\overline{\pi}} w(\pi) f(\pi \mid e_H) d\pi$$

subject to

$$\int_{\underline{\pi}}^{\overline{\pi}} v(w(\pi)) f(\pi \mid e_H) d\pi - g(e_H) \geq \overline{u}$$

$$\int_{\underline{\pi}}^{\overline{\pi}} v(w(\pi)) f(\pi \mid e_H) d\pi - g(e_H)$$

$$\geq \int_{\underline{\pi}}^{\overline{\pi}} v(w(\pi)) f(\pi \mid e_L) d\pi - g(e_L)$$

Let $\gamma \geq 0$ and $\mu \geq 0$ denote the multipliers on the two constraints.

The Kuhn-Tucker first order condition implies that at every $\pi \in [\underline{\pi}, \overline{\pi}]$,

$$
\begin{aligned}
f(\pi \mid e_H) &= \gamma v'(w(\pi)) f(\pi \mid e_H) \\
+\mu[v'(w(\pi))(f(\pi \mid e_H) &- f(\pi \mid e_L))].
\end{aligned}
$$

Suppose $\gamma = 0$.

Under our assumptions, there exists an open segment of profit levels in $[\underline{\pi}, \overline{\pi}]$ such that $f(\pi \mid e_H) < f(\pi \mid e_L)$, so that for such levels of π,

$$v'(w(\pi)) \leq 0,$$

a contradiction.

Hence, $\gamma > 0$ and the individual rationality constraint is binding.

Suppose $\mu = 0$. Then:

$$\gamma v'(w(\pi)) = 1, \forall \pi \in [\underline{\pi}, \overline{\pi}]$$

so that the optimal scheme would be a fixed wage scheme.

However, under any fixed wage scheme, the manager would always choose e_L as it yields lower disutilty and so it contradicts the hypothesis that the scheme implements e_H.

Hence, $\mu > 0$ and the incentive compatibility constraint is binding.

The FOC can be re-written as:

$$\frac{1}{v'(w(\pi))} = \gamma + \mu[1 - \frac{f(\pi \mid e_L)}{f(\pi \mid e_H)}].$$

$[\frac{f(\pi|e_L)}{f(\pi|e_H)}]$ is the ratio of the "likelihood" of getting profit level π when effort level is e_L to that when effort level is e_H.

The condition says that the wages should vary with profit according to changes in the likelihood ratio.

Monotone Likelihood Ratio Property: $\frac{f(\pi|e_L)}{f(\pi|e_H)}$ is decreasing in π.

This is not implied by first order stochastic dominance.

If MLRP holds, the optimal $w(\pi)$ is increasing in π.

Otherwise, it may be non-monotonic.

Finally, recall that in the full information case where effort is observable, a fixed wage payment could implement e_H and that wage was $w^*_{e_H} = v^{-1}(\overline{u} + g(e_H))$.

With effort unobserved, the optimal scheme $w(\pi)$ for implementing e_H is not a fixed wage scheme and since

$$E[v(w(\pi))] - g(e_H) = \overline{u}$$

it follows from Jensen's inequality that

$$v(E(w(\pi))) > E[v(w(\pi))] = g(e_H) + \overline{u}$$

so that

$$E(w(\pi)) > v^{-1}(\overline{u} + g(e_H)) = w^*_{e_H}.$$

For the manager to get reservation utility $\overline{u}$ with a risky compensation scheme, the mean wage must be higher (to compensate for the variance) than in the risk-less fixed wage case.

This immediately implies that if the owner tries to implement e_H he will be making higher expected payment to the manager than under observability of effort.

His own expected profit from implementing e_H must be lower than under observability.

To decide which effort to implement, the owner compares whether

$$\int_{\underline{\pi}}^{\overline{\pi}} \pi f(\pi \mid e_H) d\pi - \int_{\underline{\pi}}^{\overline{\pi}} \pi f(\pi \mid e_L) d\pi \gtreqless E[w(\pi) \mid e_H] - w^*_{e_L}$$

In contrast, when effort is observable (first best), the owner decides which effort to implement according to whether

$$\int_{\underline{\pi}}^{\overline{\pi}} \pi f(\pi \mid e_H) d\pi - \int_{\underline{\pi}}^{\overline{\pi}} \pi f(\pi \mid e_L) d\pi \gtreqless w^*_{e_H} - w^*_{e_L}$$

Since, $E[w(\pi) \mid e_H] > w^*_{e_H}$, it follows that

* if in the first best situation, the owner implements e_L, then he will choose to do so in the second best situation through the same fixed wage scheme as in the first best case. So, there is no inefficiency in that case.

* if in the first best situation, the owner implements e_H, then there are two possibilities:

(a) he will still find it optimal to do so in the second best case; in which case he will choose a non-constant wage scheme $w(\pi)$ where $E[w(\pi) \mid e_H] > w^*_{e_H}$ and will make less expected profit than in the first best case.

(b) he will no longer find it optimal to implement e_H in the second best case and will choose the fixed wage $w^*_{e_L}$ to implement e_L earning the same level of expected profit as he would have earned in the first best case if he chosen to implement e_L instead of e_H.

Since e_H is the optimal effort level in the first best case, he earns less expected profit than in the first best case.

In both (a) and (b), the worker will still earns the same expected utility as in the first best case viz., $\overline{u}$.

Thus, there is inefficiency resulting from non-observability and the second best payoff is lower for the owner.