

Strategic segmentation of a market

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Abstract

When competing firms target information towards specific consumers through direct marketing activities, complete segmentation of markets can result. We analyze a two-stage duopoly where, prior to price competition, each firm targets information to specific consumers and only consumers informed by a firm can buy from it. This has the effect of endogenously determining market segments in a model of ‘sales’. In equilibrium, pure local monopoly emerges; firms target and sell to mutually exclusive market segments. When the cost of marketing approaches zero, market shares reflect relative production efficiency (equal shares when firms are symmetric); this may not be the case when marketing cost is high. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The last two decades have been characterized by a rapid increase in direct marketing activities of firms. These include a wide range of promotional and selling activities where firms identify, target and directly contact potential

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consumers through telephone, direct mail, sales representatives, personalized newspaper inserts etc. The New York based Direct Marketing Association estimates that in 1996, 58% of all marketing expenditure in the U.S. was for some form of direct marketing (The Economist, 1997). Businesses have developed extensive databases containing vast amount of information about actual and potential customers leading to a rapid increase in the ability of firms to target potential customers with extreme precision. When competing firms decide which specific consumer lists they wish to target, important strategic considerations arise. In particular, the extent of overlap in the sections of consumers targeted by firms with similar promotions and product information determines the extent of market segmentation and monopoly power. This paper examines the extent and pattern of market segmentation that can result from strategic direct marketing and targeted promotional activities by competing firms.

We analyze a simple model of homogenous good duopoly where, *ex ante*, consumers are unaware of the existence of different firms operating in the market. Firms simultaneously indulge in promotional direct marketing which is targeted towards specific consumers informing them about their existence. Only consumers who are targeted by the direct marketing strategy of a specific firm become aware of it and may choose to buy from this firm.¹ In the next stage, firms engage in price competition. Firms may, if they so wish, completely segment the market by targeting disjoint sets of customers leading to pure local monopoly. At the other extreme, target sets of firms may be identical in which case we have the classical Bertrand outcome. We analyze the sub-game perfect equilibria of this two stage game.

Beginning with the model of sales by Varian (1980), there has developed a significant literature dealing with competition in markets which are exogenously segmented. Narasimhan (1988) and Deneckere et al. (1992) analyze the outcomes of price competition in a homogenous good symmetric duopoly where some consumers are loyal to a particular firm; equilibrium in this market necessarily involves mixed strategies and the firm with the larger captive segment is less aggressive in price competition.² In our paper, the size of the captive segments of competing firms is endogenous.

Following the classic article by Butters (1977), there have been a number of studies which analyze strategic advertising in models where consumers can buy only from firms from which they receive informative advertisement. One section of this literature models advertising as conveying information about the existence as well as the price charged by a firm.³ Another section of this literature (closer to

¹Our analysis also extends to situations in which a consumer can choose to buy the product of a firm only if the latter provides some consumer-specific infrastructural support (e.g. cable line connection).

²See also, Shilony (1977), Baye and de Vries (1992) and Fishman (1994).

³See, among others, Grossman and Shapiro (1984), Stegeman (1991), Boyer and Moreaux (1993), Stahl (1994) and Bester and Petrakis (1995).

our paper) deals with non-price promotional activities of firms and analyzes outcomes of competition where firms decide on levels of consumer awareness prior to price competition. Thus, in a two stage sequential move incumbent-entrant game, Fudenberg and Tirole (1984) show that entry deterrence may be associated with strategic under-investment in awareness creation; a high level of awareness for the incumbent's product implies a large captive market for its product which reduces its incentive to respond aggressively to its competitor. Similar strategic under-investment results are established by Ireland (1993) and Fershtman and Muller (1993) in a two stage (simultaneous move) oligopoly. The main difference between our paper and this class of multi-stage games is that we endow firms with the ability to precisely target the disseminated information to specific consumers.⁴

We show that, in equilibrium, the entire market is divided into mutually exclusive captive segments where each firm acts as a pure local monopolist. Thus, in comparison with models of non-targeted advertising, the possibility of targeted marketing increases the extent of segmentation and monopoly power. We characterize the allocations of market shares which are sustainable in equilibrium. There is a continuum of equilibrium allocations. The set of such allocations contracts when the marginal cost of informing consumers is reduced; when this cost approaches zero, the set of equilibrium allocations contracts to a unique outcome, viz. one in which each firm is a local monopolist over exactly half the market. The intuition is as follows: if informing consumers is relatively costless and a firm does not leave a reasonable share of the market for its rival, then the latter finds it optimal to encroach on the firm's target group of consumers and compete aggressively in prices thus reducing its expected profit. On the other hand, if the cost of informing additional consumers is large, the marginal cost of encroaching on the territory covered by a rival firm becomes high and therefore, a firm might choose to accommodate an unequal division of the market. The result can be generalized to the case of asymmetric duopoly.

Section 2 sets up the model formally. Section 3 outlines the equilibrium profit from price competition in the second stage. Section 4 analyses the equilibria of the reduced form game in the first stage and characterizes the set of subgame perfect outcomes. Section 5 provides an outline of how the results generalize when we allow for firms with asymmetric production costs.

2. The model

Consider the market for a homogenous good with two firms – we shall refer to them as firms 1 and 2, respectively. We use the index k for a typical firm; while indices i and j distinguish between rival firms. The firms produce their output at

⁴Basu and Bell (1991) analyze a two stage game where competing lenders, who are also landlords, first hire workers for farm production and these specific workers become captive borrowers later.

constant unit cost denoted by c . There is a continuum of identical consumers whose total measure is normalized to be 1. Consumers are located uniformly on the unit interval $[0,1]$. Location is not a proxy for taste; it simply specifies the address of a consumer. Firms are fully informed about the location of all consumers. Initially, consumers are unaware of the existence of either firm. Firms target their direct marketing activities towards specific consumers – in this case, informing them about their existence. Consumers have unit demand i.e. buy either 0 or 1 unit of the commodity. The gross surplus from consuming a unit is $\theta > 0$. A consumer who becomes aware of the existence of only one firm, buys from that firm if the price charged does not exceed θ . If a consumer becomes aware of both firms, she buys from the firm offering the lower price, provided it does not exceed θ (if both prices are equal, she randomizes between the firms with equal probability). Consumers not targeted by either firm do not buy.⁵

If the cost of direct marketing is not consumer location specific and all consumers are otherwise identical, a firm could target any subset of the unit interval. For example, if both firms wish to target non-overlapping halves of the market, any two disjoint sets with measure $1/2$ whose union is the unit interval, could be used as target sets. However, all such outcomes would be equivalent in terms of the market price, profits and welfare. In order to keep the analysis and notation clean, we formally assume that firm 1 targets consumers located in the sub-interval $[0, \alpha_1]$ for some chosen $\alpha_1 \in [0,1]$, while firm 2 targets consumers on $[1 - \alpha_2, 1]$ for some chosen $\alpha_2 \in [0,1]$.⁶ Thus, the marketing decision by each firm is simply to choose a number in the unit interval. We shall refer to α_k , $k = 1,2$, as the market coverage of firm k .

The marketing cost for each firm k is given by the function $\mu f(\alpha_k)$, where $\mu > 0$ and f is a continuously differentiable convex function on the unit interval, $f(0) = 0$ and $f'(x) > 0$ on the unit interval. We allow for the case where f is linear which represents a situation where the cost of informing a consumer is constant (independent of how many other consumers are informed or their location). However, in much of the strategic advertising literature, it is assumed the marginal cost of informing is increasing in the number of consumers informed (this would be consistent with the situation described in footnote 6). We also assume that:

$$\mu f'(1) + c < \theta$$

This implies that the marginal cost of producing and selling a unit to a consumer is

⁵In the marketing literature, it is fairly well recognized that consumers' choice set (or 'evoked set') depends on their awareness and may not include all products and firms in the market (see Kotler, 1997).

⁶This is quite reasonable if firms differ in their cost of marketing to specific consumers and for any firm, the cost of marketing differs across consumers. For example, if firms differ in their geographical location and send sales representatives to reach spatially dispersed consumers.

always less than her willingness to pay for it. The game proceeds in two stages. In the first stage, firms 1 and 2 simultaneously choose their market coverage α_1 and α_2 . In the next stage, they simultaneously set prices p_1 and p_2 , $p_k \geq c$. The payoff to each firm is its expected profit, net of marketing cost. The solution concept used is subgame perfection.

3. Price competition

In this section, we outline the Nash equilibria of the subgames in the second stage where firms set prices, given their decisions about market coverage. Consider any (α_1, α_2) , where $\alpha_1 + \alpha_2 > 0$. Let $n_i = \min(\alpha_i, 1 - \alpha_j)$, $j \neq i$, denote the size of the captive segment of firm i , $i = 1, 2$. Further, let $m = \max(\alpha_1 + \alpha_2 - 1, 0)$ denote the size of the contested segment of consumers. Fig. 1 depicts the contested and captive segments in a situation where $\alpha_1 + \alpha_2 > 1$, $0 < \alpha_k < 1$. Let $\pi_k, k = 1, 2$, denote the expected profit of firm k in the product market, gross of marketing cost.

Given n_k and m , let \underline{p}_k denote the critical price level such that if firm k charges a price below this critical level, its profit, even when it sells to all the consumers covered by it, is less than what it can earn by selling only to its captive market segment at monopoly price i.e.

$$(\underline{p}_k - c)(n_k + m) = (\theta - c)n_k \tag{1}$$

so that

$$\underline{p}_k = c + [(n_k / (n_k + m))(\theta - c)], \quad k = 1, 2 \tag{2}$$

A firm k will never charge a price below \underline{p}_k with positive probability. Note that $\underline{p}_k \geq c$. If firm k has no captive segment ($n_k = 0$), then $\underline{p}_k = c$. On the other hand, if there is no contested segment of consumers ($m = 0$), then \underline{p}_k is equal to θ , the monopoly price. If $\underline{p}_i < \underline{p}_j$, then firm i is willing to undercut its rival at prices lower than what the latter is willing to charge; firm i is the more aggressive price competitor. Further, $\underline{p}_i < \underline{p}_j$ if and only if $n_i < n_j$. The firm with a larger captive market is less aggressive in price competition.

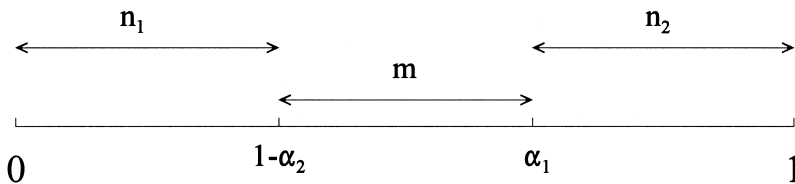


Fig. 1. Captive and contested segments of consumers when $\alpha_1 + \alpha_2 > 1$. n_k : size of the captive segment for firm k ; m : size of the contested segment.

If $m=0$, then the unique equilibrium outcome is that of local monopoly with prices $p_1=p_2=\theta$ and profits $\pi_k=(\theta-c)n_k$, $k=1,2$. If $m=1$, then $n_1=n_2=0$, the market coverages of both firms are identical and we have a case of classical Bertrand competition with profits $\pi_1=\pi_2=0$. If $0<m<1$, then there is no equilibrium in pure strategies as firm i undercuts firm j if p_j is high but prefers to charge the monopoly price and just serve the captive market if $p_j < \underline{p}_i$. There exists a mixed strategy equilibrium. Narasimhan (1988) has rigorously characterized the unique mixed strategy equilibrium when $0<m<1$ (see also Deneckere et al., 1992).

If $\underline{p}_i \leq \underline{p}_j$ (i.e. $n_i \leq n_j$), then the support of the equilibrium strategies for both firms can be shown to be equal to the interval $[\underline{p}_j, \theta]$. Furthermore, it can be shown that if $\underline{p}_i < \underline{p}_j$, then firm j charges the monopoly price θ with strictly positive probability; except for this, neither firm's equilibrium price distribution has any other mass point. Thus firm j , which is less aggressive in price competition is undercut with probability one when it charges price θ and therefore, its expected equilibrium profit is just what it would earn if it sold only to its captive segment at monopoly price, i.e. $n_j(\theta-c)$. Firm i , which is the more aggressive price competitor, earns expected profit equal to what it gets if it sells to all the (n_i+m) consumers covered by it at price \underline{p}_j , the lower bound of the price support. To summarize:

Proposition 3.1: *Let $m, n_k, k=1,2$ be as defined above. Further, suppose that $n_i \leq n_j$. The support of equilibrium price strategies for both firms is identical and equal to $[p_j, \theta]$. The (unique) Nash equilibrium expected profits (gross of advertisement cost) for the two firms are:*

$$\pi_i = n_i[(n_i + m)/(n_j + m)](\theta - c) \text{ and } \pi_j = n_j(\theta - c)$$

4. Strategic market coverage: reduced form game

In this section, we consider the reduced form game in the first stage where both firms simultaneously choose their market coverage, i.e. (α_1, α_2) , and their payoff is the Nash equilibrium profit from the resulting price subgame (as described in Proposition 3.1), net of marketing cost. Let R_i denote the net payoff to firm i . Then

$$R_i(\alpha_i, \alpha_j) = \pi_i(\alpha_i, \alpha_j) - \mu f(\alpha_i) \quad (3)$$

Suppose that firm i chooses some $\alpha_i \in [0,1]$. If firm j chooses $\alpha_j \in (1-\alpha_i)$, then the market is perfectly segmented in the next stage resulting in pure local monopoly and firm j earns monopoly profit on its captive market of size $(1-\alpha_i)$. On the other hand, if $\alpha_j > (1-\alpha_i)$ then it covers a larger market but faces non-trivial price competition in stage 2. If the chosen value of α_j is greater than both α_i and $(1-\alpha_i)$ then we have a situation where there is price competition for a segment of

consumers which are informed by both firms, the captive segment of firm j is larger than that of firm i so that firm j is the less aggressive price-competitor and its expected (gross) profit is exactly what it would get by selling only to its captive territory $(1 - \alpha_i)$ at monopoly price (see Proposition 3.1). Taking into account the additional marketing cost involved in reaching consumers in excess of $(1 - \alpha_i)$, firm j would then earn strictly lower net profit compared to what it could get by setting α_j equal to $(1 - \alpha_i)$. Therefore, given α_i , firm j never chooses values of α_j greater than $\max\{(1 - \alpha_i), \alpha_i\}$.

An immediate implication of this is that if firm i follows a timid marketing strategy which covers less than half the market, i.e. $\alpha_i \leq 1/2$, then any choice of $\alpha_j > (1 - \alpha_i)$ by firm j implies that $\alpha_j > \alpha_i$ and so the unique best response of firm j is to choose $\alpha_j = (1 - \alpha_i)$, i.e. accommodate and cover the part of the market not covered by firm i . Thus, the allocation $\alpha_1 = \alpha_2 = 1/2$ where each firm acts as a local monopolist over exactly half the market is an equilibrium outcome for all values of μ . We will see that this is also the limiting point of the set of equilibria as $\mu \downarrow 0$.

Let the critical value γ be implicitly defined by:

$$\gamma = [(\theta - c) / \{2(\theta - c) - \mu f'(1 - \gamma)\}] \tag{4}$$

It is easy to check that, $0 < \gamma < 1$ and that:

$$\gamma > \frac{1}{2} \text{ and } \gamma \downarrow 1/2 \text{ as } \mu \downarrow 0$$

We claim that: a firm finds it optimal to intrude into the territory claimed by its rival if and only if the latter aims for a market coverage exceeding γ .

Proposition 4.1: *In the reduced form first stage game, the best response function for firm j , $j = 1, 2$, $i \neq j$, is given by $\alpha_j(\alpha_i) = 1 - \alpha_i$, if $0 \leq \alpha_i \leq \gamma$, while for $\gamma < \alpha_i \leq 1$, $\alpha_j(\alpha_i) > 1 - \alpha_i$ and satisfies:*

$$0 = (\theta - c)(2\alpha_j(\alpha_i) - 1) + \mu f'(\alpha_j(\alpha_i))\alpha_i \tag{5}$$

(where γ is as defined by Eq. (4)).

Proof. We have seen that if firm j chooses $\alpha_j > (1 - \alpha_i)$, then it must choose α_j lying in the segment $((1 - \alpha_i), \alpha_i)$ and if it does so, $n_j \leq n_i$ and firm j 's expected net profit will be given by:

$$R_j(\alpha_j) = [\alpha_j \{((1 - \alpha_j) / \alpha_i)(\theta - c)\}] - \mu f(\alpha_j)$$

Taking the derivative of $R_j(\alpha_j)$ and evaluating it at the point $\alpha_j = (1 - \alpha_i)$ yields:

$$R'_j(1 - \alpha_i) = (\theta - c)[2 - (1/\alpha_i)] - \mu f'(1 - \alpha_i)$$

which implies that $R'_j \leq 0$ at $\alpha_j = 1 - \alpha_i$ if and only if $\alpha_i \leq \gamma$. If $\alpha_i > \gamma$ then the

optimal action $\alpha_j(\alpha_i) > (1 - \alpha_i)$ is obtained by setting $R'_j(\alpha_j) = 0$ which yields Eq. (5). ■

From the expression for the best response function, one can check that the best response function $\alpha_j(\alpha_i)$ is strictly decreasing in α_i on $[0, 1]$. A representative set of best response functions on the (α_1, α_2) -space are depicted in Fig. 2. The line AB is the set of pure monopoly allocations of the market, i.e. the set of (α_1, α_2) where $\alpha_1 + \alpha_2 = 1$. The best response function of firm 1 is given by BET and that for firm 2 is given by AE'S. As can be readily observed, the set of equilibrium allocations (the intersection of the best response functions) is given by the segment EE'. Thus, all equilibrium outcomes involve the emergence of pure local monopoly. The point E corresponds to the equilibrium allocation $(\gamma, 1 - \gamma)$ which is most favourable to

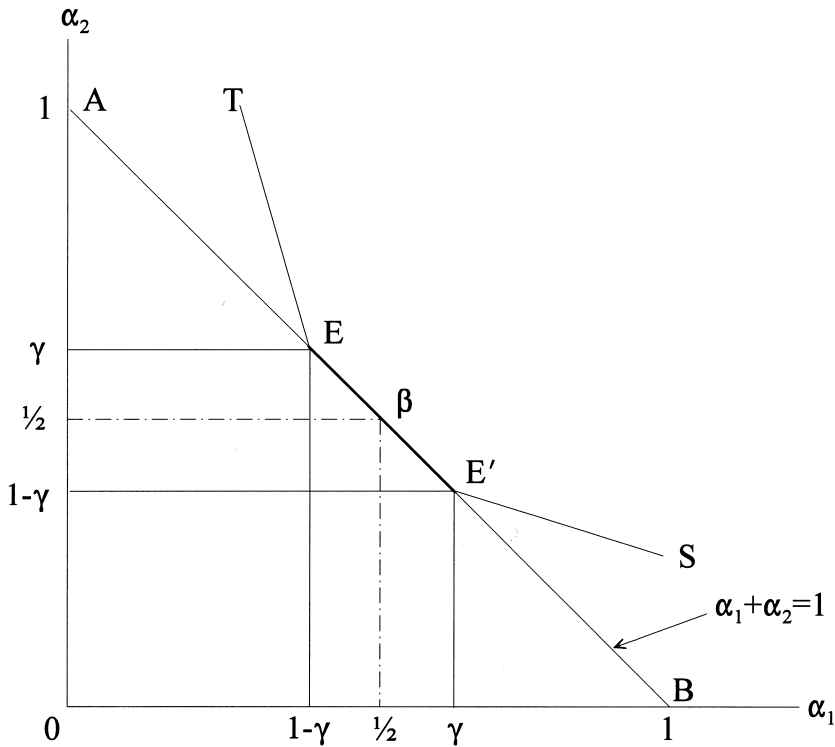


Fig. 2. Best response functions and equilibria in the (α_1, α_2) space for the (reduced form) first stage game where firms decide on their market coverage. BET, best response of firm 1; AE'S, best response of firm 2; EE', set of equilibria.

firm 1 and least favourable to firm 2; the reverse is true for the point E' representing allocation $(1-\gamma, \gamma)$. The point β is at the midpoint of EE' representing allocation $(\frac{1}{2}, \frac{1}{2})$.

Proposition 4.2 [Main Result]. *The set of pure-strategy equilibria in the (reduced form) first stage game is exactly equal to $\{(\alpha_1, \alpha_2): \alpha_1 + \alpha_2 = 1, \alpha_i \in [1-\gamma, \gamma], \alpha_j \in [1-\gamma, \gamma]\}$ which always includes the particular allocation $(1/2, 1/2)$. This set contracts as μ decreases and converges, as $\mu \downarrow 0$, to the unique point $(1/2, 1/2)$.*

Proof. From Proposition 4.1, if $\alpha_i \leq \gamma$ choosing $\alpha_j = 1 - \alpha_i$ is the unique best response of firm j . So any market allocation (α_1, α_2) where $\alpha_1 + \alpha_2 = 1$, $\alpha_1 \in [1-\gamma, \gamma]$, $\alpha_2 \in [1-\gamma, \gamma]$ is an equilibrium and in fact, these are the only equilibrium outcomes with $\alpha_i \leq \gamma$ for some i . Next, consider (α_1, α_2) where $\alpha_i = a > \gamma$ for some i . The best response for firm j is then $\alpha_j(\alpha_i) = a^*$ (say). Observe that $a^* > 1 - a$. As the best response of firm i $\alpha_i(\cdot)$ is strictly decreasing, $\alpha_i(a^*) < \alpha_i(1 - a)$. However, $a > \gamma > \frac{1}{2}$ implies $1 - a < \frac{1}{2}$, so that $\alpha_i(1 - a) = a$. Thus, $\alpha_i(a^*) < a$ which implies that there is no equilibrium allocation (α_1, α_2) where $\alpha_i > \gamma$ for some i . ■

The set of equilibria are Pareto-efficient and every equilibrium involve complete segmentation of the market; in stage 2, firms simply choose monopoly price over their territory. For firm 1, the equilibrium outcome yielding the highest payoff is $(\alpha_1 = \gamma, \alpha_2 = 1 - \gamma)$ and the lowest payoff is yielded by the outcome $(\alpha_1 = 1 - \gamma, \alpha_2 = \gamma)$; the reverse is true for firm 2. Interpret μ as a parameter that affects the marginal cost of advertisement. As μ increases, γ increases which implies that the set of equilibrium allocations expands. In terms of Fig. 2, the interval EE' becomes larger as E moves up and E' moves down on the line AB. On the other hand, as μ is reduced, the set of equilibria contracts and, in the limit as $\mu \downarrow 0$, the set of equilibria is reduced to a unique allocation $(1/2, 1/2)$ indicated by point β in Fig. 1.⁷ Note that the allocation $(1/2, 1/2)$ is an equilibrium outcome for all $\mu > 0$. For $\mu > 0$, there always exist equilibria where one firm has a greater market share even though both firms are equally efficient. As μ increases, the largest market share that each firm can attain in equilibrium increases; in terms of Fig. 2, EE' expands with μ to eventually cover the entire line AB. Lastly, observe that every equilibrium is socially efficient; firms appropriate the entire social surplus and by

⁷If μ is exactly equal to 0, then there are equilibria other than $(1/2, 1/2)$ where firms target overlapping sets and play mixed strategy in the price subgame. Such equilibria are not robust to perturbations in μ (see Roy and Schreurs, 1994).

eliminating any overlap in their market converge, firms avoid what would be a ‘duplication’ in the social cost of sending product information to consumers.⁸

5. Extension to asymmetric duopoly

The above mentioned results easily carry over to a situation where firms have asymmetric production costs. Let c_1 and c_2 denote the unit production costs of the two firms. Assume that $c_1 < c_2$ and that $\mu f'(1) + c_2 < \theta$. First, consider the stage of price competition where each firm has a captive segment n_k , $k=1,2$ and there is a competitive segment m . Then, \underline{p}_k (the critical price level below which firm k never undercuts) is given by:

$$(\underline{p}_k - c_k)(n_k + m) = (\theta - c_k)n_k$$

As before, if $\underline{p}_i < \underline{p}_j$, firm i is the more aggressive price competitor. With equal production costs, the firm with a larger captive market is less aggressive in price competition. However, with unequal costs, even if the low cost firm has a larger captive market, it may be more aggressive. The explicit derivation of mixed strategy equilibria for this case is contained in Baye and de Vries (1992) (see also Deneckere and Kovenock, 1992, 1996). If $\underline{p}_i \leq \underline{p}_j$, then the support of the equilibrium strategies for both firms can be shown to be equal to the interval $[\underline{p}_j, \theta]$. Further, firm j , which is less aggressive in price competition is undercut with probability one when it charges price θ and therefore, its expected equilibrium profit is just what it would earn if it sold only to its captive segment at monopoly price, i.e. $n_j(\theta - c_j)$. Firm i , which is more the aggressive price competitor, earns expected profit equal to what it gets if it sells to all the $(n_i + m)$ consumers covered by it at price \underline{p}_j , the lower bound of the price support.

Consider the reduced form stage 1 game where firms choose (α_1, α_2) and define a critical allocation (β_1, β_2) :

$$\beta_k = [(\theta - c_k)/(\theta - c_1) + (\theta - c_2)], \quad k = 1,2$$

Observe that, $0 < \beta_k < 1$, $k=1,2$, and that $\beta_1 + \beta_2 = 1$. Further, note that $\beta_1 = \beta_2 = \frac{1}{2}$ if $c_1 = c_2$. In fact the allocation (β_1, β_2) represents the focal split of the market in the asymmetric case just as the split $(1/2, 1/2)$ does in the symmetric case. It can be shown that the best response of firm j to any choice of α_i in $[0, \beta_i]$ is to set

⁸Suppose firms could perfectly price discriminate (by location) and, in the second stage, charge different prices in the contested and captive segments. This would lead to the classical Bertrand outcome in the contested segment and monopoly outcome in the captive segments. Once again, in the first stage, firms avoid any overlap in their territory, i.e. complete market segmentation occurs. The only difference is that every division of the market can be sustained as an equilibrium outcome.

$\alpha_j = 1 - \alpha_i$, i.e. accommodate and cover the part of the market not covered by firm i no matter how small the cost of informing additional consumers. Finally, there exists some critical value γ_i higher than β_i such that if α_i is below this critical value, the net profit (net of marketing cost) of firm j from not intruding into the market covered by firm i is greater than that obtained by intrusion. These critical values γ_1, γ_2 are defined by:

$$\gamma_i = [(\theta - c_i) / \{(\theta - c_i) + (\theta - c_j) - \mu f'(1 - \gamma_i)\}]$$

It can be easily verified that for $k=1,2$, $0 < \gamma_k < 1$, $\gamma_k > \beta_k$ and that $\gamma_k \downarrow \beta_k$ as $\mu \downarrow 0$. Further, $\gamma_1 + \gamma_2 > 1$. Also, $\gamma_1 = \gamma_2 = \gamma$, if $c_1 = c_2 = c$ and $\gamma_1 > \gamma_2$, if $c_1 < c_2$. The equilibrium set is as follows:

Proposition 5.1: *The set of pure-strategy equilibria in the (reduced form) game is exactly equal to $\{(\alpha_1, \alpha_2) : \alpha_1 + \alpha_2 = 1, \alpha_1 \in [1 - \gamma_2, \gamma_1], \alpha_2 \in [1 - \gamma_1, \gamma_2]\}$ which always includes the particular allocation (β_1, β_2) . This set contracts as μ decreases and converges, as $\mu \downarrow 0$, to (β_1, β_2) .*

Once again, every equilibrium involves complete market segmentation and the emergence of pure local monopoly. All the equilibria are Pareto-efficient. γ_1 and γ_2 are the largest market shares firms 1 and 2 can obtain in any equilibrium. As μ increases, both γ_1 and γ_2 increase which implies that the set of equilibrium allocations expands. On the other hand, as μ is reduced, the set of equilibria contracts and, in the limit as $\mu \downarrow 0$, it is reduced to the unique focal allocation (β_1, β_2) . In this allocation (β_1, β_2) firm 1 gets a larger market share ($\beta_1 > \frac{1}{2} > \beta_2$) and the difference between the market share of the two firms is strictly increasing in the difference between the production cost of the two firms. Interestingly enough, for μ large enough there may be an equilibrium where the relatively inefficient firm commands a larger market share. Lastly, equilibrium is social inefficient to the extent that the higher cost firm serves part of the market.

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