MATH 2343 2343-1p3.gif

C&H: Sec 1.3

4) $\frac{dx}{dt} = e^{t+x}$

$\int e^{-x}\,dx = \int e^t\,dt$

$-e^{-x} = e^t + c$

$x = -\ln(c - e^t)$

5) $\frac{dx}{dt} = tx + 4x + 3t + 12$

$= (x+3)(t+4)$

$\int \frac{dx}{x+3} = \int (t+4)\,dt$

$\ln(x+3) = \frac{1}{2}t^2 + 4t + c$

$x = ce^{\frac{1}{2}t^2+4t} - 3$

Note
$(x+3) = 0$ if $x = -3$
$\frac{d(-3)}{dt} = 0$
$\Rightarrow x = -3$ is also a solution

8) $\frac{dx}{dt} = t^2x^2$

$\int \frac{1}{x^2}\,dx = \int t^2\,dt$

$-\frac{1}{x} = \frac{1}{3}t^3 + c$

$x = -\frac{1}{\frac{1}{3}t^3 + c}$

Note:
$x^2 = 0$ if $x = 0$
$\frac{d}{dt}(0) = 0$
$\Rightarrow x = 0$ is also a solution

11) $\frac{dx}{dt} = x^2\cos t^2 \quad x(0) = 1$

$\int \frac{1}{x^2}\,dx = \int \cos t^2\,dt$

$\int_1^x \frac{1}{r^2}\,dr = \int_0^t \cos s^2\,ds$

$-\frac{1}{r}\Big|_1^x = \text{"}$

$-\frac{1}{x} + 1 = \text{"}$

$x = \left(1 - \int_0^t \cos s^2\,ds\right)^{-1}$

Note:
$x = 0$ is a solution but does not satisfy the IC.
Not a solution to the IVP

13) $\frac{dx}{dt} = t\cos(x^{-1/2}) \quad x(1) = 2$

$\int \frac{1}{\cos(x^{-1/2})}\,dx = \int t\,dt$

$\int_2^x \frac{1}{\cos(r^{-1/2})}\,dr = \int_1^t s\,ds$

$\text{"} = \frac{1}{2}t^2 - \boxed{\frac{1}{2}}$

Implicit sol.

15) $\frac{du}{dt} = \frac{t^2+1}{u^2+4} \quad u(0) = 1$

$\int (u^2+4)\,du = \int (t^2+1)\,dt$

$\int_1^u (r^2+4)\,dr = \int_0^t (s^2+1)\,ds$

$\frac{1}{3}r^3 + 4r\Big|_1^u = \frac{1}{3}s^3 + s\Big|_0^t$

$\frac{1}{3}u^3 + 4u - \frac{1}{3}t^2 - t - \frac{13}{3} = 0$

Implicit Sol.