

C4H: Sec 1.6

$$20) \quad \frac{dx}{dt} + 2tx = t$$

$$u = e^{\int 2t dt} = e^{t^2} \quad \text{Check!}$$

$$\frac{d}{dt}(e^{t^2} x) = te^{t^2}$$

$$e^{t^2} x = \frac{1}{2} e^{t^2} + c$$

$$x = \frac{1}{2} + ce^{-t^2} \quad \underbrace{\hspace{1cm}}_{x_p} \quad \underbrace{\hspace{1cm}}_{x_h}$$

$$23) \quad (t^2+1) \frac{dx}{dt} + 2tx = 1$$

$$\frac{dx}{dt} + \frac{2t}{t^2+1} x = \frac{1}{t^2+1}$$

$$u = e^{\int \frac{2t}{t^2+1} dt} = e^{\ln(t^2+1)} = (t^2+1)$$

$$\frac{d}{dt}((t^2+1)x) = (t^2+1) \frac{1}{t^2+1} = 1$$

$$(t^2+1)x = t + c$$

$$x = \frac{t}{t^2+1} + \frac{c}{t^2+1} \quad \underbrace{\hspace{1cm}}_{x_p} \quad \underbrace{\hspace{1cm}}_{x_h}$$

$$24) \quad \frac{du}{dt} = 3(u-1)$$

$$\frac{du}{dt} - 3u = -3$$

$$I.F. = e^{\int -3 dt} = e^{-3t}$$

$$\frac{d}{dt}(e^{-3t} u) = -3e^{-3t}$$

$$e^{-3t} u = e^{-3t} + c$$

$$u = 1 + ce^{3t}$$

OR solve by Sep of Var

$$\int \frac{du}{u-1} = \int 3 dt$$

$$\ln(u-1) = 3t + c$$

$$u-1 = ce^{3t}$$

$$u = ce^{3t} + 1$$

$$39) \quad \frac{dx}{dt} + e^t x = 3$$

$$u = e^{\int e^t dt} = e^{e^t}$$

$$\frac{d}{dt}(e^{e^t} x) = 3e^{e^t}$$

$$e^{e^t} x = 3 \int^t e^{e^s} ds + c$$

$$x = ce^{-e^t} + 3e^{-e^t} \int^t e^{e^s} ds$$

$$45) \quad 7t \frac{dx}{dt} + x = e^t$$

$$\frac{dx}{dt} + \frac{1}{7t} x = \frac{1}{7t} e^t$$

$$u = e^{\int \frac{1}{7t} dt} = e^{\frac{1}{7} \ln t} = e^{\ln t^{1/7}} = t^{1/7}$$

$$\frac{d}{dt}(t^{1/7} x) = t^{1/7} \frac{1}{7t} e^t$$

$$= \frac{1}{7t^{6/7}} e^t$$

$$t^{1/7} x = \frac{1}{7} \int^t \frac{1}{s^{6/7}} e^s ds + c$$

$$x = ct^{-1/7} + \frac{1}{7} t^{-1/7} \int^t s^{-6/7} e^s ds$$