Math 3313: Exponential growth and decay

(1) (*Based on Sec. 1.8 #9*)
(a) An organism living in a pond reproduces at a rate r_1 proportional to the population size. Organisms also die off at a rate r_2 proportional to the population size. Formulate and solve the differential equation for the biomass of organisms in the pond if initially there is 1 kg.
(b) How long will it take for the biomass in the pond to triple.
(c) Suppose that in addition to normal birth and death process, organisms are continuously added at a rate of k g/yr. Now how long does it take for the biomass to double?

(2) (*Based on Sec. 1.8 #10*)
(a) A bacteria population is reproducing in a large vat of nutrients according to an exponential growth law that would cause the population to double in 0.5 h. However, bacteria are continuously siphoned off at a rate of 5 g/h. Initially, there are 10 g of bacteria. How many bacteria are there after 2 hours?
(b) What should the rate of siphoning bacteria be for the population to remain at equilibrium of 10 g?

(3) *Based on Sec. 1.8 #19*)
A radioactive isotope sits unused in the laboratory for 10 years at which time it is found to contain only 80% of the original amount.
(a) What is the half-life of the material.
(b) How many additional years will it be when only 10% of the original amount remains?

(4) (*Based on Sec. 1.8 #27*)
(Assume continuous compounding.) You invest \$2000 in an account that pays 6% annually on the amount in excess of \$500.
(a) Formulate and solve the appropriate differential equation to determine the amount you have after 10 years?
(b) When does you balance double?
(c) Suppose during the 10 year period you make withdrawals at the constant annualize rate of \$200 / year. Now how much money do you have after 10 years?
(d) Suppose that you invest/deposit additional funds into the account at the time dependent rate of $d(t) = 1 + \cos(\frac{24\pi}{t}$ (\$/ year) (there is a peak deposit every month). Determine (but do not solve) the differential equation for this case?