MATH 2343 2343-1p9.gif

C&H: Sec 1.9

1.9 #1 $V(t) = 300$ gal for all time.

$Q(t)$ = Amount in units of lbs.

$C(t)$ = Concentration lbs/gal.

$Q(0) = 0.2 \frac{lb}{gal} \cdot 300 \text{ gal} = 60 \text{ lb}$

$\left[\frac{dQ}{dt}\right]_{in} = 2 \frac{gal}{min} \cdot 0.4 \frac{lb}{gal} = 0.8 \frac{lb}{min}$

$\left[\frac{dQ}{dt}\right]_{out} = +2 \frac{gal}{min} \cdot \frac{Q(t)\, lb}{V(t)\, gal} = \frac{2}{300} Q \frac{lb}{min}$

$= \frac{Q(t)}{150} \frac{lb}{min}$

$\frac{dQ}{dt} = \left[\frac{dQ}{dt}\right]_{in} - \left[\frac{dQ}{dt}\right]_{out} = 0.8 - \frac{Q}{150}$

1st ORDER, NONHOMO., CONSTANT COEFF.

SOLVE USING METHOD OF UNDETER. COEFF.

$Q_h$: $\frac{dQ}{dt} = -\frac{1}{150} Q \Rightarrow Q(t) = c_0 e^{-\frac{1}{150}t}$

$p$: $\frac{dQ}{dt} + \frac{1}{150} Q = 0.8$

Guess $Q = A$ constant

$0 + \frac{A}{150} = 0.8 \Rightarrow A = 120$

$Q(t) = c_0 e^{-\frac{1}{150}t} + 120$

$Q(0) = c_0 + 120 = 60 \quad c_0 = -60$

$Q(t) = -60 e^{-\frac{1}{150}t} + 120$

$C(t) = \frac{Q(t)}{V(t)} = \frac{Q(t)}{150}$

1.9 #5a Volume of Tank:

$V(0) = 500$ gal

$\left[\frac{dV}{dt}\right]_{in} = 6 \frac{gal}{min}, \quad \left[\frac{dV}{dt}\right]_{out} = 1 \frac{gal}{min}$

$\frac{dV}{dt} = \left[\frac{dV}{dt}\right]_{in} - \left[\frac{dV}{dt}\right]_{out} = 6 - 1 = 5 \frac{gal}{min}$

$V(t) = 5t + C, \quad V(0) = C = 500, \quad V(t) = 5t + 500$

Quantity of Iodine:

$Q(0) = 10$ lb

$\left[\frac{dQ}{dt}\right]_{in} = 0 \frac{lb}{gal} \cdot 6 \frac{gal}{min} = 0$

$\left[\frac{dQ}{dt}\right]_{out} = 1 \frac{gal}{min} \cdot \frac{Q(t)\, lb}{V(t)\, gal} = \frac{Q(t)}{V(t)} \frac{gal}{min}$

$\frac{dQ}{dt} = \left[\frac{dQ}{dt}\right]_{in} - \left[\frac{dQ}{dt}\right]_{out} = -\frac{Q(t)}{V(t)}$

$\frac{dQ}{dt} = -\frac{Q(t)}{5t+500}$

1st order, Homogeneous / Solve using SEP.

$\frac{1}{Q} dQ = -\frac{1}{5t+500} dt$

$\ln Q(t) - \ln Q(0) = -\frac{1}{5}\left(\ln(5t+500) - \ln(500)\right)$

$\ln\left(\frac{Q(t)}{Q(0)}\right) = -\frac{1}{5} \ln\left(\frac{5t+500}{500}\right)$

$Q(t) = Q(0) \left(\frac{1}{100}t + 1\right)^{-1/5}$

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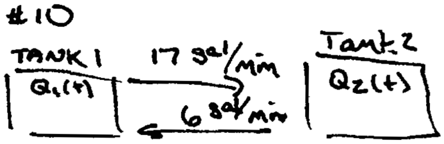
$Q(t) = 10 \left(\frac{t}{100} + 1\right)^{-1/5}$

$Q(100) = 10(1+1)^{-1/5} \approx 8.7$ lb

$C(100) = Q(100)/V(100) = \frac{8.7}{1000}$ lb/gal

1.9 #10




Volume: Tank 1:

$\frac{dV_1}{dt} = \left[\frac{dV}{dt}\right]_{in} - \left[\frac{dV}{dt}\right]_{out} = 6 - 17 \frac{gal}{min} = -11$

$V_1(t) = -11t + C, \quad V_1(0) = 100, \quad V_1(t) = 100 - 11t$

$\frac{d}{dt} = \left[\frac{dV}{dt}\right]_{in} - \left[\frac{dV}{dt}\right]_{out} = 17 - 6 \frac{gal}{min} = 11$

$V_2(t) = 11t + C, \quad V_2(0) = 400, \quad V_2(t) = 400 + 11t$

AMOUNT TANK 1

$\left[\frac{dQ_1}{dt}\right]_{in} = 6 \frac{gal}{min} \cdot \frac{Q_2\, lb}{V_2\, gal} = 6 \frac{Q_2}{V_2}$

$\left[\frac{dQ_1}{dt}\right]_{out} = 17 \frac{gal}{min} \cdot \frac{Q_1\, lb}{V_1\, gal} = 17\, Q_1/V_1$

$\frac{dQ_1}{dt} = [\;]_{in} - [\;]_{out} = \frac{6 Q_2}{400+11t} - \frac{17 Q_1}{-11t+100}$

AMOUNT TANK 2

$\left[\frac{dQ_2}{dt}\right]_{in} = 17 \frac{gal}{min} \cdot \frac{Q_1}{V_1} = +\frac{17 Q_1}{V_1}$

$\left[\frac{dQ_2}{dt}\right]_{out} = 6 \frac{gal}{min} \cdot \frac{Q_2}{V_2} = \frac{6 Q_2}{V_2}$

$\frac{dQ_2}{dt} = \frac{+17 Q_1}{100-11t} - \frac{6 Q_2}{400+11t}$