Math 3313: Differential Equations
First-order ordinary differential equations

Thomas W. Carr

Department of Mathematics
Southern Methodist University
Dallas, TX

Outline

Outline

Math 2343: Introduction

Separable ODEs

Linear, non-homogeneous

Graphical analysis

Numerical Approximation

Applications

What's it about?

The change in $x(t)$ w/rt t is given by the function $f(x, t)$.

$$\frac{dx(t)}{dt} = f(x, t) \tag{1}$$

Goals of this class.

- What is $x(t)$?
  Solution methods?
- Where did the differential equation come from?
  Modeling
- Interpretation
  What does the solution $x(t)$ say about the "physics"?

Some terms

Ordinary Differential Equation (ODE)

Has only 1 independent variable t.

$$\frac{dx}{dt} = f(x, t) \tag{2}$$

Partial Differential Equation (PDE)

Has only 2 or more independent variables (ex. time & space).

$$\frac{\partial x}{\partial t} = \frac{\partial^2 x}{\partial z^2} \qquad \text{(Diffusion)} \tag{3}$$

1st order ODE $\rightarrow$ highest derivative is 1. $\frac{dx}{dt} = f(x, t)$

2nd order ODE $\rightarrow$ highest derivative is 2. $\frac{d^2 x}{dt^2} = f(x, \frac{dx}{dt}, t)$

Linear vs. Nonlinear

Linear n^{th} order

$$a_n(t)\frac{d^n x}{dt^n} + a_{n-1}(t)\frac{d^{n-1} x}{dt^{n-1}} + \ldots + a_1(t)\frac{dx}{dt} + a_0(t)x = f(t) \quad (4)$$

x, $\frac{dx}{dt}$, ... $\frac{d^n x}{dt^n}$ appear linearly.
No x^2, $\sin(x)$, $x\frac{dx}{dt}$, ...
t doesn't matter. t^2, $\sin(t)$, ... are OK.

Nonlinear

$$\frac{dx}{dt} = \sin(x), \quad \left(\frac{dx}{dt}\right)^2 + x\frac{dx}{dt} = 2.$$

$\sin(x)$, $(\frac{dx}{dt})^2$, $x\frac{dx}{dt}$ are nonlinear functions of x.

Solutions must satisfy the ODE

ex. Is $x(t) = ce^{-3t}$ a solution to

$$\frac{dx}{dt} = -3x ? \tag{5}$$

Substitute and check!

ex. Is $x(t) = c_1 \sin 2t + c_2 \cos 2t$ a solution to

$$\frac{d^2 x}{dt^2} + 4x = 0? \tag{6}$$

Substitute and check!

Make it a habit to check!
Find x then substitute.

Explicit vs. Implicit

Explicit: when we can solve for $x = F(t)$.

Implicit: Left with $G(x(t), t) = 0$.

You don't have $x =$ something. Instead, it's defined implicitly by the function G.

ex. Let $G(x, t) = x^3 + x - t + \frac{1}{t} + c$. Show that $G = 0$ is a solution to

$$t^2 \frac{dx}{dt} = \frac{t^2 + 1}{3x^2 + 1} \tag{7}$$

Substitute and check!

Solving implies "integrating"

ex.

$$\frac{dx}{dt} = \sin t \qquad \text{(1st order, linear)} \tag{8}$$

What function x has derivative $\sin t$?

$$x(t) = \int \sin t \; dt + c = -\cos t + c.$$

ex.

$$\begin{aligned}
\frac{d^3x}{dt^3} &= 1 \qquad \text{(3rd order, linear)} \tag{9} \\
\frac{d^2x}{dt^2} &= t + c_1 \\
\frac{dx}{dt} &= \frac{1}{2}t^2 + c_1 t + c_2 \\
x &= \frac{1}{6}t^3 + \frac{c_1}{2}t^2 + c_2 t + c_3.
\end{aligned}$$

Every time we integrate we pick up a constant.

The constants and Initial Conditions

The solution to an nth order ODE will have n constants.
Specify the c_j with n initial conditions (ICs)

ex.

$$\frac{dx}{dt} = \cos \gamma t \quad AND \quad x(t = \frac{\pi}{2\gamma}) = 3. \tag{10}$$

Integrate and solve for c!

The *general solution*: $x(t) = \frac{1}{\gamma} \sin \gamma t + c$

The solution to the *initial value problem*: $x(t) = \frac{1}{\gamma} \sin \gamma t + (3 - \frac{1}{\gamma})$

ex.

$$\frac{dx}{dt} = te^{t^2}, \quad x(0) = 1. \tag{11}$$

Integrate (definite vs. indefinite integration)

Outline

What is "separable"?

Given

$$\frac{dx}{dt} = f(t) \qquad \text{Integrate}$$

$$\frac{dx}{dt} = f(x,t) \qquad \text{Need other methods (if at all doable)}$$

$$\frac{dx}{dt} = f(x,t) = g(x)h(t) \qquad \text{SEPARABLE}$$

$f(x,t)$ is the product of a function of x (g) with a function of t (h).

ex.

$$f(x,t) = tx \qquad \rightarrow \quad g(x) = x, \ h(t) = t$$
$$f(x,t) = x^2 e^t \qquad \rightarrow \quad g(x) = x^2, \ h(t) = e^t$$
$$f(x,t) = \sin(xt) \qquad \rightarrow \quad ???$$

Separation of variables

1. Given:

$$\frac{dx}{dt} = g(x)h(t)$$

2. Separate x and t:

$$\frac{1}{g(x)}\frac{dx}{dt} = h(t)$$

3. Relabel $1/g = p(x)$:

$$p(x)\frac{dx}{dt} = h(t)$$

4. Integrate w/rt t

$$\int p(x)\frac{dx}{dt}\,dt = \int h(t)\,dt$$

5. Integrate w/ substitution

Let $u = x(t) \Rightarrow \quad \frac{du}{dt} = \frac{dx}{dt} \Rightarrow \quad du = \frac{dx}{dt}\,dt$

$$\int p(u)\,du = \int h(t)\,dt$$

antider of $p(u)|_{u=x}$ = antider of $h(t)$

May or may not be able to integrate.

Separation: examples

ex. Find the general solution to

$$\frac{dx}{dt} = -6tx \qquad (12)$$

Separate and solve.

ex.

$$\frac{dx}{dt} = (x-1)^2 \sin t \qquad (13)$$

Separate and solve.

ex. Find the solution to the initial value problems below.

$$t^2 \frac{dx}{dt} = \frac{t^2+1}{3x^2+1}, \quad x(1) = 2. \qquad (14)$$

Separate and solve.

ex.

$$\frac{dx}{dt} = te^{t^2+x^2}, \quad x(t_0) = x_0 \qquad (15)$$

Separate and solve.

Linear, constant coefficient

Special case of separable ODE

$$\frac{dx}{dt} = kx \tag{16}$$

Linear: $\frac{dx}{dt}$ and x are linear functions of x.
Constant coefficient: 1 and k.

$$
\begin{aligned}
\frac{1}{x}dx &= k\,dt \\
\ln x &= kt + c \\
x &= e^{kt+c} \\
x &= e^c e^{kt} \\
x &= \tilde{c} e^{kt}
\end{aligned}
$$

Linear, constant-coefficient ODEs always have e^{rt} as solutions.

Linear, constant coefficient: examples

ex.

$$\frac{dx}{dt} = 5x \tag{17}$$

Substitute and find r.

ex.

$$\frac{dx}{dt} - kx = 0 \tag{18}$$

Substitute and find r.

Outline

What is "linear, nonhomogenous"?

Given

$$\frac{dx}{dt} = f(t) \qquad \text{Integrate}$$

$$\frac{dx}{dt} = f(x,t) = g(x)h(t) \qquad \text{separate}$$

$$\frac{dx}{dt} = f(x,t) = kx \qquad \text{Const. coeff.} \rightarrow e^{rt}$$

Now

$$\frac{dx}{dt} = f(x,t) \quad \Rightarrow \quad \frac{dx}{dt} + p(t)x = f(t) \qquad (19)$$

$p(t)$: variable coefficient. Function of t.
$f(t)$: nonhomogenous/forcing term. Function of t.

Some properties

The $f(t)$ prevents separation.

$$\frac{1}{x}\frac{dx}{dt} = -p(t) + \frac{f(t)}{x}$$

The solution to a linear n^{th} order nonhomog. ODE has 2 parts.

$x(t) = x_h(t) + x_p(t) =$ homogeneous sol. $+$ particular sol.

$$x_h: \quad \frac{dx_h}{dt} + p(t)x_h = 0 \quad (f = 0).$$

$$x_p: \quad \frac{dx_p}{dt} + p(t)x_p = f(t)$$

Check. Let $x = x_h + x_p$. Substitute into original. Use the above to cancel terms.

Integrating factor method

Put ODE into standard form

Step 1: $\dfrac{dx}{dt} + p(t)x = f(t)$

Multiply by I.F. $u(t)$ (unknown)

$u(t)\dfrac{dx}{dt} + u(t)p(t)x = u(t)f(t)$

Choose u such that

$$u(t)p(t) = \dfrac{du}{dt}$$

Make replacement

$$u(t)\dfrac{dx}{dt} + \dfrac{du}{dt}x(t) = u(t)f(t)$$

This is the result of product rule.

$$\dfrac{d}{dt}(ux) = u\dfrac{dx}{dt} + \dfrac{du}{dt}x$$

Make replacement

Step 3: $\dfrac{d}{dt}(u\,x) = uf$

Integrate

Step 4: $\displaystyle\int \dfrac{d}{dt}(u\,x)\,dt = \int u\,f\,dt$

$$u\,x = \int u\,f ds + c$$

Solve for x.

The method:
Choose u so we can integrate.

$$\dfrac{du}{dt} = p(t)u.$$

$$\dfrac{1}{u}du = p(t)dt$$

Step 2: $u = e^{\int p(t)dt}$

A first example

ex.

$$\frac{dx}{dt} = 3x + e^{2t}, \quad x(0) = 5$$

Apply IF method.

Summary of IF Method

1. Standard form: $\frac{dx}{dt} + p(t)x = f(t)$
2. If is: $u = e^{\int p(t)\,dt}$
3. ODE becomes: $\int \frac{d}{dt}(u\,x)\,dt = \int u\,f\,dt$
4. Integrate

Turn something you didn't know how to solve into something you do (integration). Price is needing to find $u(t)$.

Some more IF examples

ex.

$$t^2\frac{dx}{dt} + tx = t\sin t, \quad x(1) = 2 \tag{20}$$

Apply IF method.

ex.

$$\frac{dx}{dt} + t^4 x = 1 \tag{21}$$

Apply IF method.

Outline

Online analysis tools

- GeoGebra Slope Field Plotter:
  www.geogebra.org/m/W7dAdgqc
- Bluffton Univ. Slope and Direction Fields:
  bluffton.edu/homepages/facstaff/nesterd/java/slopefields.html
- Interactive Differential Equations:
  www.aw-bc.com/ide/

First, some board work.

Direction Fields

$$\frac{dx}{dt} = t^2$$

- $f(t, x) = t^2$ Non-negative.
  $x(t)$ never decreases.

- $f(t, x) = 0$? $t^2 = 0$? $\Rightarrow t = 0$.
  When $t = 0$, $x(t)$ is horizontal.

- $f(t, x) = 1$?: $t^2 = 1$? $\Rightarrow t = \pm 1$.
  When $t = \pm 1$, $x(t)$ has slope 1.

- Integrate to find solution:
  $x(t) = \frac{1}{3}t^3 + c$.
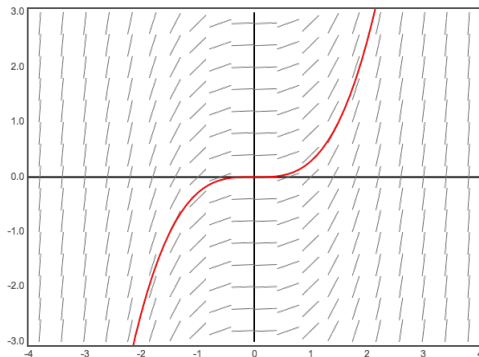
Note missing axis labels!
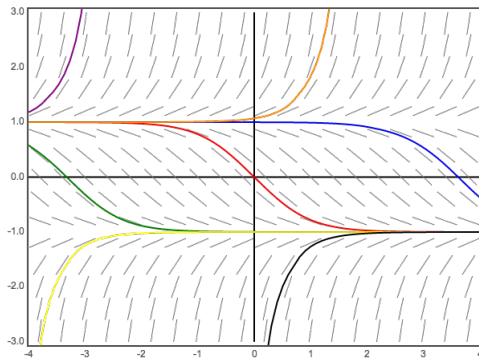
(Blufton's Slope and Dir. Fields tool)

Direction Fields

$$\frac{dx}{dt} = x^2 - 1$$

- If $x < -1$ or $x > 1$, then $f > 0$ so x increases.
- If $-1 < x < 1$, then $f < 0$ so x decreases.
- If $x = \pm 1$, then $f = 0$ so x is at Equilibrium.
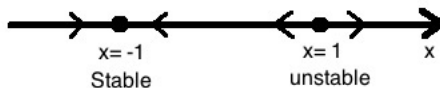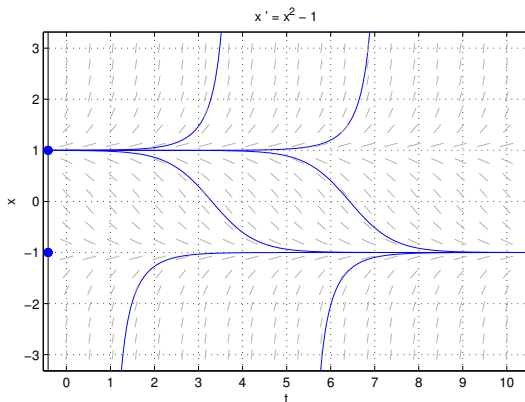- Solve by Sep of Var $x(t) = \frac{1 + ce^{2t}}{1 - ce^{2t}}$
- Singular when $1 - ce^{2t} = 0$. Finite time blowup.

Note missing axis labels!
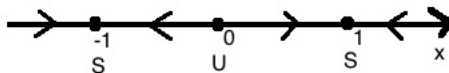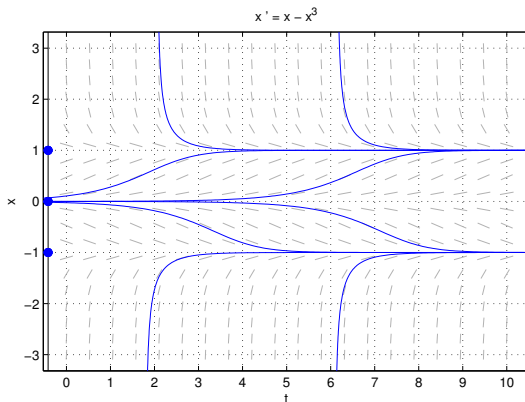(Blufton's Slope and Dir. Fields tool)

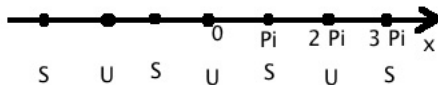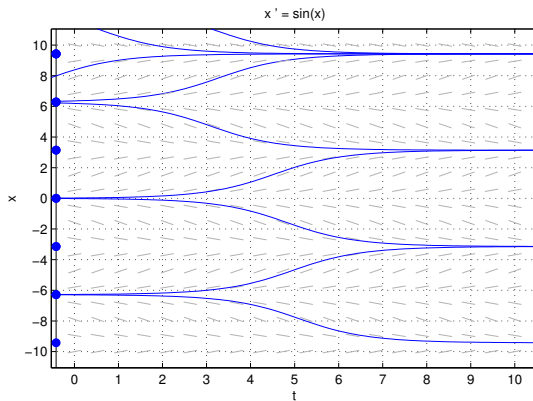# Autonomous ODEs & Equilibrium

First, some board work.

$x' = x^2 - 1$

Autonomous ODEs: example 2

First, some board work.

Autonomous ODEs: example 3

$x' = \sin(x)$

Existence & Uniqueness
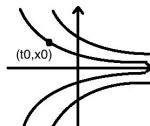
Before we start solving....

- How do we know if there is a solution to find? Existence.
- If we find a solution, how do we know if it is the only one? Uniqueness.

ex.

ODE: $\frac{dx}{dt} = -x \quad \Rightarrow \quad x(t) = ce^{-t}$: (a family of solution curves)

IC: $x(t_0) = x_0 \quad \Rightarrow c = x_0 e^{t_0} \Rightarrow x(t) = x_0 e^{t_0 - t}$

(a specific curve passing through a specific point (t_0, x_0))

Every point (t, x) has one and only one solution curve passing through it.

- If no solution curve: *Does not exist.*
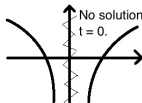- If more than one: *Not unique.*

E& U: Theorem

Given $\frac{dx}{dt} = f(x, t)$. If both $f(x, t)$ and $\frac{\partial f}{\partial x}(x, t)$ are continuous in a region containing (t_0, x_0), then there exists a unique solution through (t_0, x_0). *(f and its partial with respect to x must be continuous.)*

ex.

$$\frac{dx}{dt} = \frac{1}{t} \tag{22}$$

f is discontinuous at $t = 0$ so theorem fails at $t = 0$.

Solve: $x(t) = \ln|t| + c$. Undefined at $t = 0$.

For $t \neq 0$, where the theorem is satisfied, there is one and only one solution through each point.

E& U: Example

ex.

$$\frac{dx}{dt} = \frac{x}{t} \tag{23}$$

Examine f and $\partial f/\partial x$. Then solve.

ex.

$$\frac{dx}{dt} = \sqrt{x^2 - t^2} \tag{24}$$

$f(x, t)$ must be real. Simulate.

ex.

$$\frac{dx}{dt} = x^{2/3} \quad \text{vs.} \quad \frac{dx}{dt} = x^{4/3} \tag{25}$$

Simulate and compare.

Outline

Math 2343: Introduction

Separable ODEs

Linear, non-homogeneous

Graphical analysis

Numerical Approximation

Applications

Analysis vs. Numerics/Simulation

Given

$$\frac{dx}{dt} = f(x, t), \quad x(t_0) = x_0$$

- What if we can't separate, can't use an I.F., isn't Exact (see text)?
- What if the problem is too complicated and an *analytical* solution is not possible? Which is MOST!
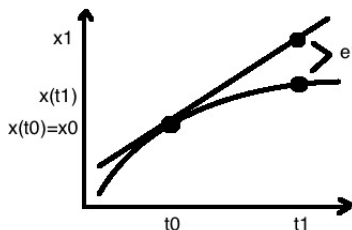- Use computer simulations to find a *numerical* approximation.

But computers can't do "calculus". How do we approximate the problem so that computers can operate on it?

Linear approximation

Given a curve $x(t)$. Suppose you know a point on the curve (t_0, x_0) The linear approximation (tangent line) is

$$x_l(t) - x_0 = m(t - t_0), \quad \text{where} \quad m = \frac{dx}{dt}(t_0). \tag{26}$$

$$x_l(t) = x_0 + \frac{dx}{dt}(t_0)(t - t_0)$$

Use the line as an approximation to the curve. At $t = t_1$:

- True value is $x(t_1)$.
- Approximate value is $x_l(t_1) = x_1$.
- Error: $e_1 = x(t_1) - x_1$

If the step size $h = t_1 - t_0$ is not too big, we expect the error to not be too big.

Euler's method

Given:

$$\frac{dx}{dt} = f(x, t), \quad x(t_0) = x_0$$

To get $x(t_1)$ use the approximation x_1:

$$x_1 = x_0 + \frac{dx}{dt}(t_0)(t_1 - t_0)$$

The derivative is given by the ODE:

$$x_1 = x_0 + f(x_0, t_0)(t_1 - t_0)$$

Go to a new point when $t = t_2$.
Use the linear approx. again.

$$x_2 = x_1 + f(x_1, t_1)(t_2 - t_1)$$

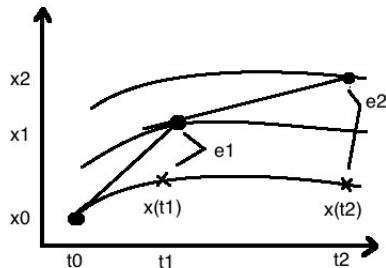Repeat, repeat, …

$$x_{n+1} = x_n + f(x_n, t_n)(t_{n+1} - t_n)$$

If fixed stepsize: $h = t_{n+1} - t_n$.

$$x_{n+1} = x_n + h f_n \qquad (27)$$

- f is given by the ODE, so known.
- You pick the times t_n, so known.
- The x_n are repeated approxs.
- Approx. based on approx!

    While $x_0 = x(t_0)$, $x_1 \neq x(t_1)$.

Error Euler's method is "Order h"

$$|e_n| = |x(t_n) - x_n| = Mh$$

Euler examples

For each problem below:

- Solve analytically then evaluate at the specified points.
- Solve numerically using Euler's method.

ex.

$$\frac{dx}{dt} = 5 + 2x, \quad x(0) = 0, \quad t = 0,\ 0.1,\ 0.2,\ 0.3$$

Compare analytical and numerical solutions.

ex.

$$\frac{dx}{dt} = 3x^2, \quad x(0) = 1, \quad t = 0,\ 0.2,\ 0.4,\ 0.6$$

Compare analytical and numerical solutions.

Issues

- To go long times, need more steps. Error can accumulate.

- To reduce error, reduce the stepsize *h*. Now computer takes a long time.

- Perhaps better methods. Instead of using linear (tangent) approx, use quadratic approx, or polynomial approx, of weighted averages of derivatives, or . . . .

- Implement error "correction." Take a step, estimate error, devise a scheme to eliminate the error.

- Better methods and error correction require more work by the computer. Now the computer takes more time.

- Buy a faster computer.

- If you don't have an analytical solution to compare against, how do you know the numerical method gives a correct result? What is the numerical solution converging to?

- Issues, caveats, issues, caveats, issues,. . .

- A deep knowledge of these issues and solutions, i.e., the fields of Scientific Computer and Numerical Analysis, gets you jobs.

euler.m

$\frac{dx}{dt} = 5 + 2x, \quad x(0) = 0$

Default: $h = 0.1$, $N = 100$, $tf = 10$.

Change $h = 0.2 \Rightarrow tf = 20$.

Exp. growing solution gets large.

Change $h = 0.1$, $N = 10$ for better view.

Change $h = 0.2$, $N = 5$.

Can see tangent lines.

Change $h = 0.3$, $N = 4$.

Note:

- When $h = 0.1$, $x(1) \approx 13$.

- When $h = 0.3$, $x(1) \approx 10$.

- Which is more accurate?
  $x(1) = -\frac{5}{2} + \frac{5}{2} e^2 = 15.9 \ldots$

Change $h = 0.01$, $N = 100$.

Very accurate but more steps. (slow?)

$\frac{dx}{dt} = 5 - 2x, \quad x(0) = 0$

What do we expect? Always ask yourself, what do you expect?

Default: $h = 0.1$, $N = 100$, $tf = 10$.

Goes to steady state at $\frac{5}{2}$.

Change $h = 0.2$.

Change $h = 0.5$. Not smooth.

Change $h = 0.6$. Overshoot.

Change $h = 0.8$. Oscillations.

Change $h = 1.0$. UNSTABLE!

- Accuracy issues.

- Stability issues.

- We can do better than Euler.

- Different methods have different advantages and disadvantages.

- In matlab, ode23 and ode45 are good all-purpose solvers.

- Interested in details? MATH 3315

Outline

Modeling process

Given some problem to solve.

- May be from science, engineering, economics, finance,. . .

Model (describe) the problem with a differential equation.

- This can be quite difficult and time consuming.

Solve the DE-model

- Exactly
- Approximately
- Numerically/computationally
- All of the above

Evaluate the solution

- Does the solution described previously observed behavior?
- Should the model be modified?

- Is the model good in some restricted set of cases?
- Can the model predict behavior not yet observed?

Are there other ways to model the problem? DEs are just one tool of many.

Philosophy on the application of mathematics:

- It is not "exact."
- Requires judgment and imagination.
- Requires knowledge of both the application and mathematics.
- Requires collaboration and communication across disciplines.

Growth/Decay Rate

There is a branch of mathematical biology called "Population Dynamics," where the competition between species is studied. This is important in environmental resource management. "Epidemiology" is very similar in that there is a competition between those who are susceptible, those who are infected and those who are recovered, from a particular disease.

$$\frac{dP}{dt} = \textit{Rate of change} \text{ of the population.}$$

$$\frac{\frac{dP}{dt}}{P} = \frac{1}{P}\frac{dP}{dt} = \frac{\text{Rate of change}}{\text{population}} = \textit{Growth/Decay Rate}$$

Growth rate measures the Rate of change with respect to the population size, i.e., the *relative rate of change*. The distinction is important.
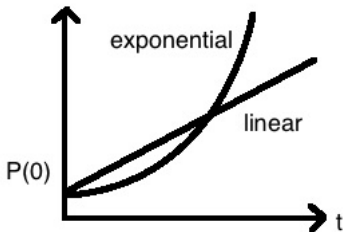
Exponential vs. linear

Suppose $\frac{dP}{dt} = k$.

- There are always k new individuals in a given time, independent of P. For example, no matter how many have been admitted, Bush Stadium's turnstile gates allow only a fixed number to enter over a given time.

- Solve by integration.

- $P(t) = kt + C$, apply IC $C = P(0)$, so $P(t) = P(0) + kt$.

- Results in linear growth.

Suppose $\frac{1}{P}\frac{dP}{dt} = k$ or $\frac{dP}{dt} = kP$.

- The rate of change depends on the population size. For example, this reflects the fact that with more members in a population there can be more births.

- Solve by separation.

- $P(t) = Ce^{kt}$, apply IC $C = P(0)$, so $P(t) = P(0)e^{kt}$.

- Results in exponential grown

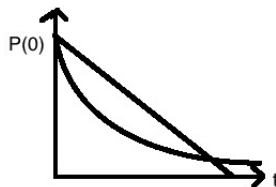Which model is more accurate depends on the application.

Decay and Death

Decay is just the negative of growth.
$\frac{dP}{dt} = -k$ so $P(t) = P(0) - kt$.

- Linear decrease.
- People leave Bush Stadium at a fixed rate.

$\frac{dP}{dt} = -kP$ so $P(t) = P(0)e^{-kt}$.

- Exponential decay.
- The more members of a population, the more deaths there are.

Money in savings account

P = amount (principal). $\frac{dP}{dt}$ = change in time.
How can the amount change?

- Interest: "Interest rate" = "Growth rate" = r_1

$$\begin{aligned}
\frac{dP}{dt} &= \quad \text{"interest rate" times } P \\
&= r_1 P \\
\Rightarrow P(t) &= P_0 e^{rt}
\end{aligned}$$

- Deposits & withdrawals: on average, r_2 dollars/day.

$$\begin{aligned}
\frac{dP}{dt} &= r_2 \\
\Rightarrow P(t) &= r_2 t + P_0
\end{aligned}$$

Money in savings account

Together.... Interest AND deposits+withdrawals?

- Must start with new ODE.
- DO NOT ADD THE SOLUTIONS FROM ABOVE.

New ODE with both proceses.

$$\frac{dP}{dt} = r_1 P + r_2, \quad P(0) = P_0 \tag{28}$$

Solve and note effect of compound interest!

A few more applications

Radioactive decay
Experimental observation: The rate of decay of a radioactive material
is proportional to the number of atoms present.

- Half-life
- Doubling time (money in the bank)

Model and solve

Single-species population
Birth, death, deposits and withdrawals.

- Equilibrium
- Fish are positive

Model and solve

Argon LASER

And two more applications

Your money-market checking account comes with an interest rate of 2% (r = 0.02 1/day). On average you withdray $3 dollars/day. Initially, you have $1000 in the account. When does your balance increase tenfold?
Model and solve.

Thermal cooling: the rate of change of the surface temperature of an object is proportional to the difference between the temperature of the object and it's surroundings. (Newton's law of cooling)
Modeling and solving.

And mixture (tank) applications

Concerned with the amount of a given substance present in a solution as a function of time. Our goal is to formulate and solve a differential equation for the quantity $Q(t)$ of interest. Consider a box of sand:
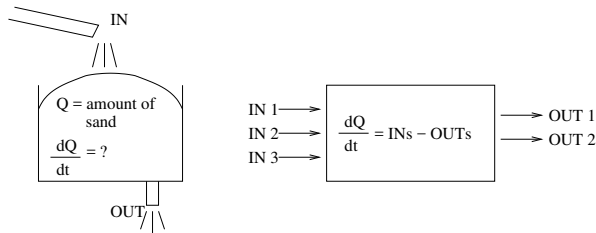
LAW of MASS BALANCE

The rate of change of Q = rate of sand in - rate of sand out.

$$\frac{dQ}{dt} = \left[\frac{dQ}{dt}\right]_{in} - \left[\frac{dQ}{dt}\right]_{out}$$

Similar to $\frac{dP}{dt}$ = (Births - Deaths) or (Deposits - Withdrawals).

A problem with just flow

First, a simple example with just water, not a mixture.

ex. Consider a tank that can hold 1000 gal of water. Water is being pumped into the tank at a *rate* of 10 gal/min. Water is pumped out of the tank at *rate* of 8 gal/min. Initially, there are 200 gal in the tank. Formulate and solve an ODE for the amount of water in the tank.

Now with stuff in the flow

Suppose there is "stuff" mixed into the water, i.e., there is a *concentration* of "stuff" in the volume of water. How do we determine the *amount* quantity of "stuff" that is in the mixture?

$$\text{Concentration} = \frac{\text{Quantity}}{\text{Volume}} \quad \Rightarrow \quad C(t) = \frac{Q(t)}{V(t)}$$

Both the amount $Q(t)$ and the volume $V(t)$ may be functions of time. Hence, the concentration $C(t)$ also changes in time.

How do we get the rate of change of the *quantity/amount* of stuff?

$$\frac{dQ}{dt} = \text{flow rate} \cdot \text{concentration} \quad (29)$$

$$\frac{mass}{time} = \frac{volume}{time} \cdot \frac{mass}{volume}$$

Two mixture problems

ex. A room has a volume of 800 ft^3. The air in the room contains chlorine at an initial concentration of 0.1 g/ft^3. Fresh air enters the room at a rate of 8 ft^3/min. The air in the room is well mixed and flows out of the door at the same rate that it flows in.
Find the concentration of chlorine as a function of time.

ex. A well circulated pond contains 1 million L of water. It contains pollutant at a concentration of 0.01 kg/L. Pure water enters from a stream at 100 L/h. Water evaporates from the pond (leaving the pollutant behind) at 10 L/h and water flows out a pipe at 80 L/h.
How many days will it take for the pollution concentration to drop to 0.001 kg/L?