1) a) Define any 2 vectors

$r_1 = (3,2,3) - (0,0,0) = \langle 3,2,3 \rangle$

$r_2 = (1,2,1) - (0,0,0) = \langle 1,2,1 \rangle$

$n = r_1 \times r_2 = \begin{vmatrix} i & j & k \\ 3 & 2 & 3 \\ 1 & 2 & 1 \end{vmatrix} = \langle -4,0,4 \rangle$

$n \cdot (r - r_0) = 0$

$\langle -4,0,4 \rangle \cdot (x-0, y-0, z-0) = 0$

$-x + z = 0$

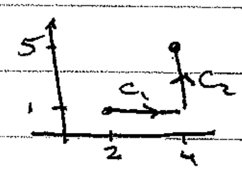
b) Area $= \frac{1}{2} |r_1 \times r_2|$

$= \frac{1}{2}(4^2 + 4^2)^{1/2} = 2\sqrt{2}$

c) Pick any vectors in plane.

e.g. $(3,2,3) \cdot (-4,0,4) = 0$

2) a)



c_1: $x = t \quad y = 1 \quad t \in [2,4]$

$r = \langle t,1 \rangle \quad r' = \langle 1,0 \rangle$

c_2: $x = 4 \quad y = t \quad t \in [1,5]$

$r = \langle 4,t \rangle \quad r' = \langle 0,1 \rangle$

$\int_c = \int_{c_1} + \int_{c_2}$

$= \int_2^4 (t \cdot 1, t^2) \cdot (1,0)\,dt$

$+ \int_1^5 (4 \cdot t, 16) \cdot (0,1)\,dt$

$= \int_2^4 t\,dt + \int_1^5 16\,dt$

$\frac{1}{2} t^2 \Big|_2^4 + 16t \Big|_1^5$

$\frac{16}{2} - 2 + 80 - 16 = 70$

2b) C_3: $y - 1 = \frac{5-1}{4-2}(x-2)$

$= 2x - 4$

$y = 2x - 3$

let $x = t \quad y = 2t - 3$

$t \in [2,4]$

$r = \langle t, 2t-3 \rangle$

$r' = \langle 1,2 \rangle$

$\int_{C_3} = \int_2^4 \langle t(2t-3), t^2 \rangle \cdot \langle 1,2 \rangle\,dt$

$= \int_2^4 2t^2 - 3t + 2t^2\,dt$

$= \int_2^4 4t^2 - 3t\,dt$

$= \frac{4}{3} t^3 - \frac{3}{2} t^2 \Big|_2^4$

$= \frac{4^4}{3} - \frac{3}{2} 16$

$- \frac{32}{3} + \frac{3}{2} \cdot 4$

$= 170/3$

c) $\int_{C_4} = \int_{C_1} + \int_{C_2} - \int_{C_3}$

$= 70 - \frac{170}{3} = \frac{40}{3}$

d) let $x = t \quad t \in [1,3]$

$y = t^3$

$r = \langle t, t^3 \rangle$

$r' = \langle 1, 3t^2 \rangle$

$|r'| = (1 + 9t^4)^{1/2}$

$\int_1^3 (t^3 + t^3)(1 + 9t^4)^{1/2}\,dt$

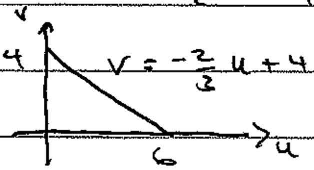
$2 \int_1^3 t^3 (1 + 9t^4)^{1/2}\,dt$

$2\left(\frac{2}{3}\right)\frac{1}{36}(1 + 9t^4)^{3/2} \Big|_1^3$

$\frac{1}{27}(1 + 3^6)^{3/2} - \frac{1}{27} 10^{3/2}$

3) $x = u \quad y = v$

$z = 3 - \frac{1}{2}u - \frac{3}{4}v$



$r = \langle u, v, 3 - \frac{1}{2}u - \frac{3}{4}v \rangle$

$\frac{\partial r}{\partial u} = \langle 1, 0, +\frac{1}{2} \rangle$

$\frac{\partial r}{\partial v} = \langle 0, 1, -\frac{3}{4} \rangle$

$\left| \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right| = \left| \langle +\frac{1}{2}, \frac{3}{4}, 1 \rangle \right| = \sqrt{\frac{29}{16}}$

$$\int_0^6 \int_0^{-\frac{2}{3}u+4} u v^2 \left(3 - \frac{1}{2}u - \frac{3}{4}v\right)^3 \sqrt{\frac{29}{16}}\, dv\, du$$

$$\int_0^4 \int_0^{-\frac{3}{2}(v-4)} \text{"} \; du\, dv$$

4 a) $x = u\cos v$, $y = u \sin v$ $\Big\}$ $u \in [0,1]$, $v \in [0, 2\pi]$ Disk

$z = u$ ~ gives cone

$r = \langle u\cos v, u \sin v, u \rangle$

$\frac{\partial r}{\partial u} = \langle \cos v, \sin v, 1 \rangle$

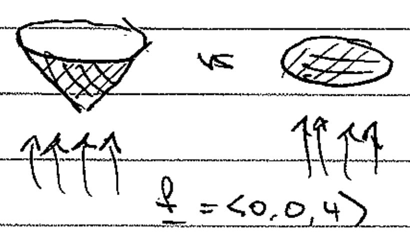
$\frac{\partial r}{\partial v} = \langle -u \sin v, u\cos v, 0 \rangle$

$\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} = \langle -u\cos v, u \sin v, u \rangle$

$$\int_0^{2\pi} \int_0^1 \langle 0, 0, 4 \rangle \cdot \langle -u\cos v, u\sin v, u \rangle\, du\, dv$$

$$\int_0^{2\pi} \int_0^1 4u\, du\, dv$$

$$2\pi \; 2u^2 \Big|_0^1 = 4\pi$$

b) 

Both surfaces present the same cross-sectional area to $\underline{f}$. ∴ Same flux

Both $= 4\pi$

5) $\langle 2x, -6y, 8z \rangle = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$

$\frac{\partial f}{\partial x} = 2x \quad f = x^2 + c_1(z, y)$

$\frac{\partial f}{\partial y} = 0 + \frac{\partial c_1}{\partial y} = -6y$

$c_1 = -3y^2 + c_2(z)$

$f = x^2 - 3y^2 + c_2(z)$

$\frac{\partial f}{\partial z} = 0 + 0 + \frac{dc_2}{dz} = 8z$

$c_2 = 4z^2 + c_3$

$f = x^2 - 3y^2 + 4z^2 + c_3$

b) Because $\underline{F} = \nabla f$ $\underline{F}$ is conservative.

C is closed.

∴ $\oint = 0$

c) $\underline{F}$ is conservative.

⇒ Path independent

$= f\big|_{(1,1,1)} - f\big|_{(0,0,0)}$

$= (1 - 3 + 4) - 0 = 2$