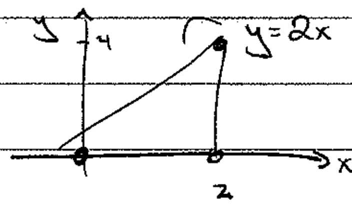
1) $\dfrac{\partial u_2}{\partial x} - \dfrac{\partial u_1}{\partial y} = 2xy + 2x$



$$\int_0^2 \int_0^{2x} (2xy + 2x)\, dy\, dx$$

$$xy^2 + 2xy \Big|_0^{2x}$$

$$4x^3 + 4x^2 - 0$$

$$\int_0^2 4x^3 + 4x^2\, dx$$

$$x^4 + \tfrac{4}{3}x^3 \Big|_0^2$$

$$16 + \tfrac{4}{3}\cdot 8 - 0 = \tfrac{80}{3}$$

2) $\nabla \times \underline{u} = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ y-z & z-x & x-y \end{vmatrix} = (-1-1)\underline{i} - (1+1)\underline{j} + (-1-1)\underline{k}$

$= \langle -2, -2, -2 \rangle$

$\hat{n}\,|_{\text{plane}} : \langle 1, 0, 1 \rangle$ w/ $dS = r\,dr\,d\theta$

– or –

let $x = u\cos v$, $y = u\sin v$ $\Big\}$ $v \in [0, 2\pi]$, $u \in [0, 1]$

$z = 1 - x = 1 - u\cos v$

$\underline{r} = \langle u\cos v, u\sin v, 1 - u\cos v \rangle$

$\underline{r_u} = \langle \cos v, \sin v, -\cos v \rangle$

$\underline{r_v} = \langle -u\sin v, u\cos v, u\sin v \rangle$

$\underline{r_u} \times \underline{r_v} = \langle u, 0, u \rangle$

$$\int_0^{2\pi}\int_0^1 \langle -2, -2, -2 \rangle \cdot \langle u, 0, u \rangle\, du\, dv$$

$$\int_0^{2\pi}\int_0^1 (-4u)\, du\, dv$$

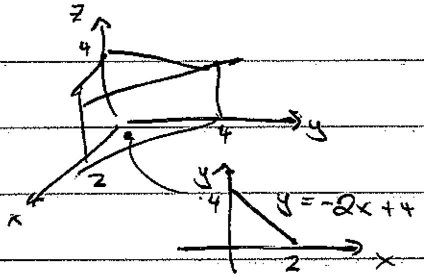
$$2\pi\, (-2u^2)\Big|_0^1 = -4\pi$$

3a) $\nabla \cdot u = 2+3-4 = 1$

$= \iiint_V 1\, dV = \text{volume of sphere}$

$= \frac{4}{3}\pi r^3 \Big|_{r=2} = \frac{32}{3}\pi$

b) $\nabla \cdot u = 1-1+3 = 3$



$= \iiint_V 3\, dV$

$= \int_0^4 dz \int_0^2 dx \int_0^{-2x+4} dy \cdot 3$

$= \int_0^4 dz \int_0^2 dx\; 3(-2x+4)$

$= \int_0^4 dz\; 3(-x^2+4x)\Big|_0^2$

$= \int_0^4 dz\; 3(-4+8)$

$= 12 \int_0^4 dz = 48$

4a) $\frac{du}{dt} = ku \quad u(t_0) = u_0$

$\frac{1}{u}\, du = k\,dt$

$\ln u = kt + c$

$u = ce^{kt}$

$u(t_0) = ce^{kt_0} = u_0$

$u = u_0 e^{k(t-t_0)}$

b) $\frac{du}{dt} + \alpha u = \beta$

$r = e^{\int \alpha dt} = e^{\alpha t}$

$\frac{d}{dt}(e^{\alpha t} u) = \beta e^{\alpha t}$

$e^{\alpha t} u = \frac{\beta}{\alpha} e^{\alpha t} + c$

$u = ce^{-\alpha t} + \beta/\alpha$

5) $u_{tt} + 2u_t = u_{xx}$

SS: $0 = u_{xx}$

$u = c_1 x + c_2$

$u(0,t) = c_1 \cdot 0 + c_2 = 1 \quad c_2 = 1$

$u(1,t) = c_1 + \cancel{c_2}^{1} = 0 \quad c_1 = -1$

$u(x,t) = -x + 1$

6) $u_t - 2u_x = -u$

a) $\frac{dt}{ds} = 1 \quad t = s$

$\frac{dx}{ds} = -2 \quad x = -2s + x_0 \Rightarrow x = -2t + x_0$

$x_0 = x + 2t$

$\frac{du}{ds} = -u \quad u = u_0 e^{-s}$

b) $u(x(s), t(s)) = u_0 e^{-s}$

Sol: $u(x(0), t(0)) = u(x_0, 0) = u_0$

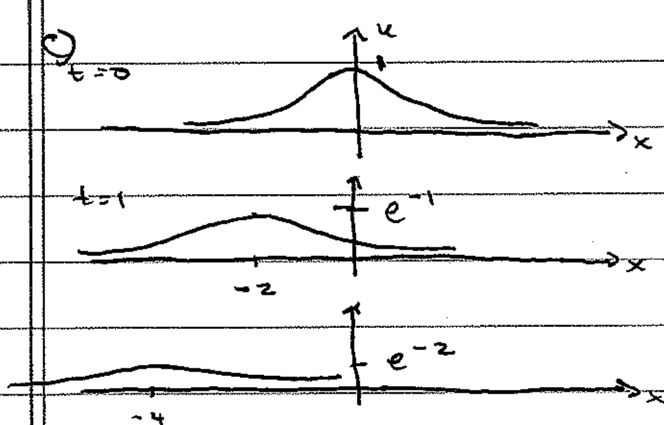
IC: $u(x_0, 0) = e^{-x_0^2}$

$\Rightarrow u_0 = e^{-x_0^2}$

$u(x,t) = e^{-x_0^2} e^{-s}$

$= e^{-(x+2t)^2} e^{-t}$

c)



d) Moves to right instead of left.

Moves w/ speed $c = 1$ ($\Rightarrow$ $\frac{1}{2}$ as fast.

Amplitude is constant

$\Rightarrow u = u_0 \Rightarrow e^{-(x-ct)^2}$