MATH 3337 h-divgradcurl.gif

DIV GRAD CURL

1a) $T = \cos x \cosh y$

$-\nabla T = (+\sin x \cosh y, -\cos x \sinh y)$

$\nabla^2 T = -\cos x \cosh y + \cos x \cosh y = 0$

$\nabla T|_p = (-1 \cdot \cosh 1,\ 0 \cdot \sinh 1)$

$= (-1, 0)$

b) $T = \sin(x+z)$

$-\nabla T = (-\cos(x+z), 0, -\cos(x+z))$

$\nabla^2 T \neq 0 \Rightarrow$ Not a valid function for an actual temperature field.

$\nabla T|_p = (\cos\frac{\pi}{4}, 0, \cos\frac{\pi}{4})$

$= (\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})$

* Temp DECREASE $\Rightarrow -\nabla T$

2) a) $f_x = yz \Rightarrow f = xyz + c_1(y,z)$

$f_y = xz + \frac{\partial c_1}{\partial y} = xz$

$\frac{\partial c_1}{\partial y} = 0$

$c_1 = c_2(z)$

$f = xyz + c_2(z)$

$f_z = xy + \frac{dc_2}{dz} = xy$

$\frac{dc_2}{dz} = 0$

$c_2 = c_0$

$f = xyz + c_0$

2b)

3) $\underline{u} = e^x \underline{i} + ye^{-x} \underline{j} + 2z\sinh x\, \underline{k}$

$\nabla \cdot u = e^x + e^{-x} + 2\sinh x$

$= e^x + e^{-x} + 2(\frac{1}{2})(e^x - e^{-x})$

$= 2e^x$

2b) $f_x = -3e^{-3x}y^2$

$f = e^{-3x}y^2 + c(y)$

$f_y = 2ye^{-3x} + \frac{dc}{dy}$

$\Rightarrow \frac{dc}{dy} = 0 \Rightarrow c(y) = \text{constant}$

$f = e^{-3x}y^2 + c_0$

4) $\underline{u} = x\underline{i}$

$\frac{d\underline{r}}{dt} = (\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}) = (x, 0, 0)$

$\frac{dx}{dt} = x \Rightarrow x = c_1 e^t$

$\frac{dy}{dt} = 0 \Rightarrow y = c_2$

$\frac{dz}{dt} = 0 \Rightarrow z = c_3$

$\underline{r} = (c_1 e^t, c_2, c_3)$

$\nabla \cdot \underline{u} = \frac{\partial x}{\partial x} + \frac{\partial 0}{\partial y} + \frac{\partial 0}{\partial z} = 1 + 0 + 0 = 1$

$\nabla \cdot \underline{u} \neq 0 \Rightarrow$ compressible.

5) $\underline{u} = (x, y, -z)$

$x = c_1 e^t$

$y = c_2 e^t$

$z = c_3 e^{-t}$

$\underline{r} = (c_1 e^t, c_2 e^t, c_3 e^{-t})$

$$\nabla \times \underline{u} = \left|\begin{pmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ x & y & -z \end{pmatrix}\right| = \underline{i}(0-0) - \underline{j}(0-0) + \underline{k}(0-0) = 0$$

$\nabla \times \underline{u} = 0 \Rightarrow$ IRROTATIONAL.

6) $\underline{u}$ is conservative.

$\Rightarrow \underline{u} = \nabla f$ for some f.

$$\nabla \times \nabla f = \left|\begin{pmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ f_x & f_y & f_z \end{pmatrix}\right|$$

$= \underline{i}(f_{zy} - f_{yz})$

$- \underline{j}(f_{zx} - f_{xz})$

$+ \underline{k}(f_{yx} - f_{xy})$

Assume f is smooth enough and we can swap the order of differentiation ie $f_{zy} - f_{yz} = 0$

$\Rightarrow \nabla \times (\nabla f) = 0$

All irrotation flows are conservative and derivable from a scaler/potential function.

$\Rightarrow$ Gravity is irrotational