MATH 3337 h- Lineintegrals.

LINE INTEGRALS

1 a) $f = x^3 + y$

$\underline{r} = (x, y) = (3t, t^3)$

$f(t) = 27t^3 + t^3 = 28t^3$

$\frac{d\underline{r}}{dt} = (3, 3t^2)$

$\left|\frac{d\underline{r}}{dt}\right| = 3(1+t^4)^{1/2}$

$\Rightarrow \int_0^1 28 \cdot 3 \cdot t^3 (1+t^4)^{1/2} dt$

$= 14(2^{3/2} - 1) \approx 25.6$

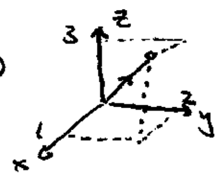
b) $f = xy^{2/5}$

$\underline{r} = (\frac{t}{2}, t^{5/2})$

$\left|\frac{d\underline{r}}{dt}\right| = \frac{1}{2}(1+25t^3)^{1/2}$

$\Rightarrow \int_0^1 \frac{1}{2} t^2 \frac{1}{2} (1+25t^3)^{1/2} dt$

$= \frac{1}{450}(26^{3/2} - 1)$

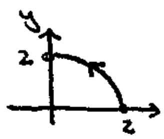
2) 

$\underline{r} = (t, 2t, 3t)$ $t \in [0,1]$

$\frac{d\underline{r}}{dt} = (1, 2, 3)$

$\left|\frac{d\underline{r}}{dt}\right| = \sqrt{14}$

$\int_c xyz\, ds = \int_0^1 t(2t)(3t)\sqrt{14}\, dt = \frac{3}{2}\sqrt{14}$

3) a) 

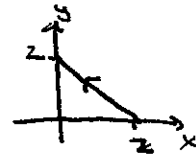
$\underline{r} = (x, y) = (2\cos t, 2\sin t)$

$t \in [0, \pi/2]$

$\frac{d\underline{r}}{dt} = (-2\sin t, 2\cos t)$

$\int_c \underline{F} \cdot \underline{r}\, ds = \int_0^{\pi/2} (-8\cos t \sin^2 t + 32\cos^3 t \sin^2 t) dt$

Have fun.

b) 

$y = -x + 2$

let $x = 2 - t$

$t \in [0, 2]$

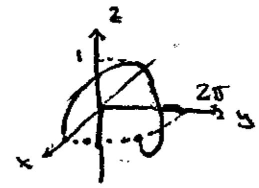
then $y = t$

$\underline{r} = (2-t, t)$ $\frac{d\underline{r}}{dt}(-1, 1)$

$\Rightarrow \int_0^2 ((2-t)t, (2-t)^2 t^2) \cdot (-1, 1) dt$

$= \int_0^2 (-2t + 5t^2 - 4t^3 + t^4) dt$

$= 4/15$

c) 

Spiral in (xz) growing in z direction.

$\int_0^{2\pi} (2\cos t - t, t - 2\sin t, 2\sin t - 2\cos t) \cdot (-2\sin t, 1, 2\cos t) dt$

$= -8\pi + 2\pi^2$

d) $\int_0^1 (e^t, e^{-t^2}, e^t) \cdot (1, 2t, 1) dt$

$= 2e - \frac{1}{e} - 1$

4) $W = \int \underline{F} \cdot dr$

$= \int_a^b \underline{F} \cdot \frac{dr}{dt} dt$

$r = \langle 4\cos t, 2\sin t \rangle$ $t \in [0, 2\pi]$

$r' = \langle -4\sin t, 2\cos t \rangle$

$\Rightarrow \int_0^{2\pi} -8\sin t + 10\cos t\, dt$

$= 8\cos t - 10\sin t \Big|_0^{2\pi}$

$= 0$

No net work because on 1/2 path the object moves w/ the force and on the other 1/2 it moves against. Like going up a hill, then down.