MATH 3337 h-pdeintro.gif

Intro to PDEs

Logan 1.1

1) $\frac{\partial}{\partial t}\left(\frac{1}{\sqrt{4\pi k t}} e^{\frac{-x^2}{4kt}}\right) = \frac{-2\pi k}{(4\pi k t)^{3/2}} e^{\frac{-x^2}{4\pi k t}} + \frac{x^2}{4kt^2\sqrt{4\pi k t}} e^{\frac{-x^2}{4\pi k t}}$

$\frac{\partial^2}{\partial x^2}(\ \cdot\ ) : \frac{-2}{4kt\sqrt{4\pi k t}} e^{-\frac{x^2}{4\pi k t}} + \frac{4x^2}{(4kt)^2}$

w/ algebra $\frac{\partial}{\partial t}u = k\frac{\partial^2 u}{\partial x^2}$

Figures on other page

3) $u_{xx} = 0$

$u_x = c_1(t)$

$u = c_1(t)x + c_2(t)$

$u(0,t) = 0 + c_2(t) = t^2$

$u(1,t) = c_1(t) + t^2 = 1$

$c_1(t) = 1 - t^2$

$u(x,t) = (1-t^2)x + t^2$

4) $u(x,t) = \frac{1}{2c}\int_{x-ct}^{x+ct} g(\xi)\,d\xi$

$\frac{\partial u}{\partial t} = \frac{1}{2c}\Big[\int_{x-ct}^{x+ct} \underbrace{\frac{\partial g(\xi)}{\partial t}}_{0} d\xi + g(x+ct)\underbrace{\frac{\partial}{\partial t}(x+ct)}_{+c} + g(x-ct)\underbrace{\frac{\partial}{\partial t}(x-ct)}_{-c}\Big]$

$= \frac{1}{2}[g(x+ct) - g(x-ct)]$

$\frac{\partial^2 u}{\partial t^2} = \frac{c}{2}[g'(x+ct) + g'(x-ct)]$

$\frac{\partial^2 u}{\partial x^2} = \frac{1}{2c}[g'(x+ct) + g'(x-ct)]$

$c^2\frac{\partial^2 u}{\partial x^2} = \frac{c}{2}[\ \cdot\ ] = \frac{\partial^2 u}{\partial t^2}$

5) $\frac{\partial}{\partial t}(e^{at}\sin bx) - k\frac{\partial^2}{\partial x^2}(e^{at}\sin bx) = 0$

$a e^{at}\sin bx + kb^2 e^{at}\sin bx = 0$

$(a + kb^2)e^{at}\sin bx = 0$

Must have $a + kb^2 = 0$

6) $u_{xt} + 3u_x = 1$

let $v = u_x$

$v_t = u_{xt}$

$\Rightarrow v_t + 3v = 1$

Solve w/ int. factor.

$\frac{\partial}{\partial t}[e^{3t}v] = 1\cdot e^{3t}$

$e^{3t}v(x,t) = \frac{1}{3}e^{3t} + c_1(x)$

$v(x,t) = \frac{1}{3} + c_1(x)e^{-3t}$

$u_x(x,t) = \frac{1}{3} + c_1(x)e^{-3t}$

$u = \frac{1}{3}x + e^{-3t}\underbrace{\int c_1(x)dx}_{\text{Some new function of } x} + c_2$

$u = \frac{1}{3}x + e^{-3t}c_3(x) + c_2$

7) $u_t = u_x^2 + u_{xx}$

let $w = e^u$

$w_t = \underbrace{e^u}_{w} u_t \Rightarrow u_t = \frac{w_t}{w}$

Similarly

$u_x = \frac{w_x}{w}$

$u_{xx} = \frac{w_{xx}w - w_x^2}{w^2}$

$\frac{w_t}{w} = \cancel{\left(\frac{w_x}{w}\right)^2} + \left(\frac{w_{xx}w - \cancel{w_x^2}}{w^2}\right)$

$= \frac{w_{xx}w}{w^2}$

$w_t = w_{xx}$

Alternatively $u = \ln w$

& $u_t = \ldots$

$u_x = \ldots$

Better to avoid the ln function though

- The point of 6) & 7) is to note that often the "method" to solve a problem is to change it to a different problem. The idea being the latter is easier or already known.