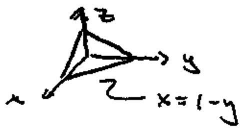
MATH 3337 h-surface.gif

SURFACE INTEGRALS

1a) By observation of plane: $\underline{N} = (1,1,1)$

$S: x+y+z=1$ 1st octant



let $\left.\begin{array}{l} x=u \\ y=v \end{array}\right\}\begin{array}{l} 0\le u\le 1-v \\ 0\le v\le 1 \end{array}$

$z = 1-u-v$

$\underline{r} = (u, v, 1-u-v)$

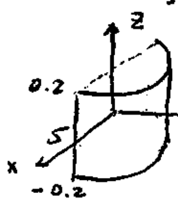
$\left|\frac{\partial \underline{r}}{\partial u} \times \frac{\partial \underline{r}}{\partial v}\right| = |(1,1,1)| = \sqrt{3}$

$\Rightarrow \int_0^1 \int_0^{1-v} (\cos u + \sin v)\sqrt{3}\, du\, dv$

$= (2 - \cos 1 + \sin 1)\sqrt{3}$

1b) $S: \underline{r} = (5\cos u, 5\sin u, v)$

$0 \le u \le \pi, -0.2 \le v \le 0.2$



Cut of a side of a cylinder

$\underline{r}_u = (-5\sin u, 5\cos u, 0)$

$\underline{r}_v = (0,0,1)$

$|\underline{r}_u \times \underline{r}_v| = 5$

$\Rightarrow 5 \cdot 625 \int_{-0.2}^{0.2} \int_0^{\pi} (\sin^4 u + \cos^4 u)\, du\, dv$

$5 \cdot 625\,(0.4) \underbrace{\int_0^{\pi} (\sin^4 u + \cos^4 u)\, du}_{\text{Use tables}}$

$5 \cdot 625\,(0.4)\left(\frac{3\pi}{4}\right) = 937.5\pi$

2a) $S: x+y+z=1$ Restrictions are the same as 1a), ie, "first quadrant."

$\underline{r} = (u, v, 1-u-v)$

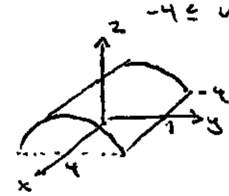
$\underline{r}_u \times \underline{r}_v = (1,1,1)$

$\int_0^1 \int_0^{1-v} (u^2, e^v, 1) \cdot (1,1,1)\, du\, dv$

$= e - {}^{17}/_{12}$

2b) $S: \underline{r} = (u, \cos v, \sin v)$

$-4 \le u \le 4 \quad 0 \le v \le \pi$



Cut of side of cylinder

$\underline{r}_u = (1,0,0)$

$\underline{r}_v = (0, -\sin v, \cos v)$

$|\underline{r}_u \times \underline{r}_v| = (0, -\cos v, -\sin v)$

$\underline{f} \cdot \underline{n} = (\sinh(\cos v \sin v), 0, \cos^4 v) \cdot (0, -\cos v, -\sin v)$

$\Rightarrow -\int_{-4}^{4} \int_0^{\pi} \sin v \cos^4 v\, du\, dv = -\frac{16}{5}$

2c) $S: x^2 + 4y^2 = 1$

let $x = 2\cos u \quad y = \sin u \quad z = v$

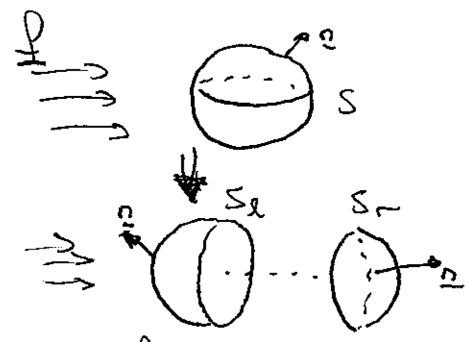
$\underline{r} = (2\cos u, \sin u, v)$

$\underline{r}_u \times \underline{r}_v = (\cos u, 2\sin u, 0)$

$\Rightarrow \int_0^h \int_0^{\pi/2} \cos u \sin^3 u + 16 \sin u \cos^3 u\, du\, dv$

$h \cdot 17/4$

3) $\underline{f}$ is in the x dir.



On S_l: $\underline{f}$ is "opposite" $\underline{n}$ $\Rightarrow$ net flux is neg as into S.

On S_r: $\underline{f}$ is "same dir" as $\underline{n}$ $\Rightarrow$ net flux is pos as out of S.

In other word, for every bit of fluid that goes in, same amount goes out.

Net flux is 0!