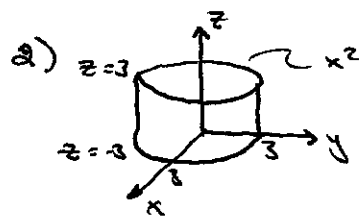


VOLUME INTEGRALS

$$\begin{aligned}
 1) \quad & \int_{-2}^2 \int_{-3}^3 \int_{-1}^1 (x^2 + y^2 + z^2) dx dy dz \\
 &= 2 \cdot 2 \cdot 2 \int_0^2 \int_0^3 \int_0^1 (x^2 + y^2 + z^2) dx dy dz \\
 &\quad \text{due to even symmetry in } x, y \text{ and } z \\
 &= 8 \int_0^2 \int_0^3 \left[\frac{1}{3} x^3 + (y^2 + z^2) x \right]_0^1 dy dz \\
 &= 8 \int_0^2 \left[\frac{1}{3} y + \frac{1}{3} y^3 + z^2 y \right]_0^3 dz \\
 &= 8 (10z + z^3) \Big|_0^2 = 224
 \end{aligned}$$

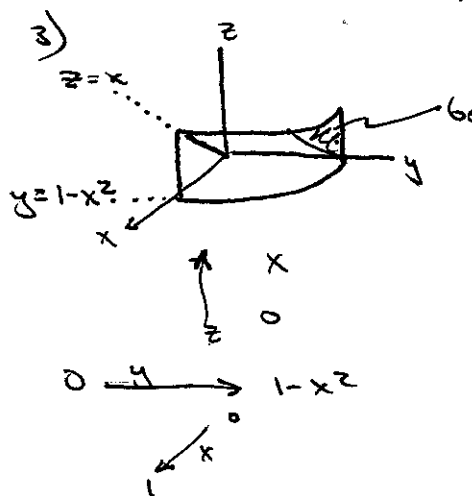
2)  $x^2 + y^2 = 9$
 $z = 3$
 $z = -3$

$$\int_{-3}^3 \iint_A \frac{1}{3} (x^2 + y^2)^2 dx dy dz$$

use polar coordinates
 $x^2 + y^2 = r^2 \quad dx dy = r dr d\theta$

$$\begin{aligned}
 &= \int_{-3}^3 \int_0^{2\pi} \int_0^3 \frac{1}{3} r^4 r dr d\theta dz \\
 &= \int_{-3}^3 \int_0^{2\pi} \frac{1}{18} 3^6 d\theta dz
 \end{aligned}$$

$$6 \cdot 2\pi \frac{1}{18} 3^6 = 486\pi$$

3)  $z = x$
 $y = 1 - x^2$
 gets cut off?

$$\int_0^1 \int_0^{1-x^2} \int_0^x 4z dz dy dx$$

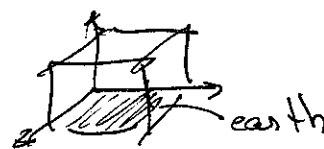
$$2z^2 \Big|_0^x$$

$$2x^2 y \Big|_0^{1-x^2}$$

$$\int_0^1 2x^2 (1 - x^2) dx$$

$$\frac{2}{3} x^3 - \frac{2}{5} x^5 \Big|_0^1 = \frac{4}{15}$$

4) $\rho = 1.225 - 0.000113 z \text{ kg/m}^3$
 $= a - bz$



$$\begin{aligned}
 m &= \int_0^{1000} \int_0^{1000} \int_0^{1000} (a - bz) dx dy dz \\
 &= \int_0^{1000} (a - bz) dz \int_0^{1000} dy \int_0^{1000} dx \\
 &\quad (az - \frac{1}{2} z^2) \Big|_0^{1000} (10^6)
 \end{aligned}$$

$$(a \cdot 10^3 - \frac{b}{2} 10^4) \cdot 10^6$$

Sub for a & b

$$1.1655 \text{ kg. !}$$