# Define the basis functions

```
> phi[n]:=sin( (n*Pi*x)/l );
```

*Note that* $sin(0) = 0$ *and* $sin(l) = 0$, *i.e., the basis function is zero at* $x = 0$ *and* $x = l$.

$$\phi_n := \sin\left(\frac{n\,\pi\,x}{l}\right) \qquad (1.1)$$

# Example 1

$f(x)$ *is a function that is zero at* $x = 0$ *and* $x = 1$.

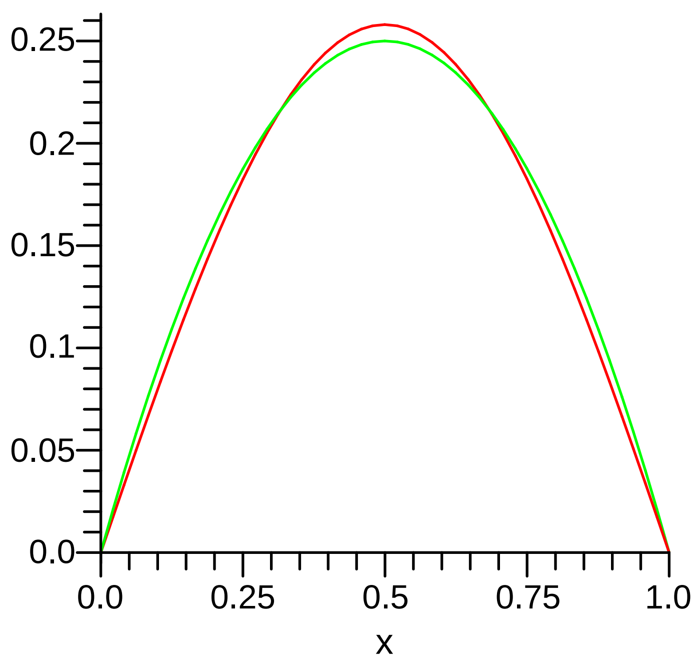
```
> f:= x*(1-x);
```

$$f := x\,(1 - x) \tag{2.1}$$

*Below is a* "for" *loop to calculate the coefficients of the Fourier series.*
*imax* = *number of terms in teh series to compute.*
*lvalue* = *sets the length of the interval* $(l = lvalue)$.

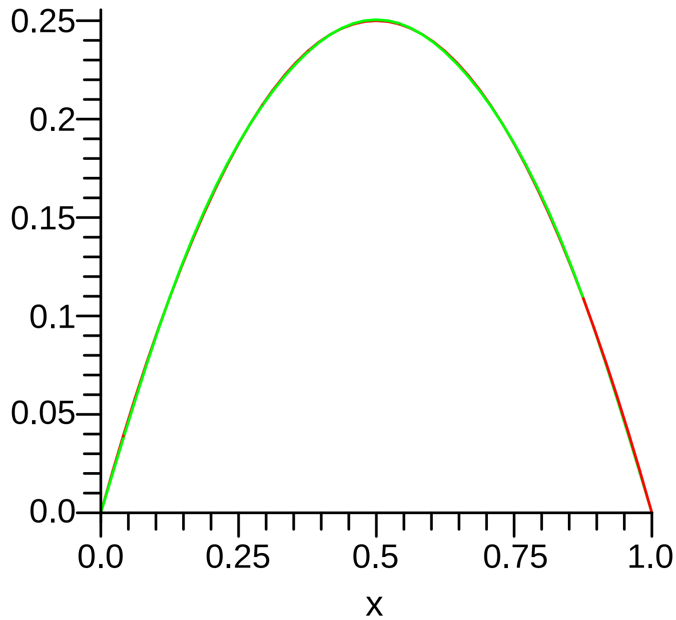
```
> imax:= 1:
  lvalue:= 1:
  for i from 1 by 1 to imax do
    a[i]:= evalf(
            subs(l=lvalue,
             int( f*subs(n=i,phi[n]),x=0..l)/int(subs(n=i,phi[n]
  ^2), x=0..l))):
  end do:
  unassign('i'):
  fseries:= subs(l=lvalue, sum( a[i]*subs(n=i,phi[n]), i=1..imax))
  ;
  plot({f,fseries}, x=0..1);
```

$$fseries := 0.2580122754\,\sin(\pi\,x)$$



*The result below is for imax* = 5. *The coefficients of* $n=2$ *and* $n=4$ *are* 0.

$$fseries := 0.2580122754 \sin(\pi x) + 0.009556010199 \sin(3 \pi x) + 0.002064098203 \sin(5 \pi x)$$



# Example 2

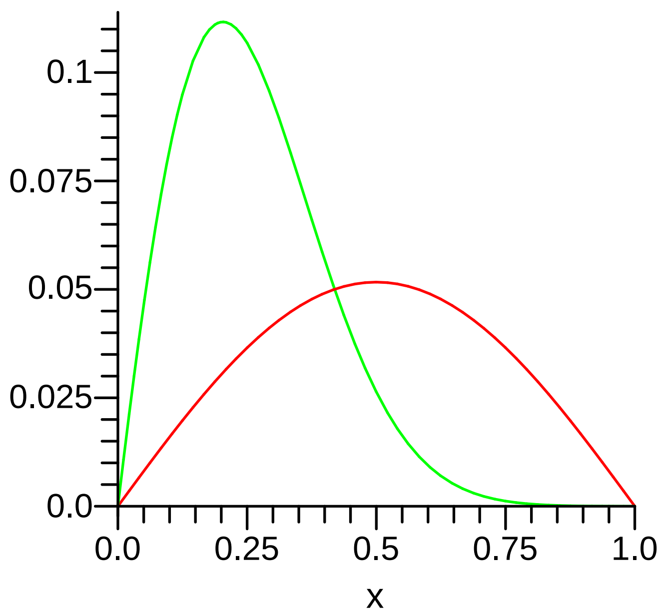
*Similar to example* 1 *the function is zero at the end points. However, it is not symmetrix about the center.*

```
> f:= exp(-(3*x)^2)*x*(1-x);
```

$$f := e^{(-9\,x^2)}\,x\,(1-x) \qquad (3.1)$$

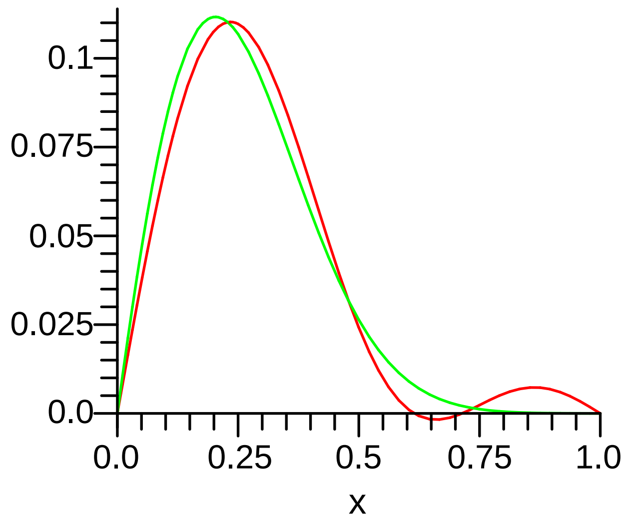
```
> imax:= 1:
  lvalue:= 1:
  for i from 1 by 1 to imax do
    a[i]:= evalf(
            subs(l=lvalue,
             int( f*subs(n=i,phi[n]),x=0..l)/int(subs(n=i,phi[n]
  ^2), x=0..l))):
  end do:
  unassign('i'):
  fseries:= subs(l=lvalue, sum( a[i]*subs(n=i,phi[n]), i=1..imax))
  ;
  plot({f,fseries}, x=0..1);
```

$$fseries := (0.05167728534 + 0.\,I)\,\sin(\pi\,x)$$

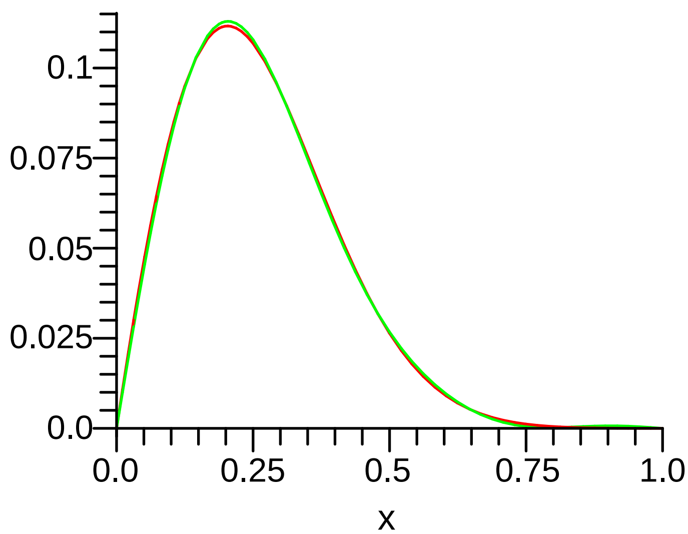


*Three term series.*

$$fseries := (0.05167728534 + 0.\,I)\sin(\pi x) + \left(0.05367436974 - 2.539296380\;10^{-12}\,I\right)\sin(2\pi x) + \left(0.02753731894 + 2.285366743\;10^{-12}\,I\right)\sin(3\pi x)$$




*Five term series.*



# Example 3

*f is zero at the left end but nonzero at the right end. No matter how many terms are in the Fourier sine series, the sum will always be zero at the right end. Thus, we expect some* "mismatch."

```
> f:= x;
```

$$f := x \qquad (4.1)$$

```
> imax:= 50:
  lvalue:= 1:
  for i from 1 by 1 to imax do
    a[i]:= evalf(
            subs(l=lvalue,
             int( f*subs(n=i,phi[n]),x=0..l)/int(subs(n=i,phi[n]
  ^2), x=0..l))):
  end do:
  unassign('i'):
  fseries:= subs(l=lvalue, sum( a[i]*subs(n=i,phi[n]), i=1..imax))
  :
  plot({f,fseries}, x=0..1);
```

*The fit of the series to the function is decent at* $x = 0$. *However, at* $x = 1$ *there is substantial* "ringing." *This is referred to as Gibbs phenomena. We can reduce the interval over which the ringing occurs by taking more and more terms. However, there will always be some overshoot and undershoot at the end. This is an unavoidable result of truncating the series, i.e., only taking a finite number of terms.*