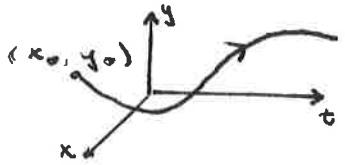
BIG PICTURE

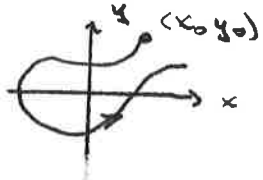
$\dot{\underline{x}} = \underline{f}(\underline{x})$ (2D)

Solutions



projection ⟹

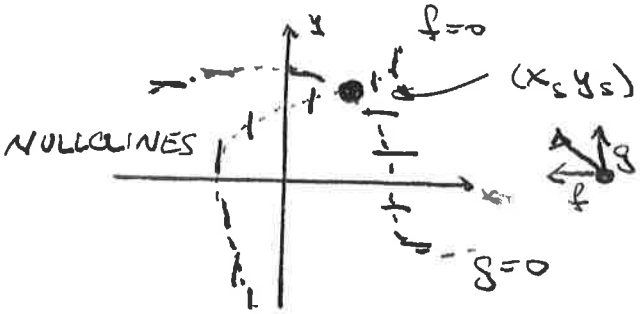
Phase Plane



In the PPlane if

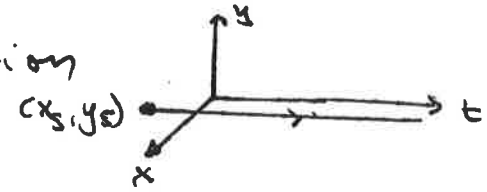
$\dot{x} = 0$ if $f(x,y) = 0$ ⇒ No horz motion ⇒ all vertical

$\dot{y} = 0$ if $g(x,y) = 0$ ⇒ No vert. " ⇒ " horz.



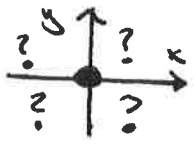
At any pt the rel. vector of the trajectory has components f & g.

EP/FP : $f = 0$ AND $g = 0$. No motion



LITTLE PICTURE

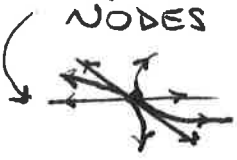
$\dot{\underline{x}} = \underline{\underline{A}} \cdot \underline{x}$ has EP at $(x_s, y_s) = (0,0)$



A.S. & U SPIRALS

CENTER

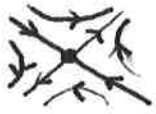
A.S. & U NODES



STARS

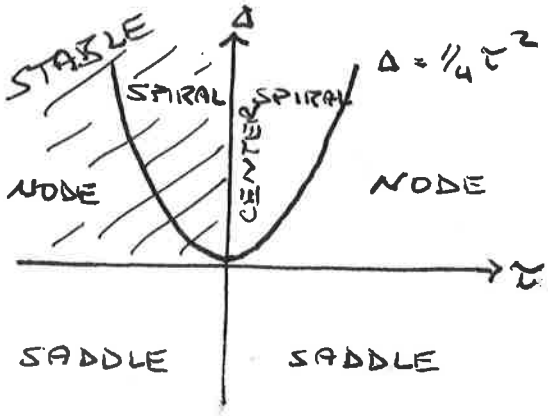
DEG. NODES

SADDLES



Eigenvalues

$\lambda^2 - \tau\lambda + \Delta = 0$



LINEARIZATION

How do we connect the BIG PICTURE to the LITTLE PICTURE?

$$\dot{\underline{x}} = \underline{f}(\underline{x}) \xrightarrow[\text{LINEARIZE}]{} \dot{\underline{u}} = \underline{\underline{A}} \cdot \underline{u}$$

① Identify EP/FP (x_{1s}, x_{2s})

② Shift origin: $\underline{x} = \underline{x}_s + \underline{u}$ $\Big\{ \begin{array}{l} x_1 = x_{1s} + u_1 \\ x_2 = x_{2s} + u_2 \end{array}$

③ Taylor series (2D)

④ Zoom in: $u_1^2, u_1 u_2, u_2^2 \ll 1$