MATH 4325

• $\dot{x} = \begin{pmatrix}2&0\\0&-1\end{pmatrix}\cdot x$

$\dot{x}_1 = 2x_1 \Rightarrow x_1 = c_1 e^{2t}$

$\dot{x}_2 = -x_2 \qquad x_2 = c_2 e^{-t}$

5.1.2) $\frac{dy}{dx} = \frac{\frac{d}{dt}(c_2 e^{-t})}{\frac{d}{dt}(c_1 e^{at})} = \frac{c_2}{|a|c_1} e^{(|a|-1)t}, \quad |a|>1$

$\lim_{t\to\infty} dy/dx \to \infty$ Vertical

$\lim_{t\to-\infty} dy/dx \to 0$ Horizontal.

• $\dot{x} = \begin{pmatrix}-1&1\\0&-1\end{pmatrix}\cdot x$

$\dot{x}_2 = -x_2 \Rightarrow x_2 = c_2 e^{-t}$

$\dot{x}_1 = -x_1 + c_2 e^{-t}$

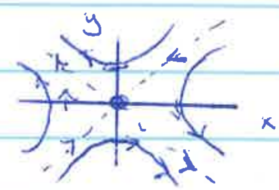
x_1 = Homo + part.

$x_{1p} = Ate^{-t}$

$\Rightarrow A = c_2$

$x_1 = c_1 e^{-t} + c_2 t e^{-t}$

5.1.8) a)



b) $\frac{dy}{dx} = \frac{x}{y}$

$y\,dy = x\,dx$

$\frac{1}{2}y^2 - \frac{1}{2}y_0^2 = \frac{1}{2}x^2 - \frac{1}{2}x_0^2$

$x^2 - y^2 = x_0^2 - y_0^2 = C$

• $\dot{x} = \begin{pmatrix}1&2\\-2&1\end{pmatrix}\cdot x$

$\ddot{x}_1 = \dot{x}_1 + 2\dot{x}_2$

$\ddot{x}_1 = \dot{x}_1 + 2(-2x_1 + x_2)$

$= \dot{x}_1 - 4x_1 + \dot{x}_1 - x_1$

$\ddot{x}_1 - 2\dot{x}_1 + 5x_1 = 0$

$r^2 - 2r + 5 = 0$

$r = 1 \pm 2i$

$x_1 = e^t(c_1\cos 2t + c_2 \sin 2t)$

$x_2 = \frac{1}{2}(\dot{x}_1 - x_1)$ ugly.

5.1.1) $\frac{dv}{dx} = \frac{-\omega^2 x}{v} \qquad v^2 + \omega^2 x^2 = C$

(Integrate as in 5.1.8)

$v^2 \sim$ KINETIC ENERGY

$\omega^2 x^2 \sim$ POTENTIAL "

$\therefore$ TOTAL ENERGY $E(t)$ is

$E(t) = C$

$\frac{dE}{dt} = 0$

(2.3.4) a) $\frac{\dot{N}}{N} = F(N) = r - a(N-b)^2$

$\frac{dF}{dN} = -a2(N-b) = 0$

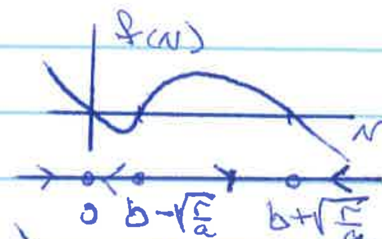
$\exists$ a crit pt $N = b \Rightarrow b > 0$

$\frac{d^2F}{dN^2} = -2a$ want a max so $a > 0$

$\frac{\dot{N}}{N} = 0$ when $N = b \pm \sqrt{\frac{r}{a}}$

want these $> 0 \Rightarrow b - \sqrt{\frac{r}{a}} > 0$

$F(b) = r$ growth $\Rightarrow r > 0$

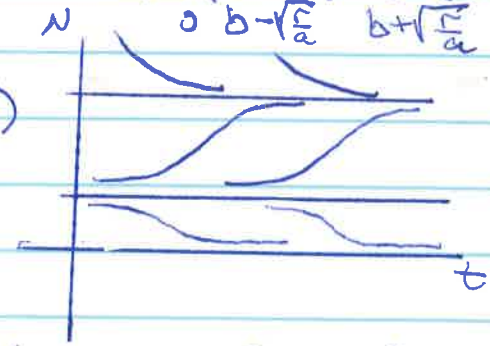


b) $\dot{N} = N F(N) = f(N)$

$f = 0$ when $N = 0,\ b \pm \sqrt{\frac{r}{a}}$

fix points

c)



d) Here we require a min initial population for long term nonzero population. For logistic 0 is unstable and all small pops grow.