Math 4325: Maps

(1) (*Based on Strogatz 8.7.9*)

$$\dot{r} = r(1-r), \quad \dot{\theta} = 1$$

(a) Solve the differential equation for $r(t)$.
(b) Consider the surface of section defined by $\theta = \pi/2$, and use the solution from (a) to define a Poincare' map.
(c) Find the fixed points of the Poincare' map and analyze their linear stability.
(d) Sketch the cobweb diagram illustrating the evolution of a few initial conditions.

(2) (*Based on Strogatz 10.1.9*)

$$x_{n+1} = \frac{3x_n}{1+x_n}$$

(a) Find the fixed points and analyze their linear stability.
(b) Use the provided blank cobweb template to carefully (use a ruler and be accurate) plot the evolution of the following initial conditions: $x_0 = -0.5, 0.5$ and 4.

(3)
Do Strogatz 10.1.12. A couple of comments:

- *Superstable* means $f'(x) = 0$ (see text). . I suggest plotting $f(x)$ to understand what would happen in the cobweb diagram and why these points are *superstable*
- "Numerically iterate" means to substitute x_0 into the map and compute x_1. Then substitute x_1 to get x_2, etc. You will have to use a calculator or software.

(4) *(Based on Strogatz 10.2.6)*

$$x_{n+1} = r\cos(x_n)$$

Use the provided blank cobweb templates to carefully (use a ruler and be accurate) sketch the dynamics of the map for $r = 1, 2$ and 4. See if you can find a P2 orbit and a chaotic orbit.

(5)
Do Strogatz 10.3.4 (a & b only)

(5)
Do Storgatz 10.3.11