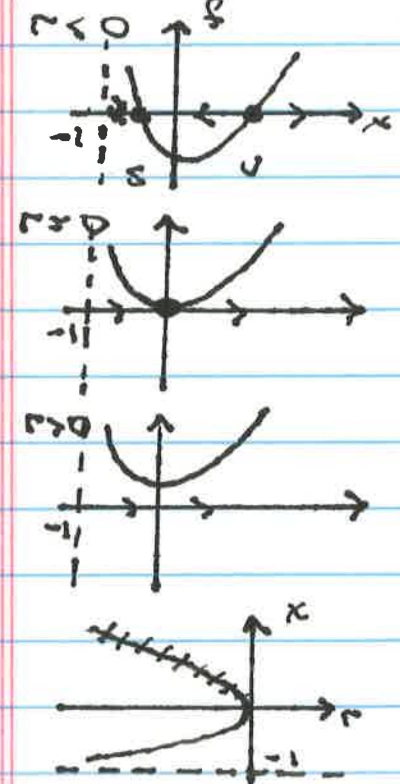
MATH 4325

(3.1.3) $\dot{x} = r + x - \ln(1+x)$



When $r=0$ there is a EP $x_s = 0$

Checking L.S.

$$f'(x_s) = 1 - \frac{1}{1+x_s}\Big|_{x_s=0} = 0$$

$\therefore$ $(x_s, r_c) = (0,0)$ is a potential bif. pt.

Let $(x, r) = (0,0) + (y, \mu)$

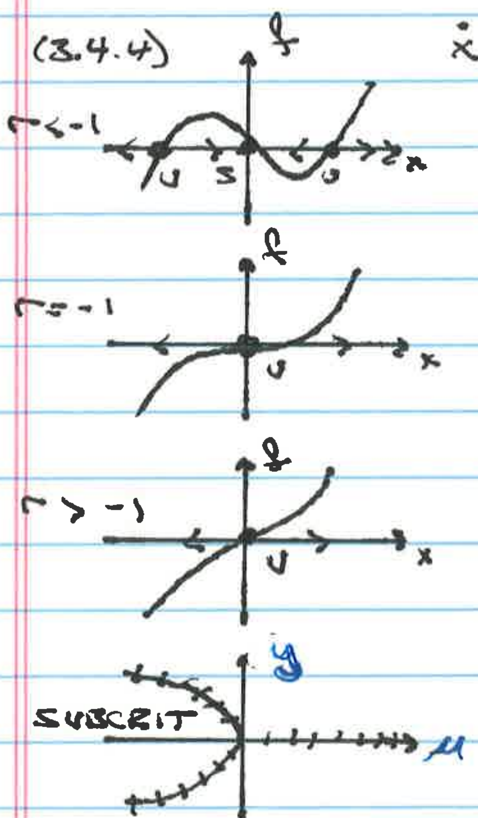
$$\dot{y} = \mu + y - \ln(1+y)$$

$$= \mu + \cancel{y} - (\underbrace{\cancel{\ln 1}}_{0} + \cancel{y} - \tfrac{1}{2}y^2 + \dots)$$

$$\dot{y} = \mu + \tfrac{1}{2}y^2$$

SADDLE NODE

(3.4.4) $\dot{x} = x + \frac{rx}{1+x^2}$



$f(0) = 0 \Rightarrow x_s = 0$

$f'(0) = 1 + r = 0$ when $r = -1$

$\therefore$ $(x_s, r_c) = (0,-1)$ is pot bif pt.

Let $(x, r) = (0,-1) + (y, \mu)$

$$\dot{y} = y + \frac{(-1+\mu)y}{1+y^2}$$

$$= y + (-1+\mu)y(1 - y^2 + \dots)$$

$$= \cancel{y} - \cancel{y} + \mu y - y^3 + \dots$$

$$\dot{y} = \mu y + y^3$$

PITCHFORK

(3.3.2)

$\dot P = 0 \Rightarrow P = ED$

$\dot D = 0 \Rightarrow \lambda + 1 - \cancel{D} - \lambda E P = 0$

P/E

$$P = \frac{(\lambda+1)E}{1+\lambda E^2}$$

$$\dot E = k\left(\frac{\lambda+1}{1+\lambda E^2} - 1\right)E$$

let $E = \frac{F}{\sqrt{\lambda}}$ so $E^2 = \frac{F^2}{\lambda}$

$$\dot F = k\left(\frac{\lambda+1}{1+F^2} - 1\right)F$$

EP: $F_s = 0$

$$\frac{1+\lambda}{1+F_s^2} - 1 = 0$$

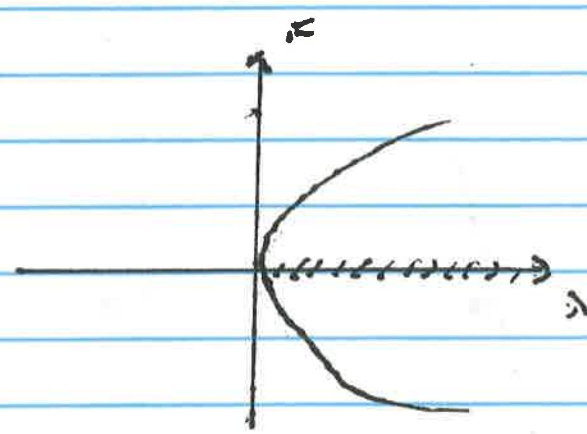
$F_s^2 = \lambda$ $(F_s = \pm\sqrt{\lambda}, \lambda > 0)$

L.S.: f' = Product rule.

$f'(F_s = 0) = k\lambda$

$\lambda > 0 \Rightarrow f' > 0$ U

$\lambda < 0 \Rightarrow f' < 0$ S



(3.7.4)

a) Examine $H \frac{N}{A+N}$

$\lim_{N \to \infty} H \cdot \frac{N}{A+N} = H$.

With many fish the harvest rate is H.

$\lim_{N \to 0}$ " $= \frac{H}{A} N$

A modifies the harvest rate for small #'s.

Also $\frac{d}{dN}\left(\frac{N}{A+N}\right) = \frac{A}{(A+N)^2} \sim \frac{1}{A}$ for $N \to 0$

Again shows how harvest rate changes as N gets small.

b) $\tau = rt \Rightarrow \frac{d}{dt} = r\frac{d}{d\tau}$. Let $x = \frac{N}{K}$

$$x_\tau = x(1-x) - \frac{H}{Kr} \frac{x}{\left(\frac{A}{K}\right)+x} \Rightarrow h = \frac{H}{Kr} \quad a = \frac{A}{K}$$

c) EP/FP $\Rightarrow f=0$: $x\left(x^2 - x(1-a) + (h-a)\right) = 0$

$x_s = 0$

$x_s = \frac{1}{2}\left[(1-a) \pm \sqrt{(1-a)^2 - 4(h-a)}\right]$

2 solutions if $h \le \frac{1}{4}(1+a)^2$, 0 otherwise

Suggests this is a S.N. bif.

d) L.S.: $f'(0) = \left(1 - \frac{h}{a}\right) = 0$ if $h = a$

N.F.: let $(x\ h) = (0, a) + (y\ \mu)$

$$y_\tau = -\frac{\mu}{a} y + \left(-1 + \frac{1}{a}\left(1 + \frac{\mu}{a}\right)\right) y^2 - \left(1 + \frac{\mu}{a}\right)\frac{1}{a^2} y^3 + \dots$$

If $a \neq 1$ dominant terms are: $y_\tau = -\frac{\mu}{a} y + \left(-1 + \frac{1}{a}\right) y^2$

Transcritical

If $a = 1$ " " " : $y_\tau = -\mu y - y^3$

Pitchfork