strog3p3p2-1.gif

Strogatz 3.3.2

$$\dot{E} = \kappa\left(\frac{\lambda+1}{1+\lambda E^2} - 1\right)E$$
$$= \kappa\lambda\left(\frac{1-E^2}{1+\lambda E^2}\right)E$$

S.S. $E = 0 \quad E = \pm 1$

Note $1 + \lambda E^2 \neq 0 \Rightarrow E^2 \neq -\frac{1}{\lambda}$

if $\lambda < 0 \quad E^2 \neq \frac{1}{|\lambda|}$

SINGULAR PT.

L.S. $f' = \kappa\lambda\left[\frac{1-(\lambda+3)E^2 - \lambda E^4}{(1+\lambda E^2)^2}\right]$

if $E=0 \quad f' = \kappa\lambda \cdot 1 \Rightarrow \lambda < 0 \Rightarrow S$

$\lambda > 0 \Rightarrow U$

$E = \pm 1 \quad f' = -2\kappa\lambda\left(\frac{1+\lambda}{(1+\lambda)^2}\right) \Rightarrow$

$\lambda > 0 \Rightarrow S$

$-1 < \lambda < 0 \Rightarrow U$

$\lambda = -1$ undefined

$\lambda < -1 \Rightarrow S$

Note: let $\lambda = -1$ then f has a removable discontinuity

$\dot{E} = \kappa \cdot \lambda \cdot 1 \cdot E$

For $\lambda = -1$, $\exists$ only 1 S.S.

strog3p3p2-2.gif

Mult by E:

$$EE' = \kappa\lambda\left(\frac{1-E^2}{1+\lambda E^2}\right)E^2$$

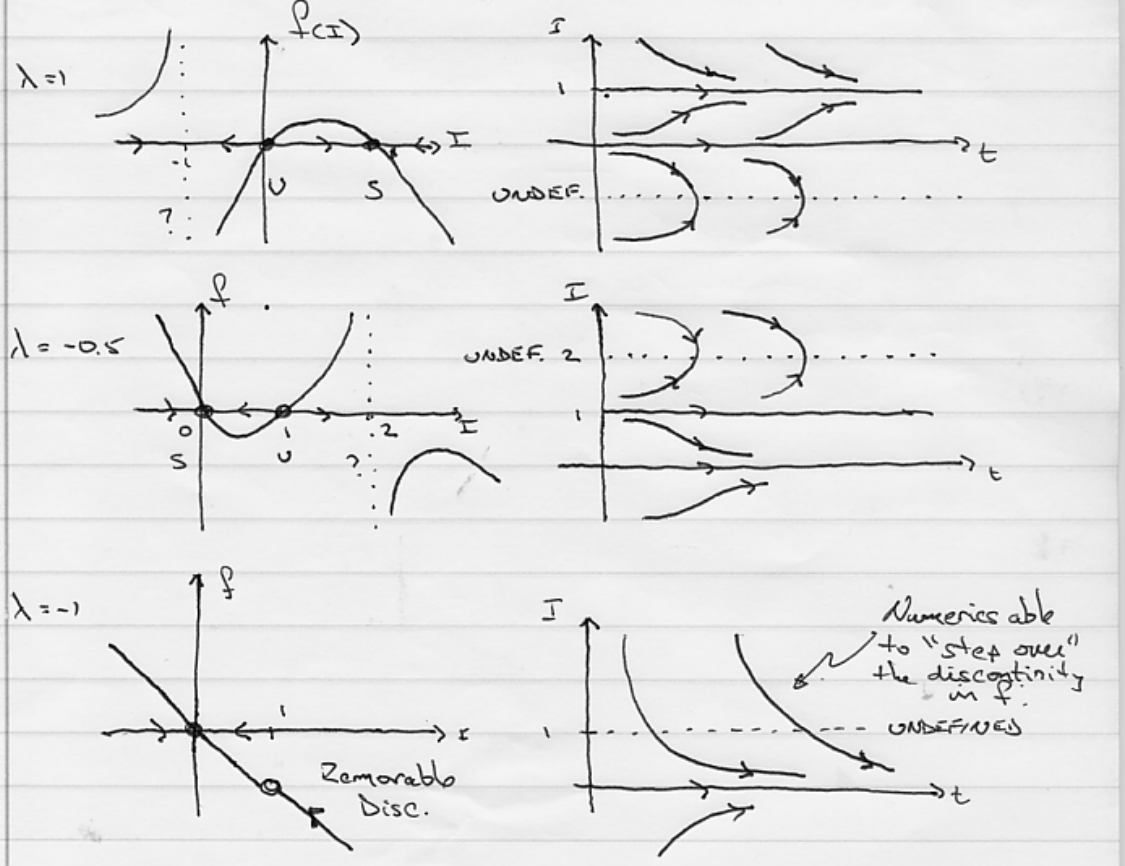
Let $I = E^2$ (Intensity = (Electric field)^2)

$$I' = 2\kappa\lambda\left[\frac{1-I}{1+\lambda I}\right]I$$

Ignore the fact that $I \geq 0$. Physically, ie, allow $I < 0$.

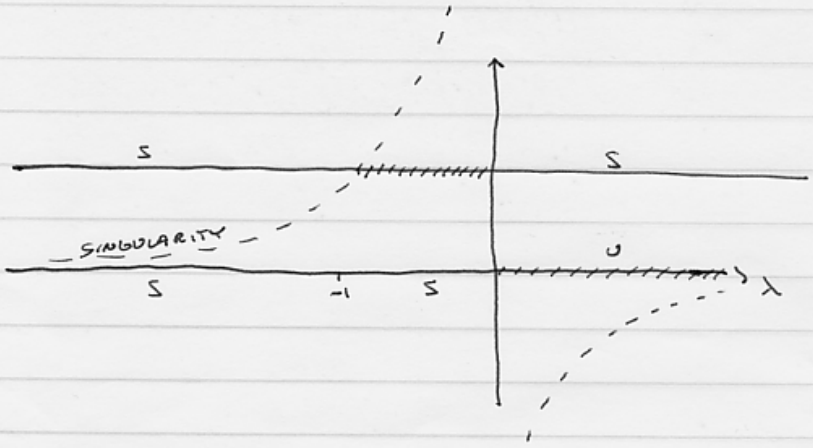
Solvable by S of V: $\frac{1+\lambda I}{(1-I)I}\,dI = 2\kappa\lambda\,dt$

$\S$ Mess.



strog3p3p2.gif

$f(I, \lambda) = 0 \Rightarrow I = I(\lambda)$



$f(I)$: