8.1.6

$\dot{x} = y - 2x$

$\dot{y} = \mu + x^2 - y$

Nullclines: $y = 2x$ $\quad y = \mu + x^2$

Graphically, μ raises and lowers the parabola so we know there will be either 0, tangent, or 2 solution/intersections. $\Rightarrow$ Saddle-Node Bif

Can verify value of μ using L.S. Alternatively,

$\frac{dy}{dx} = \frac{d}{dx}(2x) = 2$ = slope of 1st NC

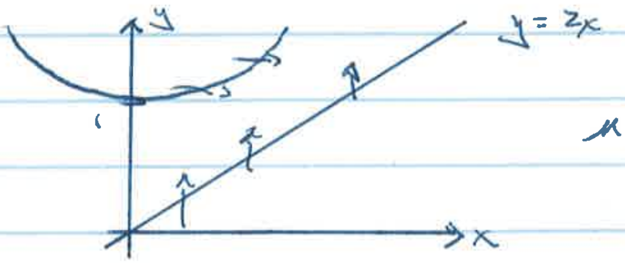
$\frac{dy}{dx} = \frac{d}{dx}(\mu + x^2) = 2x$ = slope of 2nd NC.

When slopes equal the NC are tangent, which is the transition from 0 to 2 EP.

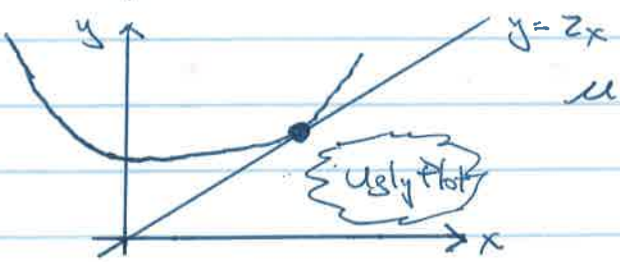
$2 = 2x \Rightarrow x = 1$

$y = 2x\big|_{x=1} = 2$

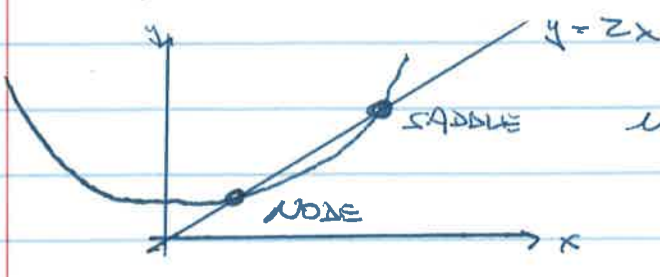
$y = \mu + x^2\big|_{x=1} = \mu + 1 \Rightarrow$ Require $\mu = 1$



$\mu > 1$



$\mu = 1$



$\mu < 1$

8.1.10

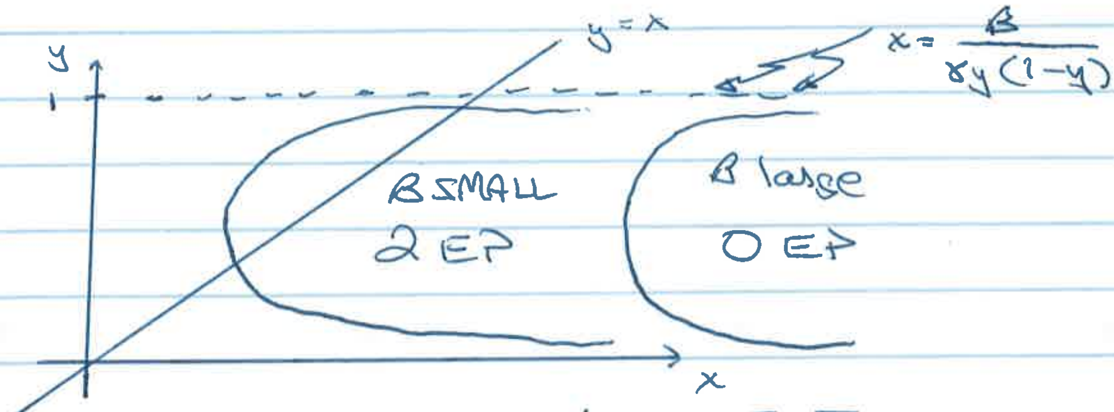
$\dot{S} = r_S S\left(1 - \frac{S}{K_S}\frac{K_E}{E}\right) \qquad \dot{E} = r_E E\left(1 - \frac{E}{K_E}\right) - P\frac{B}{S}$

a)
- r_S & r_E: Net birth rate of S & E.
- $\left(\frac{S}{K_S}\right)$: Saturation effect of the size of trees. There is an effective "natural size" (Carry capacity) K_S.
- $\left(\frac{E}{K_E}\right)$: Saturation effect of the energy reserve. The carry capacity for energy is K_E. Note, in the equation for S, if energy reserves increase the saturation effect decreases allowing for bigger trees.
- $P\left(\frac{B}{S}\right)$: Budworms decrease the energy of the tree. However, a bigger tree (S) is less susceptible.

b) let $\frac{S}{K_S} = x \quad \frac{E}{K_E} = y \quad \tau = r_S t \Rightarrow \frac{d}{dt} = \frac{d}{d\tau} r_S$

$\dot{x} = x(1 - x/y) \quad \dot{y} = \gamma y(1-y) - \frac{\beta}{x} \quad \gamma = \frac{r_E}{r_S} \quad \beta = \frac{PB}{r_S K_S K_E}$

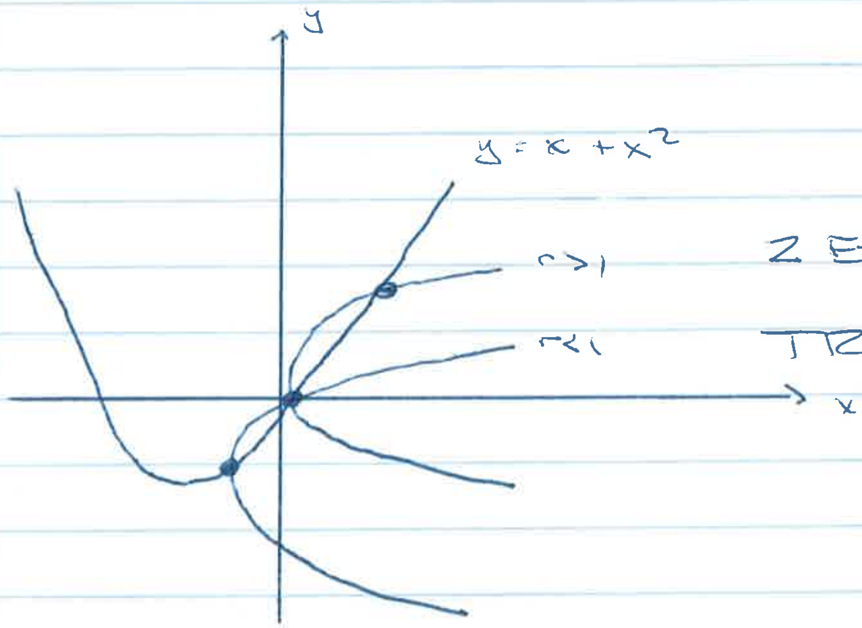
c)



$\Rightarrow$ SADDLE NODE BIF

d) Use PPLANE

$$\dot{x} = y - x - x^2$$
$$\dot{y} = rx - y - y^2$$



2 EP $\Rightarrow$ 2 EP

TRANSCRITICAL