Review of Ordinary Differential Equations

For Math 6324 - Dynamical Systems and Math 5334 - Partial Differential Equations.

- Calculus: differentiation and integration, curve sketching (max/min).
- Euler's formula:

$$e^{\pm i\theta} = cos(\theta) \pm i\sin(\theta)$$
$$\cos(\theta) = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$$
$$\sin(\theta) = \frac{1}{2}(e^{i\theta} - e^{-i\theta})$$

- The Taylor series representation of $f(x)$ for x near x_0 is

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2 + \dots$$

- First-order DE:
  - Separable
$$\frac{dy}{dx} = f(x, y) = h(x)g(y), \quad \frac{dy}{g(y)} = h(x)dx.$$
  - Linear constant-coefficient nonhomogeneous
$$\frac{dy}{dx} = kx + l$$
    Solve with method of undetermined coefficients.
  - Linear variable-coefficient nonhomogeneous
$$\frac{dy}{dx} = p(x)y + q(x)$$
    Solve with integrating factor $u(x) = \exp(-\int p(x)dx)$.
- Second-order DE:
  - Linear constant-coefficient homogeneous
$$ay'' + by' + cy = 0$$
    Let $y = e^{rx}$ to obtain the characteristic equation
$$ar^2 + br + c = 0$$
    Solve for r: real & distinct, real & repeated, complex conjugate.
  - Linear constant-coefficient nonhomogeneous
$$ay'' + by' + cy = f(x),$$
    where $f(x)$ is a polynomial, exponential, sin or cosine.
    Solve with method of undetermined coefficients.
- Systems: rewrite a 2nd order DE as a system of two DE.

$$ay'' + by' + cy = f(x)$$
$$\text{Let} z = y' \text{ so that } z' = y''$$
$$\begin{aligned} y' &= z \\ z' &= -\frac{b}{a}z - \frac{c}{a}y + \frac{1}{a}f(x) \end{aligned}$$