4335 : Exam 1

$$\alpha \frac{du}{dt} = \beta u + \gamma \qquad u(0) = \eta$$

$$\alpha \frac{du}{dt} - \beta u = \gamma$$

$$\frac{du}{dt} - \frac{\beta}{\alpha} u = \frac{\gamma}{\alpha}$$

$$V = e^{\int -\beta/\alpha \, dt} = e^{-\beta/\alpha \, t}$$

$$\frac{d}{dt}\left(e^{-\beta/\alpha \, t} u\right) = \frac{\gamma}{\alpha} e^{-\beta/\alpha \, t}$$

$$e^{-\beta/\alpha \, t} u = \frac{\gamma}{\alpha}\left(-\frac{\alpha}{\beta} e^{-\beta/\alpha \, t}\right) + c$$

$$u = -\gamma/\beta + c e^{\beta/\alpha \, t}$$

$$u(0) = -\gamma/\beta + c = \eta$$

$$c = \eta + \gamma/\beta$$

$$u(t) = (\eta + \gamma/\beta) e^{\beta/\alpha \, t} - \gamma/\beta$$

4335 EXAM #1

$u' = ru(1-u^2)$

a) $f = u(1-u)(1+u) \Rightarrow u_c = 0, +1, -1$

b) $f' = r - 3ru^2$

$f'(0) = r \qquad r \gtrless 0 \rightarrow U/S$

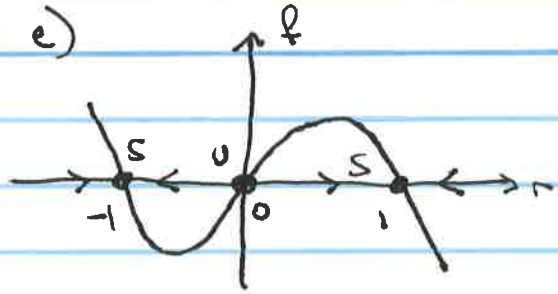
$f'(+1) = -2r \qquad r \gtrless 0 \Rightarrow S/U$

$f'(-1) = -2r \qquad r \gtrless 0 \Rightarrow S/U$

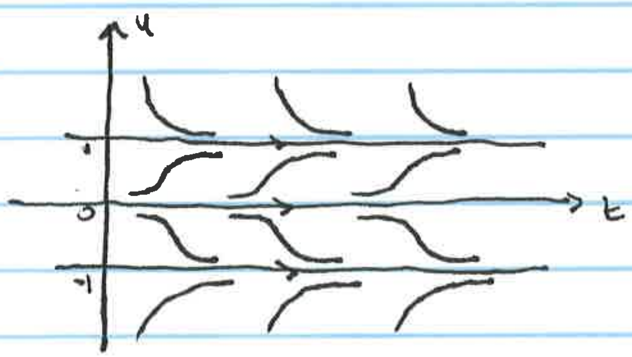
c) if $r < 0$ stability switches

$U \rightarrow S \;\&\; S \rightarrow U$

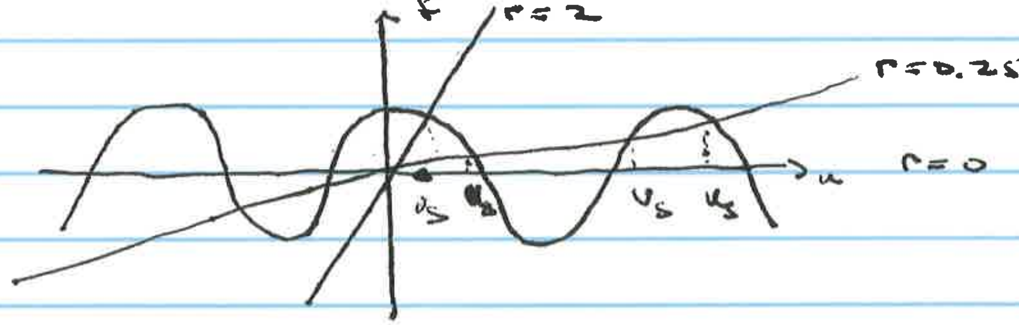
d) & e)



f)



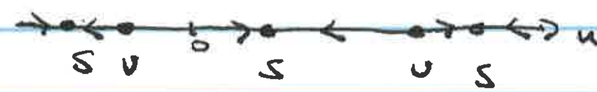
2) $u' = \cos u - r u \qquad r = 2,\ 0.25,\ 0$



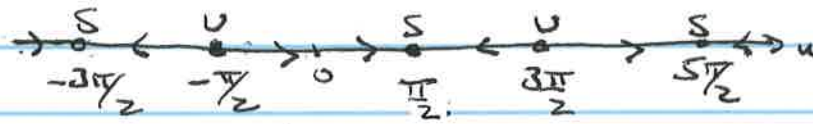
b) $r = 2$

c)

$r = 0.25$



$r = 0.0$



d) For $r=2$ there is only a single stable pos. s.s.
As r decreases, new s.s. are created in pairs due to the intersection of $\cos u$ & ru.
The stability alternates S to U to S, etc.
When $r=0$, the steady states are the roots of $\cos(u)$ and continue to alternate stability.

3) $u' = ru(1 - \frac{u}{K}) - H\frac{u}{1+u}$

a) r is the GROWTH RATE. It governs the exponential rate of increase in a small pop w/o harvesting.
K is the CARRYING CAPACITY. In the absence of harvesting the population saturates to $u = K$. due to self competition.
H controls the harvesting

b) We have

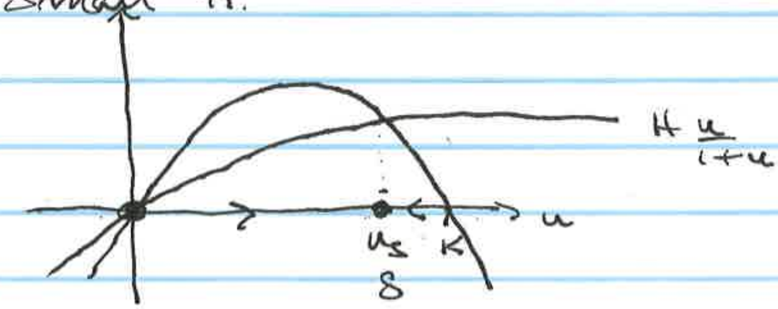
$u' \sim -H$. Here the rate of harvesting is always the same no matter how many "u" there are.

$u' \sim -Hu$ Here the Harvesting is proportional to the pop. size. Thus, if u is small the harvest rate decreases.

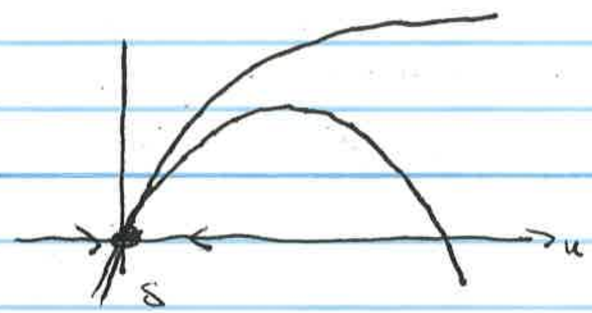
$u' \sim -H\frac{u}{1+u}$. For $u \ll 1$ $u' \sim -Hu$. Thus only a small # of u will be taken. For $u \gg 1$ this term saturates to $u' \sim -H \cdot 1$. Thus, for large u the harvest rate saturates at H instead of getting larger.

Small H.

c)



Large H.



As H increase $u_s \neq 0$ decreases. Eventually, H is large enough such that only $u_s = 0$ is a valid s.s. and it is stable.

In general, for small pop sizes the saturation effect is unimportant. For $u \ll 1$ $u' \sim -Hu$ and the dynamics is similar to constant effort.