MATH 4235

EXAM 2

(1) $u' = uv$, $v' = v^2 - 1$

a) $uv = 0$ $u = 0$ & $v = 0$ No horz

$v^2 - 1 = 0$ $v = \pm 1$ No vert.

b) If $u = 0 \Rightarrow v = \pm 1$

If $v = 0 \Rightarrow$ can't satisfy $v^2 - 1 = 0$

S.S. are $(0, 1)$ & $(0, -1)$

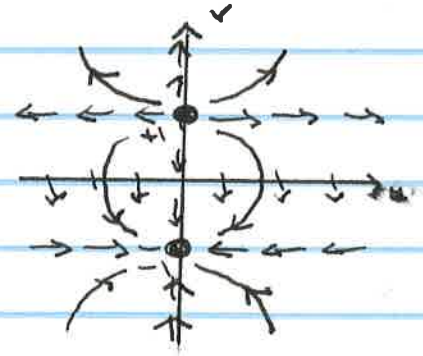
c) $Df = \begin{pmatrix} v & u \\ 0 & 2v \end{pmatrix}$

$Df|_{(0,-1)} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$

$\lambda = -1, -2$ A.S. NODE

$Df|_{(0,+1)} = \begin{pmatrix} +1 & 0 \\ 0 & +2 \end{pmatrix}$

$\lambda = +1, +2$ U. NODE



(2) $u' = -u + uv$, $v' = v - auv - bv^2$

a) $0 = u(-1 + v)$

$u = 0$ or $v = 1$

$0 = v - auv - bv^2$

If $u = 0$: $0 = v - bv^2$

$0 = v(1 - bv)$

$v = 0$ $v = 1/b$

If $v = 1$: $0 = 1 - b - au$

$u = \frac{1-b}{a}$

S.S.: $(0, 0)$

$(0, 1/b)$

$(\frac{1-b}{a}, 1)$ Coexistence

b) If $b < 1$, then $a > 0$

If $b > 1$, then $a < 0$

But this implies predation generates more prey and is unlikely

c) $Df = \begin{pmatrix} -1 + v & u \\ -av & 1 - au - 2bv \end{pmatrix}$

$Df|_{(0,0)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$\lambda = -1, 1$ SADDLE ALWAYS.

NEVER get extinction

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(3) $h' = 1 - ahz - h$

$z' = chz - z^2$

$0 = 1 - ahz - h$

$0 = chz - z^2$

$\hookrightarrow 0 = z(ch - z)$

$z = 0$ $z = ch$

If $z = 0$ No zombies

$0 = 1 - 0 - h$

$h = 1$

If $z = ch$

$0 = 1 - ach^2 - h$

Solve for h

Only care about humans surviving and wiping out zombies $\Rightarrow (h, z) = (1, 0)$

$Df = \begin{pmatrix} -az - 1 & -ah \\ cz & ch - 2z \end{pmatrix}$

$Df|_{(1,0)} = \begin{pmatrix} -1 & -a \\ 0 & c \end{pmatrix}$

$\lambda = -1, c$

Want stable $\Rightarrow c < 0$

$a - b < 0$

$a < b$

Must obliterate $\geq$ hammers faster than we generate.