DERIVING THE LOGISTIC EQU.

(Expanded version.)

$$\frac{dS}{dt} = bN - \beta\frac{SI}{N} - \mu S + \gamma I$$

$$\frac{dI}{dt} = \beta\frac{SI}{N} - (\mu+\gamma) I$$

$$N = S + I$$

If $b = \mu$, $\frac{dN}{dt} = 0 \Rightarrow N = \text{constant}.$

$\Rightarrow S = N - I$

$$\frac{dI}{dt} = \beta\frac{N-I}{N}I - (\mu+\gamma) I$$

$$= \beta\left(1 - \frac{1}{N}I\right)I - (\mu+\gamma) I$$

- Regroup terms so it looks like the LOGISTIC EQU.

  Why? Not necessary. Could analyze as is just fine.

  However, we have experience w/ LOG-Eq. and so it serves as a useful reference.

$$\frac{dI}{dt} = (\beta - (\mu+\gamma))I - \frac{\beta}{N}I^2$$

$$= (\beta-(\mu+\gamma))I\left(1 - \frac{I}{\frac{N}{\beta}(\beta-(\mu+\gamma))}\right)$$

Let: $r = \beta - (\mu+\gamma)$: linear growth rate

$K = N\left(1 - \frac{\mu+\gamma}{\beta}\right)$: carrying capacity

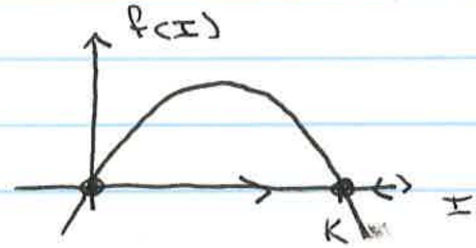
$$\frac{dI}{dt} = rI\left(1 - \frac{I}{K}\right)$$

If $r > 0 \Rightarrow \beta - (\mu+\gamma) > 0$

$$\frac{\beta}{\mu+\gamma} > 1$$

$$\Rightarrow K = N(1 - \#\text{ less than } 1) > 0$$

So if $r > 0$, $K > 0$ and we know ...



ENDEMIC STEADY STATE is STABLE

DONE!

Expect that health professionals don't know about logistic equation, stability, phase lines, etc.

They know R_0 = BASIC REPRO #.

They expect ENDEMIC if $R_0 > 1$

Can we recast our stability condition $r > 0$ into "something" > 1. Then call something R_0

$$r = \beta - (\mu+\gamma) > 0$$

$$\frac{\beta}{\mu+\gamma} > 1 \quad \nabla$$

Define $R_0 = \frac{\beta}{\mu+\gamma}$

And it makes sense.

Units: $R_0 = \frac{\beta}{\mu+\gamma} \cdot \frac{\frac{\text{inf}}{\text{person}\cdot\text{time}}}{\frac{1}{\text{time}}} = \frac{\text{inf}}{\text{person}} > 1$

$$R_0 = \frac{\text{input rate}}{\text{output rate}} \text{ of } I \text{ class} > 1$$