MATH 5331

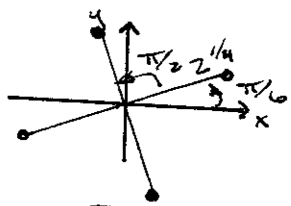
EXAM 1 SPRING 11

1) $(-1+\sqrt{3})^{1/4} =$

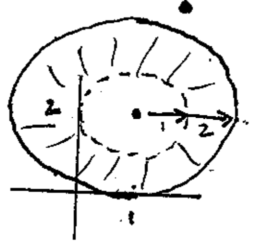
$= \left[[(-1)^2 + (\sqrt{3})^2]^{1/2} e^{i(\tan^{-1}\frac{\sqrt{3}}{-1} + 2\pi n)} \right]^{1/4}$

$= 2^{1/4} e^{i(\pi/6 + \frac{n\pi}{2})}$

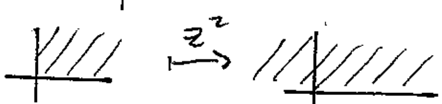
Distinct roots $\Rightarrow n = 0, 1, 2, 3$



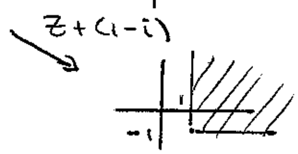
2a)



b)



c)



3a) $\lim\limits_{z\to 0} \frac{z^2}{|z|^2} = \lim\limits_{xy\to 0} \frac{x^2-y^2+i2xy}{x^2+y^2}$

let $y = m\cancel{x}$

$= \lim\limits_{x\to 0} \frac{\cancel{x^2}(1-m^2)}{\cancel{x^2}(1+m^2)} + i\frac{2\cancel{x^2}(m)}{\cancel{x^2}(1+m^2)}$

Depends on slope m. DNE.

b) $\lim\limits_{z\to 2+i} \frac{z^4}{z^3+i} = \frac{(2+i)^4}{(2+i)^3+i}$

4a) $f = \frac{1}{z} = \frac{1}{x+iy} = \frac{x-iy}{x^2+y^2}$

$u = \frac{x}{x^2+y^2}$ $u_x = \frac{y^2-x^2}{x^2+y^2}$ $u_y = \frac{-2xy}{x^2+y^2}$

$v = \frac{-y}{x^2+y^2}$ $v_x = \frac{2xy}{x^2+y^2}$ $v_y = \frac{y^2-x^2}{x^2+y^2}$

CR: $u_x = v_y$ $u_y = -v_x$ ✓ Satisfied

f is analytic except where $z=0$

$f' = -\frac{1}{z^2}$

4b) $f = (x^2+y^2) + ix$

$u = x^2+y^2$ $u_x = 2x$ $u_y = 2y$

$v = x$ $v_x = 1$ $v_y = 0$

CR: $u_x = v_y \Rightarrow x = 0$ $u_y = -v_x \Rightarrow y = -1/2$

Derivative exists only at $(0, -1/2)$

NOT ANALYTIC.

5) a) $u = x^3 - 3xy^2$

$u_x = 3x^2 - 3y^2 = v_y$

$u_y = -6xy = -v_x$

$\Rightarrow v = 3x^2y - y^3 + \phi(x)$

$v_x = \cancel{6xy} + \phi' = \cancel{6xy}$

$\phi' = 0 \Rightarrow \phi = C$

$v = 3x^2y - y^3 + C$

b) $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$

$= 6x - 6x = 0$

6a) $i^{2i} = e^{2i\log i} = e^{2i(\mathrm{Log}|i| + i\arg i)}$

$= e^{2i(\underbrace{\mathrm{Log}\,1}_{0} + i(\frac{\pi}{2} + 2\pi n))}$

$= e^{2i(i\frac{\pi}{2})} = e^{-\pi}$ PV $\Rightarrow n=0$

b) $\log(2-2i) = \mathrm{Log}|2-2i| + i\,\mathrm{Arg}(2-2i)$

$= \frac{1}{2}\mathrm{Log}\,8 - i\frac{\pi}{4}$ PV $= -\frac{\pi}{4}$

c) $(e^z)^2 + e^z + 1 = 0$

$e^z = \frac{1}{2}(-1 \pm \sqrt{1^2 - 4}) = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

$e^z = e^{\log(-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i)}$

$z = \log(-1/2 \pm \sqrt{3}i/2)$

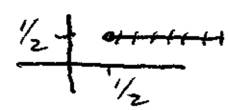
$z = \underbrace{\mathrm{Log}\left(\tfrac{1}{2}(1^2 + \sqrt{3}^2)^{1/2}\right)}_{0} + i\left(\tan^{-1}\frac{\pm\sqrt{3}}{-1} + 2\pi n\right)$

$z = i\left(\pm\frac{2\pi}{3} + 2\pi n\right)$

d) $1 + i - 2z = 1 - 2x + i(1-2y) = p + iq$

$p \le 0 \Rightarrow 1 - 2x \le 0 \Rightarrow x \ge 1/2$

$q = 0 \Rightarrow 1 - 2y = 0 \Rightarrow y = 1/2$ } BC



$f' = \frac{1}{1+i-2z}(-2)$