MATH 5331
EXAM #2

1a) $(-1)^{2/3} = e^{\frac{2}{3}(\log|-1| + i\arg(-1))}$ ($\log|-1| \to 0$)

$= e^{\frac{2}{3}i(\pi + 2\pi n)}$

b) $(1+i)^{1-i} = e^{(1-i)\log(1+i)}$

$= e^{\log(1+i)} e^{-i\log(1+i)}$

$= (1+i)\, e^{-i(\log|1+i| + i\arg(1+i))}$

$= (1+i)\, e^{-i\log\sqrt{2} + \pi/4 + 2\pi n}$

c) $i^{2i} = e^{2i(\log|i| + i\,\mathrm{Arg}(i))}$ ($\log|i| \to 0$, $\mathrm{Arg}(i) \to \pi/2$)

$= e^{-\pi}$

2a) let $x = t$, $y = t^2$ $t \in [-1, 0]$

$z = t + it^2$ $z' = 1 + i2t$

$\int_C \bar{z} + 2z\, dz = \int_{-1}^{0} (x - iy + 2x + i2y)\big|_{(t,t^2)} (1 + 2it)\, dt$

$= \int_{-1}^{0} (3t - 2t^3) + i7t^2\, dt$

$= \frac{3}{2}t^2 - \frac{1}{2}t^4 + i\frac{7}{3}t^3 \Big|_{-1}^{0}$

$= -1 + i\,7/3$

2b) C_1: $z = t$ $t \in [0,1]$ $z' = 1$

C_2: $z = (1-t) + it$ $t \in [0,1]$ $z' = -1 + i$

$\int_C f\, dz = \int_{C_1} + \int_{C_2}$

$= \int_0^1 t \cdot 1\, dt + \int_0^1 ((1-t) + it)(-1+i)\, dt$

$= \frac{1}{2}t^2\Big|_0^1 + (-1+i)t - it^2\Big|_0^1$

$= \frac{1}{2} - 1 + i - i = -\frac{1}{2}$

3a) $f = ze^{z^2}$ is ENTIRE

$\Rightarrow$ via CG $\oint f\, dz = 0$

$\Rightarrow$ Path Independent

-or- $ze^{z^2} = \frac{d}{dz}\left(\frac{1}{2}e^{z^2}\right) \Rightarrow$ Path Independent

$\Rightarrow \frac{1}{2}e^{z^2}\Big|_{-1+i}^{0} = \frac{1}{2} - \frac{1}{2}e^{(-1+i)^2}$

b) 0 by C.G.

4) $\left|\int_C f\, dz\right| \le \int_C |f|\, dz \le M \cdot L$

$L = \sqrt{2^2 + 2^2} = \sqrt{8}$

$M = \max\left|\frac{z}{z^2+i}\right|\Big|_{\text{on } C} = ?$

$\Rightarrow M \cdot L = M\sqrt{8}$

5a) $z = \pm 5i$ outside contour.

$\oint_C f\, dz = 0$

b) $= \int_{C_1} \frac{\frac{z}{z+5i}}{z-5i}\, dz + \int_{C_2} \frac{\frac{z}{z-5i}}{z+5i}\, dz$

$= 2\pi i \frac{z}{z+5i}\Big|_{z=5i} + 2\pi i \frac{z}{z-5i}\Big|_{z=-5i}$

$= 2\pi i\left(\frac{1}{2} + \frac{1}{2}\right) = 2\pi i$

c) Change dir on C_2

$= 2\pi i\left(\frac{1}{2} - \frac{1}{2}\right) = 0$

6) By C.I. Formula.

$= \frac{3!}{2\pi i} f^{(2)}(-32)$

where $f = 32$ and $f' = 0$

$\Rightarrow 0$