MATH 5331

S & S : 1.1

8.) $\dfrac{8+2i-(1-i)}{(2+i)^2} = \dfrac{33}{25} - i\dfrac{19}{25}$

10.) $\left[\dfrac{2+i}{6i-(1-2i)}\right]^2 = \left(\dfrac{6-17i}{65}\right)^2$

$= \dfrac{-253-204i}{65^2}$

15.) $i^{4k} = (i^2)^{2k} = (-1)^{2k} = 1$

for all k.

$i^{4k+1} = i^{4k} \cdot i = 1 \cdot i = i$

$i^{4k+2} = i^{4k} \cdot i^2 = 1 \cdot (-1) = -1$

$i^{4k+3} = i^{4k} \cdot i^2 \cdot i = 1 \cdot (-1) \cdot i$

$= -i$

19.) $z^3 + 5z^2 = z + 3i$

$(x+iy)^3 + 5(x+iy)^2 = (x+iy) + 3i$

$\Rightarrow x^3 - 3xy^2 + 5(x^2 - y^2) = x$

$3x^2y - y^3 + 10xy = y + 3$

21.) $(1-i)z_1 + 3z_2 = 2-3i$

$iz_1 + (1+2i)z_2 = 1$

$\Rightarrow x_1 + y_1 + 3x_2 = 2$

$-x_1 + y_1 + 3y_2 = -3$

$-y_1 + x_2 - 2y_2 = 1$

$x_1 + 2x_2 + y_2 = 0$

$\Rightarrow z_1 = (x_1, y_1) = (1, 1)$

$z_2 = (x_2, y_2) = (0, -1)$

or $z_1 = -i + (i-2)z_2$

$(1-i)[-i + (i-2)z_2] + 3z_2 = 2-3i$

$\downarrow$

$(3i+2)z_2 = 3-2i$

$z_2 = -i$

$\Rightarrow z_1 = 1+i$