MATH 5831

S&S: 2.1

1b) $g(z) = \frac{1}{z} = \frac{\bar{z}}{|z|^2} = \frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2}$

c) $h(z) = \frac{z+i}{z^2+1} = \frac{1}{z-i} = \frac{\bar{z}+i}{|z-i|^2}$

$= \frac{x + i(1-y)}{x^2 + (y-1)^2}$

2b) All of $\mathbb{C}$ except for the origin

c) All of $\mathbb{C}$ " " $(0,1)$

4a) $\omega / z = re^{i\theta}$ and given that this is a circle.

$\omega = \frac{1}{r} e^{-i\theta} = \rho e^{i\phi}$

$|\omega| = \frac{1}{r}$, counterclockwise circle

b) $\omega = \frac{1}{r} e^{-i\theta_0}$

$\arg \omega = -\theta_0$ is fixed

As $r = (0,\infty)$, $\frac{1}{r} = (\infty, 0)$

5) $f(z) = e^z = e^x e^{iy}$

a) Domain: all $\mathbb{C}$

Range: all $\mathbb{C}$ except origin

$f \neq 0$

b) $f(-z) = e^{-z} = \frac{1}{e^z} = \frac{1}{f(z)}$

c) $\mathrm{Re}\{z\} > 1 \Rightarrow z = 1 + iy$

$f = e e^{iy}$

Circle w/ radius e

d) $z = x + i\pi/4$

$f = e^x e^{i\pi/4}$

Ray w/ $\theta = \pi/4$

c) "Pie slice" $0 \le \theta \le \pi/4$

10a) $F = z + i \quad G = e^{i\pi/4} z$

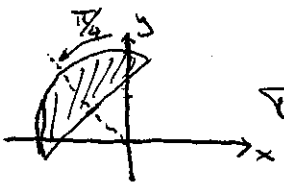
$H = z/2$

$G(F(z)) = e^{i\pi/4}(z + i)$ (1)

$= z e^{i\pi/4} + e^{i3\pi/4}$ (2)

(1) Shift up by 1

Rotate about origin by $\pi/4$



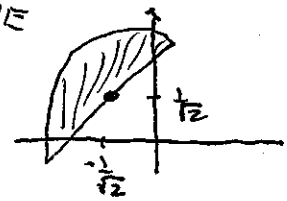
(2) Rotate about origin by $\pi/4$

Shift by $e^{i3\pi/4}$

$= \cos 3\pi/4 + i \sin 3\pi/4$

$= -\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$

SAME



Those are all supposed to be ½ disk.

b) $F(G(H(z))) = \frac{z}{2} e^{i\pi/4} + i$

reduce, rotate, shift