MATH 5331

S&S 2.2

11 a) $\lim_{z \to i} \frac{z^2+1}{z^4-1} = \lim_{z \to i} \frac{1}{z^2-1} = \frac{1}{-1-1} = -\frac{1}{2}$

e) $\lim_{\Delta z \to 0} \frac{(z_0+\Delta z)^2 - z_0^2}{\Delta z} = \frac{0}{0}$ - or - $\lim_{\Delta z \to 0} \frac{2z_0\Delta z + \Delta z^2}{\Delta z} = 2z_0$

Let $\Delta z = re^{i\theta}$

$\lim_{r \to 0} \frac{(z_0 + re^{i\theta})^2 - z_0^2}{re^{i\theta}}$

$\lim_{r \to 0} \frac{z_0^2 + 2z_0 re^{i\theta} + r^2e^{i2\theta} - z_0^2}{re^{i\theta}}$

$\lim_{r \to 0} 2z_0 + re^{i\theta} = 2z_0$

15) $\lim_{z \to 0} \frac{x^2}{x^2+y^2} + 2i$

Let $y = mx$

$\lim_{x \to 0} \frac{x^2}{x^2 + m^2x^2} + 2i = \frac{1}{1+m^2} + 2i$

limit depends on slope m.

$\therefore$ limit depends on path.

$\Longrightarrow$ DNE