- $\int_C \frac{1}{z}\,dz$ on any C connecting $z_1 = i = e^{i\frac{\pi}{2}}$ and $z_2 = -i = e^{-i\pi/2}$.

  Log z is ANALYTIC on a BRANCH $|z|>0,\ -\pi<\theta<\pi$.

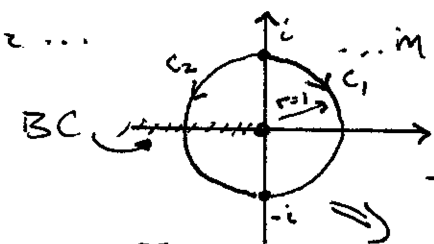
  $\therefore\ \frac{d}{dz}(\text{Log } z) = \frac{1}{z}$ on the branch.

  $\Rightarrow \int_C \frac{1}{z}\,dz = \text{Log } z_2 - \text{Log } z_1$ ... as long as we stay on the branch where Log z is ANALYTIC.

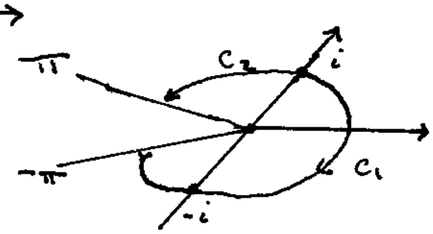
  $= \text{Log}(-i) - \text{Log}(i)$

  $= \text{Log}(1) + i(-\pi/2) - \text{Log}(1) - i(\pi/2) = \underline{\underline{-i\pi}}$

- Consider the contours C_1 & C_2 ... ...in domain D.



$\Rightarrow$



C_1: $z(t) = e^{i(\frac{\pi}{2}-t)}$ $t\in[0,\pi]$

$z' = -ie^{i(\frac{\pi}{2}-t)}$

$\Rightarrow \int_0^\pi e^{-i(\frac{\pi}{2}-t)}(-i)e^{i(\frac{\pi}{2}-t)}\,dt = \int_0^\pi (-i)\,dt$

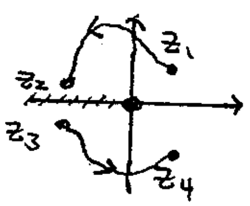
$= -i\pi$ $\rightarrow$ consistent w/ PATH INDEPENDENT RESULT BECAUSE WE DID NOT CROSS BC.

C_2: $z(t) = e^{it}$ $t\in[\pi/2, 3\pi/2]$

$z' = ie^{it}$

$\Rightarrow \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} e^{-it}\,ie^{it}\,dt = i\pi$ $\Rightarrow$ In general, crossing the BC introduces a $2\pi i$ shift.

- In general,



$C = \lim_{z_2\to re^{i\pi}} C_{12} + \lim_{z_3\to re^{-i\pi}} C_{24}$

$\int_C \frac{1}{z}\,dz = \lim_{z_2\to re^{i\pi}} \int_{C_{12}} \frac{1}{z}\,dz + \lim_{z_3\to re^{-i\pi}} \int_{C_{24}} \frac{1}{z}\,dz$

$= \lim_{z_2\to re^{i\pi}} \text{Log } z_2 - \text{Log } z_1 + \text{Log } z_4 - \lim_{z_3\to re^{-i\pi}} \text{Log } z_3$

$= \cancel{\text{Log } r} + i\pi - \text{Log } z_1 + \text{Log } z_4 - (\cancel{\text{Log } r} - i\pi)$

$= \text{Log } z_4 - \text{Log } z_1 + 2\pi i$

$\Rightarrow$ Crossing the BC introduces a $\underline{2\pi i \text{ shift}}$