Power Series

- DEF:

$$F(z) = \sum_{n=0}^{\infty} F_n(z) = \sum_{n=0}^{\infty} a_n (z-z_0)^n$$

- a_n are arbitrary
- when does it converge?
- when is a P.S. a T.S?

- CONVERGENCE.

① ∃ (there exists) an R (ROC) ∋ (such that) the P.S

CONVERGES for $|z - z_0| < R$

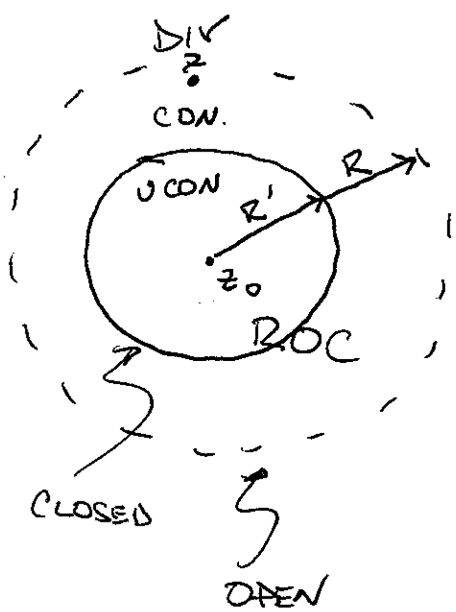
DIVERGES for $|z - z_0| > R$

② ∃ an $R' < R$ ∋ the P.S. CONVERGES UNIFORMLY on

$|z - z_0| \leq R' < R$

- UNIF. CONV: the # of terms needed ∋ $\left| \sum_{n=0}^{N} F_n - F \right| < \epsilon \quad \forall N > N_\epsilon$ does not depend on z.

N_ϵ does not depend on z.

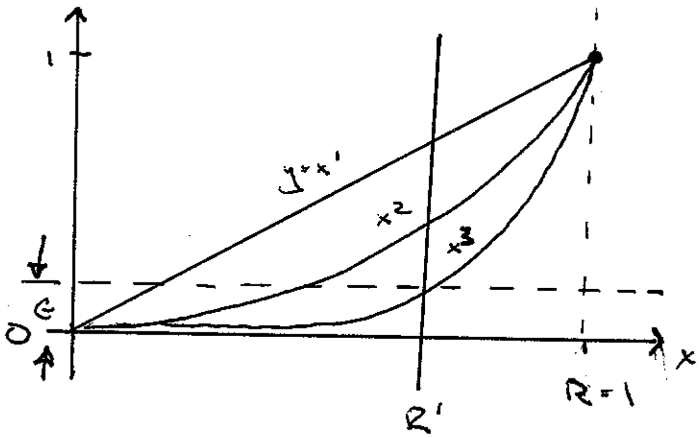


Example w/ SEQUENCE.

$$\lim_{n \to \infty} x^n = 0$$

"POINTWISE" but NOT UNIF.

on $x = [0, 1)$ ← open



- $|x^2 - 0| < \epsilon$ on $[0, R']$?

NO

- BUT

$|x^3 - 0| < \epsilon$ on $[0, R']$

and all $n = 3, 4, \ldots$

CONV UNIF on $[0, R']$

ie. PICK $N_\epsilon = 3$

- In OPEN DISK $|z - z_0| < R$

If $z = z_1$ it may take N_1 terms.

But if $z = z_2$ it may take N_2 terms.

- IN CLOSE DISK.

If when $|z - z_0| = \underline{R'}$

it takes N_ϵ terms to be ϵ close. Then certainly more terms will be ϵ close.